Shear and bulk viscosities for a pure glue matter

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Contents

Why is a glue matter? Aims

Motivation:

- New lattice data for the pure gluon EoS!
- There are lattice data for viscosities!
- Simplicity: exactly first order $\Rightarrow$ two-phase model have to work very well!

Aims:

- To improve the phenomenological quasiparticle model to reproduce the new lattice data
- To investigate the behavior of viscosity coefficients for a gluon system

The system of interacting gluons $\Leftrightarrow$ a gas of noninteracting quasiparticles with an effective mass

$$
\begin{aligned}
m_g^2(T) &= \frac{1}{2}\, g^2(T)\ T^2 \qquad \text{T-dependent !} \\
g^2(T) &= \frac{16\pi^2}{11\ \ln\left[\lambda(T - T_s)/T_c)\right]^2} \\
\varepsilon_g(T) &= \varepsilon_g^{id}(T, m_g(T)) + B(T), \\
P_g(T) &= P_g^{id}(T, m_g(T)) - B(T)
\end{aligned}
$$

The thermodynamical identity (the condition of thermodynamical consistency)

$$
T\frac{dP}{dT} - P(T) = \varepsilon(T) \quad \Rightarrow \quad \frac{dB(T)}{dT} = -\frac{\varepsilon_g^{id} - 3P_g^{id}}{m_g}\frac{dm_g}{dT}
$$

M.I. Gorenstein, S.N. Yang, Phys. Rev. **D 52**, 5206 (1995)

Glueball phase

Gluodynamics: at $T < T_c$ the matter consists of glueballs

Two lowest-lying scalar 0^{++} and tensor 2^{++} glueballs

$$m_{gb}(T) \approx \tilde{m}_{gb}(T) - 2T + \sqrt{4T^2 - \Gamma_{gb}^2(T)}$$

N. Ishii et al., Phys. Rev. **D 66**, 094506 (2002)

$$\begin{aligned} \tilde{m}_{gb}(T) &= m_{gb}^0 = const \\ \Gamma_{gb}(T) &= b_{gb}(T - T_{gb})\,\Theta(T - T_{gb}) \end{aligned}$$

F. Buisseret, EPJ **C 68**, 473 (2010)

Thermodynamic consistency – in the same way as above for gluons.

Full model and parameters

The Gibbs conditions at the transition point

$$
\begin{aligned}
T_c^g &= T_c^{gb} \equiv T_c \\
P_g(T_c) &= P_{gb}(T_c)
\end{aligned}
$$

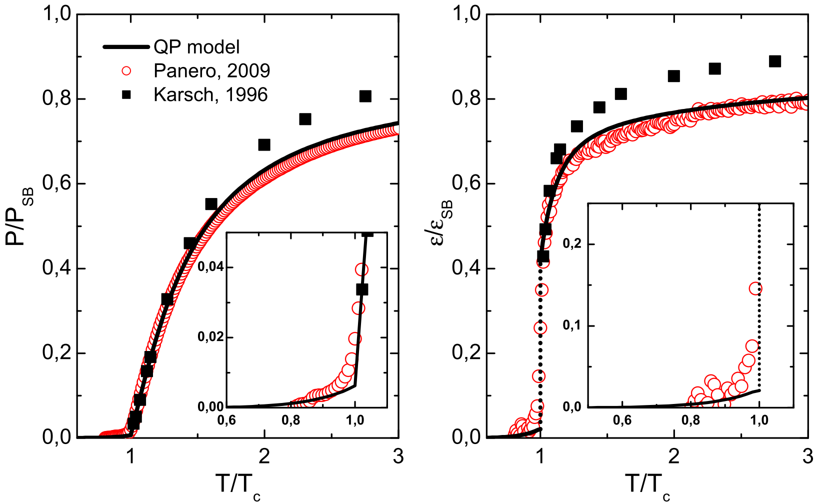
$T_c = 265$ MeV in agreement with the lattice results

Fit the new lattice data M. Panero, Phys. Rev. Lett. **103**, 232001 (2009)

$$T_s/T_c = 0.5853, \ \lambda = 3.3$$

B_0 – from intersection with glueballs to reproduce T_c-value

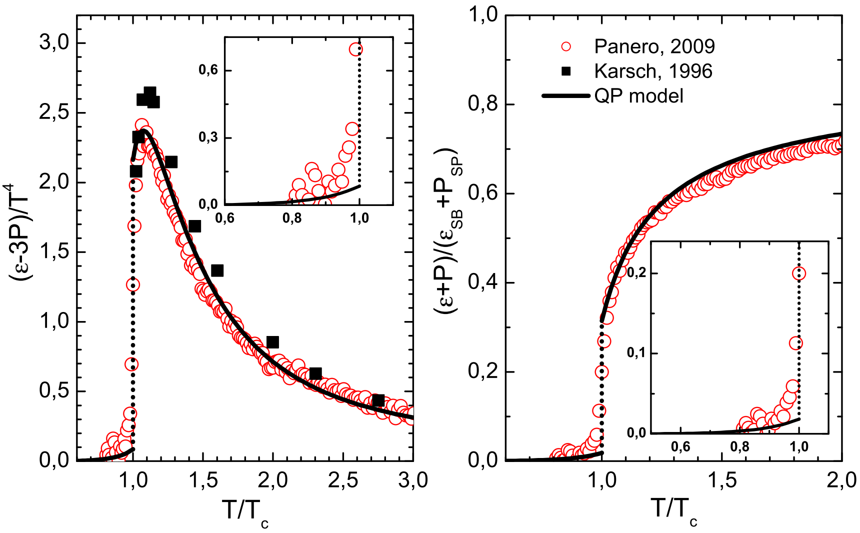
Pressure and energy density



The pressure and the energy density normalized to those in the Stefan-Boltzmann limit.

filled squares – the old Karsch's lattice results G. Boyd et al., Nucl. Phys. **B 469**, 419 (1996)

Interaction measure and entropy density



The trace anomaly and the entalpy/entropy.

$$T^{\mu\nu} = \sum_a \int d\Gamma \, \frac{p_a^\mu p_a^\nu}{E_a} F_a$$

$$p_a^\mu = (E_a(\vec{p}_a), \vec{p}_a), \quad E_a(\vec{p}) = \sqrt{\vec{p}^{\,2} + m_a^2[F_a]}$$

$$F_a^{\rm loc.eq.}(p_a, x_a) = \left[e^{p_a^\mu u_\mu / T} - 1 \right]^{-1}$$

Using Boltzmann equation and varying only F_a,

$$\delta T^{\mu\nu} = -\sum_a \int d\Gamma \left\{ \tau_a \frac{p_a^\mu p_a^\nu}{E_a^2} \; p_a^\kappa \partial_\kappa F_a \right\}_{\rm loc.eq.}$$

where $\tau_a(\vec{p})$ – the relaxation time of the given species

By definition

$$\delta T_{ij} = -\zeta \; \delta_{ij} \vec{\nabla} \cdot \vec{u} - \eta \; \left(\partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \; \partial_k u^k \right), \; i, j, k = 1, 2, 3$$

Taking derivatives $\partial_\mu F_a^{\mathrm{loc.eq.}}$, by straightforward calculations we find expression

$$\eta = \frac{1}{15\,T} \sum_a \int d\Gamma\, \tau_a \frac{\vec{p}_a^{\,4}}{E_a^2} F_a^{\mathrm{eq}} \left(1 + F_a^{\mathrm{eq}}\right)$$

Simplifying,

$$\tau_a(\vec{p}) = \tilde{\tau}_a = const$$

Glueballs: isotropic cross section $\sigma_{gb} = 30$ mb, $\tau_{gb}^{-1} = n_{gb}(T)\sigma_{gb}$

Resummation of the hard thermal loops:

$$\tilde{\tau}^{-1} \sim g^2 T \ln(1/g)$$ R.D. Pisarski, PRL **63**, 1129 (1989)

$$\tilde{\tau}_g^{-1} = N_c \frac{g^2 T}{4\pi} \ln \frac{2c}{g^2}$$ A. Peshier, W. Cassing, PRL **94**, 172301 (2005)

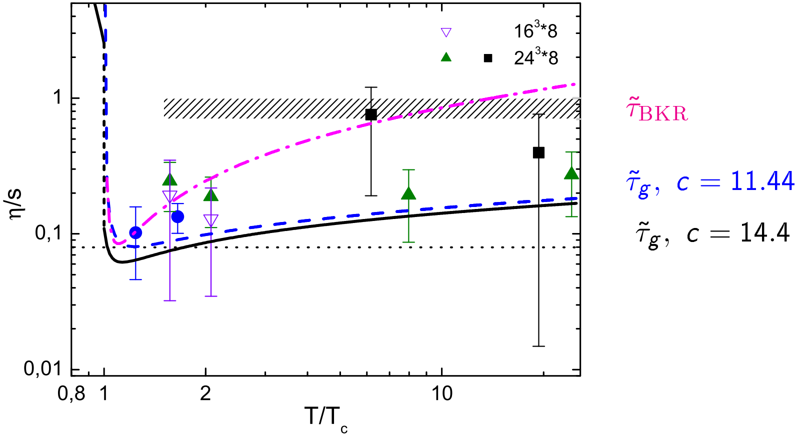
Two values of c-parameter:

$$\begin{aligned} c &= 14.4 \quad (Peshier, Cassing) \\ c &= 11.44 \; - \quad \text{the limit case } \tilde{\tau}_g^{-1} \to 0, \quad T \to T_c + 0 \end{aligned}$$

$$\tilde{\tau}_{\rm BKR}^{-1} = a_\eta/(32\pi^2)T\, g^4 \, \log(a_\eta \pi/g^2)\ ,\ a_\eta = 6.8$$

M. Bluhm et al., Nucl. Phys. **A 830**, 737c (2009)

Shear viscosity



The horizontal dotted line – $\eta/s = 1/4\pi$ bound (AdS/CFT).

Triangles and squares – the lattice data, S. Sakai, A. Nakamura, **PoS LAT2007**, 221 (2007)

Filled circles – the lattice data, H.B. Meyer, Phys. Rev. **D 76**, 101701 (2007)

The shaded region – perturbative result

Remarks on shear viscosity

$\tilde{\tau}_g$:

- perturbative regime is not achieved up to very high T
- predictions of our QP model are in a reasonable agreement with the lattice results and do not contradict perturbative estimates

$\tilde{\tau}_{\mathrm{BKR}}$:

- for $T \gtrsim 10\,T_c$ shear viscosity calculations demonstrate a noticeable growth exceeding lattice data and even a perturbative estimate $(\eta/s)_{\mathrm{pert}} \approx 0.8 - 1.0$

Expression for bulk viscosity

$$\zeta = -\frac{1}{3T} \sum_a \int d\Gamma \, \tau_a \, \frac{\vec{p}_a^{\,2}}{E_a} \, F_a^{\rm eq} \, (1 + F_a^{\rm eq}) \, Q_a,$$

where the EoS-dependent Q_a factor is given by

$$Q_a = -\left\{ \frac{\vec{p}_a^{\,2}}{3E_a} - c_s^2 \left[E_a - T \frac{\partial E_a}{\partial T} \right] \right\}$$

$c_s^2 = \frac{\partial P}{\partial \epsilon}$ – the speed of sound squared.

The Landau-Lifshitz condition

$$\delta T^{00} = \sum_a \tilde{\tau}_a \int d\Gamma E_a Q_a = 0$$

is satisfied in our QP model.

Alternative expressions for bulk viscosity

P. Chakraborty, J.I. Kapusta, arXiv: 1006.0257

$$\zeta_{\rm ChK} = \sum_a \frac{d_a}{T} \int \frac{d^3p}{(2\pi)^3} \bar{\tau}_a F_a^{eq} (1 + F_a^{eq}) Q_a^2$$

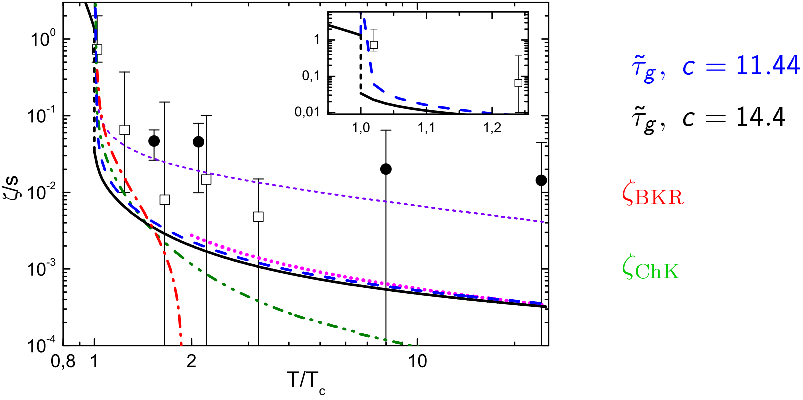
M. Bluhm et al., Nucl. Phys. **A 830**, 737c (2009)

$$\zeta_{\rm BKR} = \sum_a \frac{d_a}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a}{E_a} F_a^{eq} (1 + F_a^{eq}) Q_a \left[m_a^2(T) - T \frac{dm_a^2(T)}{dT} \right]$$

The QP interaction contributes to the energy-momentum tensor $\Rightarrow$ the second term $T dm_a^2(T)/dT$ in the square bracket.

A perturbative estimate gives

$$(\zeta/s)_{\rm pert} \approx 0.02\, \alpha_s^2, \;\; 0.06 \le \alpha_s \le 0.3$$



Empty squares – the lattice data, S. Sakai, A. Nakamura, **PoS LAT2007**, 221 (2007)

Filled circles – the lattice data, H. B. Meyer, Phys. Rev. Lett. **100**, 162001 (2008)

The dotted line – perturbative result

Thin short-dashed curve – rough approximation of lattice data

Remarks on bulk viscosity

- Values of ζ/s in our model noticeably underestimate the corresponding values on the approximating short-dashed curve. Nevertheless the behavior qualitatively is similar to that given by the approximating curve.
- $\zeta_{\rm ChK}$ yields a strong T suppression at $T \gtrsim 1.5T_c$, as compared to that given by our result.
- for $T > 1.9T_c$ $\zeta_{\rm BKR}$ becomes invalid providing negative values.

Conclusions

- For thermodynamic characteristics the quasiparticle model results are in good agreement with the latest lattice data.
- With the chosen value of the relaxation time the shear viscosity to entropy density ratio η/s fits rather well the scant lattice data.
- Although the calculated ζ/s ratio essentially underestimates the upper limits given by the corresponding lattice data, its temperature dependence is well described.

Thank you for attention!