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BOGOLIUBOV QUASIPARTICLES IN QUANTUM UNIVERSE

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A powerful apparatus of the Bogoliubov transformations is used to get conserved quantum numbers of a set of «free fields» in the Friedmann–Robertson–Walker (FRW) metric with the back-reaction of the cosmic evolution. We show how the Bogoliubov vacuum of the Heisenberg equations of motion creates «particles» detected by an observer in the comoving frame of reference at the present-day stage. The equations for coefficients of the Bogoliubov transformations reproduce the equations of states of the FRW classical cosmology in its conformal version.

We propose a new Hamiltonian scheme of construction of the unitary S-matrix, in GR, which is consistent with both the classical cosmology and the standard quantum field theory limit where General Relativity looks as the scalar version of the Weyl conformal-invariant theory with a set of predictions, including the cosmic nature of the Higgs effect in Standard Model of electro-weak and strong interactions and the negative result of CERN experiment on the search of Higgs particles.

1. INTRODUCTION

It is a great honor for us to present here our attempts to apply a powerful apparatus of the Bogoliubov transformations in quantum field theory to describe Quantum Universe consistent with the classical cosmology and to construct the corresponding unitary S-matrix in General Relativity (GR).

The first construction of the unitary [1] perturbative [2] S-matrix for General Relativity was made by Faddeev and Popov and by DeWitt [3] in an infinite flat space-time. However, this is not enough to answer the questions about the origins of creation and evolution of Quantum Universe.

In the present paper, we construct the unitary S-matrix for the Friedmann–Robertson–Walker background metric to consider creation and evolution of Universe on the level of quantum field theory. After definition of the Hamiltonian formulation of General Relativity, cosmological perturbation theory, Quantum Universe, and the Hamiltonian reduction, we show how the Bogoliubov quasiparticles help us to find the unitary S-matrix in the early Quantum Universe and to unify General Relativity with quantum field theory.

2. MODEL

We begin with the Einstein–Hilbert action in GR in terms of Dirac variables [2] $q^{ij} = ||g||g^{ij},\ N_q = q^{-1/4}N$ obtained from the Dirac–ADM parametrization of the four-dimensional metric [4]

$$ds^2 = (\varphi_0(t)/\mu)^2(N^2dt^2 - g_{ij}\breve{d}x^i\breve{d}x^j);\quad \breve{d}x^i = dx^i + N^i dt,$$

where the time metric components (a lapse function N_q and three shift vectors N^i) play the role of Lagrange multipliers in the action in the first-order formalism

$$W^E = \int\limits_{t_1}^{t_2} dt\left(\int d^3x[-\pi_{ij}\dot{q}^{ij} - N_q\mathcal{H} - N^i\mathcal{P}_i] - \dot{\varphi}_0 P_0 + \frac{P_0^2}{4V}\right),$$

$\mathcal{H}, \mathcal{P}_i$ are densities of the Dirac Hamiltonian and momentum in which the Planck constant $\mu = M_P(3/8\pi)^{1/2}$ is multiplied by a homogeneous scale factor of the Friedmann–Robertson–Walker background metric $\varphi_0(t)/\mu$, treated as the zero Fourier harmonic of logarithm of determinant of the space metric and the zero-approximation of cosmological perturbation theory;

$$V := \int d^3x N_q^{-1} := V_0 N_0^{-1}$$

is the zero Fourier harmonic of a lapse function in a finite volume $V_0 = \int d^3x$. The reparametrization-invariant version of a cosmological perturbation theory is constructed with the homogeneous cosmological lapse function (N_0). Both these homogeneous components of the metric determine two invariant evolution parameters: the dynamic evolution parameter φ_0 and the conformal time ($dT = N_0 dt$) measured by an observer in his comoving frame of reference [5].

The problem of initial data and classical and quantum Hamiltonian dynamics of all fields in GR including matter ones are conventionally formulated in terms of the so-called Lichnerowicz conformal-invariant variables [6, 7] with a separated scale factor, the power of which coincides with the conformal weight of a corresponding field.

3. MEASURABLE QUANTUM UNIVERSE

When interactions are neglected ($H_T = \int d^3x N_q\mathcal{H} \simeq H_0;\ N_q \simeq N_0$), the GR action supplemented by the matter fields is reduced to the action of the well-known system of «free» conformal fields (transverse gravitons, photons, massive vector fields, and massive and massless fermions) in a finite space-volume where all masses (including the Planck one) are expressed in terms of the cosmic scale-factor φ_0 [5].

We define a Quantum Universe as the FRW-Universe filled in by these «free» conformal fields and described by this well-known action. The problems are only to reduce this constrained system to an equivalent unconstrained one and to quantize these fields. The reduction means explicit resolving the energy constraint with respect to the momentum of the cosmic scale-factor P_0 which gives a negative contribution to the constraint.

As a result, we get the unconstrained version of «free» theory where the cosmic scale factor φ plays the role of the dynamic (internal) evolution parameter, and its momentum P_0 converts into the reduced Hamiltonian ($H_0^R = 2\sqrt{V_0 H_0}$) of the corresponding evolution of the sector of «Dirac observables» [5].

However, the unconstrained dynamics is not sufficient to determine the geometrical interval of the proper time measured by a watch of an observer in the comoving frame. Moreover, as we have shown in our papers [5–7], an observer can choose two different standards of his measurements for the same unconstrained dynamics: absolute and relative. The Einstein theory ($\mathcal{L}^E = -\mu^2 R/6$) can also be considered as the scalar version of the Weyl conformal theory [6, 7] ($\mathcal{L}^W = -\varphi^2 R/6 + \varphi\varphi_{;\alpha}^{;\alpha}$) where the determinant of the three-dimensional metric in GR multiplied by the Planck constant is considered as the Lichnerowicz variable of the Weyl scalar field (i.e., an integrable measure of a change of the length of a vector in its parallel transport).

The unconstrained dynamics of both these theories is the same, but not the times on watches of observers: A Weyl observer measures the conformal time $dT = N_0 dt$, while an Einstein observer measures the Friedmann world time $dT_f = (\varphi_0/\mu) dT$.

The evolution of the Universe (identified, in observational cosmology, with the cosmological «expansion» of the Universe for an observer with the absolute standard of his measurements) is the evolution of a measurable geometrical interval (invariant proper time) with respect to the «dynamic evolution parameter»: $\varphi_0'(T) := d\varphi_0/dT$. This Universe evolution $\varphi_0' = \sqrt{\rho_0}$; $\rho_0 = H_0/V_0$ follows from the extended theory equation for the scale-factor momentum which goes beyond the scope of the sector of the Dirac observables restricted by the unconstrained system.

The equation for dynamic evolution of the measurable time contains the energy density ρ_0 which is treated as the measurable quantity in astrophysics and observational cosmology and as the object of numerous discussions about the dark matter and hidden mass detected by an observer in his comoving frame.

As quantity ρ_0 is the observable energy density, we define «particles» as field variables which diagonalize ρ_0.

We define the Bogoliubov quasiparticles as field variables which diagonalize the equations of motion and mark states of the Universe by integrals of motion (i.e., quantum numbers in the corresponding quantum theory) [5].

Explicit solution of the equations of diagonalization for coefficients of the Bogoliubov transformation means the construction of an equivalent diagonalized system for which a new internal evolution parameter coincides with the conformal time T measured by a Weyl observer in his comoving frame.

The equations for the coefficients of the Bogoliubov transformations were derived in paper [5] in terms of the quasiparticle energy, the Bogoliubov vacuum expectation value of the number of «particles» detected by an observer in his comoving frame, and the Hubble parameter ($H_{Hab} = \varphi_0'/\varphi_0$). These equations were explicitly solved in two limits: at the beginning of the Universe, and at the present-day stage.

At the beginning of the Universe in the state of the Bogoliubov (i.e., squeezed) vacuum, we get the density of measurable gravitons which corresponds to the well-known anisotropic stage with the Misner wave function of Quantum Universe [8]. The anisotropic stage is changed by the stage of inflation-like increase of the cosmic scale factor with respect to the time measured by an observer with the relative standard of his measurement. At these stages, the Bogoliubov quasiparticles strongly differ from the detected and observed particles.

At the present-day stage, when we can neglect back-reaction of the Universe, the Bogoliubov quasiparticles coincide with observed particles, so that the measurable energy of matter in the Universe is a sum of relativistic energies of all particles in it. The wave function of Quantum Universe is nothing but the product of oscillator wavefunctions. Neglecting masses, we get the conformal version of the radiation stage for an observer with the relative standard. Neglecting momenta, we get the conformal version of the dust stage, where an observer with the relative standard observes the Hubble law of the «accelerating» Universe with $q := \varphi_0''\varphi_0/\varphi_0'^2 = +1/2$.

According to the global equation for the cosmic scale factor φ_0 discussed before, its value can be expressed in terms of astrophysical data of the observational cosmology, the density of matter, and the Hubble parameter $\varphi_0 = \sqrt{\rho_0}/H_{Hab} = \mu\Omega_{\rm exp}^{1/2}$. The present-day value of the Weyl scalar field (which forms all masses including the Planck mass) coincides with the value of the Newton coupling constant of gravity in agreement with astrophysical data $0.1 < \Omega_{\rm exp} < 2$ [7].

4. QUANTUM GRAVITY

The consistent description of the cosmic evolution by the local energy of particles in the Universe excludes the application of the standard scheme [2,3] of quantization (with the nonlocal energy and noninvariant coordinate time) and requires a time-reparametrization-invariant scheme based on the group of diffeomorphisms of the comoving frame of reference [9]. In this case, resolving the global energy constraint is sufficient to determine unambigiuosly the local lapse function N_q, the Dirac local Hamiltonian $(H_T = \int d^3x N_q \mathcal{H} = H_0 + H_{\rm int})$, and the reduced Hamiltonian $H^R = 2\sqrt{VH_T} = H_0^R + H_{\rm int}^R$.

We consider the infinite time and space-volume limit of the S-matrix element

$$S[T_1 = T_0 - \Delta T | T_2 = T_0 + \Delta T] =$$

$$= \langle {\rm o}ut\ (T_2)| T \exp\left\{ -i \int\limits_{\varphi(T_1)}^{\varphi(T_2)} d\varphi (H_{int}^R) \right\} |(T_1)\ {\rm i}n\rangle.$$

In the infinite volume limit, the Hamiltonian of interaction of GR can be expressed in terms of the conformal time T (measured by a Weyl observer in an out-state) with the «back-reaction» form factor for physical processes observed under the «laboratory» conditions when the cosmic energy is much higher than the deviation of the free energy due to creation and annihilation of real and virtual particles in the laboratory experiments [9].

We can get the conventional quantum-field-theory representation of an S-matrix element if we neglect the «back-reaction» form factor (which removes a set of ultraviolet divergences), when the measurable time of the laboratory experiments is much smaller than the age of the Universe T_0, but it is much greater than the reverse «laboratory» energy, so that the infinite time limit is valid [9]. This matrix element corresponds to the Faddeev–Popov functional integral with the initial Hamiltonian $H_T(\varphi = \mu)$, and conformal time $N_q dt = N_0 dt = dT$, and without the time component of the Faddeev–Popov determinant.

5. CONFORMAL UNIFIED THEORY

The quantum field theory chooses the conformal variables and measurable quantities, including the conformal time. In QFT, the Einstein General Relativity looks like a scalar version of the Weyl conformal-invariant theory [7, 10] with the Lagrangian density $\mathcal{L}_{CUT} = \mathcal{L}_{\mathcal{W}} + \mathcal{L}^c_{SM}$, where $\mathcal{L}^c_{SM}$ is the conformally-invariant part of the Standard Model without the Higgs potential. In this theory, the Weyl scalar field forms both the Planck mass (in agreement with the present-day astrophysical data) and masses of elementary particles (in agreement with the principle of equivalence) [7].

In the Weyl theory, the Higgs mechanism of the formation of particle masses becomes superfluous and, moreover, it contradicts the equivalence principle, as in this case the Planck mass and masses of particles have different nature and are formed by different fields.

The Weyl geometrization of the modulus of the Higgs field [7] removes the Higgs potential with its problems of tremendous vacuum energy, monopole creation, and the domain walls [11]. The Higgs potential could not be restored by the Coleman–Weinberg perturbation theory, as the vertices with the scalar field are absorbed by definition of «particles». Instead of the Coleman–Weinberg effective Higgs potential, the interactions of matter fields with the scalar field lead to the above-considered picture of cosmic creation and evolution of the Universe in Conformal Unified Theory in the space-time with the Weyl geometry. The role of the Higgs scalar field is played by the Weyl scalar one as a measure of the change of the length of a vector in its parallel transport. The Weyl scalar field converts into the dynamic evolution parameter (like the cosmic scale factor), forms the Newton interaction, and looses its particle-like excitations like the time component of an electromagnetic field in QED. In the conformal theory, we obtain the σ-version of the Standard Model [7, 10] without Higgs particles and with the «back-reaction» form factor to be free from the ultraviolet divergences for the precision calculations.

Thus, the construction of a unitary S-matrix for Quantum Universe by the Bogoliubov transformations leads to the standard quantum field theory at the present-day stage provided the measurable time is the conformal time of a Weyl observer, and General Relativity is the scalar version of the Weyl conformal-invariant theory with the set of prediction, including

the Hoyle–Narlikar version of observational cosmology [6, 7], where the physical reason of red shift is the change of masses of elementary particles in the process of evolution of the Universe;

the cosmic mechanism of formation of both the masses of elementary particles and the Planck mass by the Weyl scalar field (which does not contradict the present-day astrophysical data);

the creation of the Universe from the vacuum of dynamic equations of motion with the squeezed vacuum inflation at the beginning of the Universe;

and the negative result of CERN experiment on the search for Higgs particles.

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