

THE INTERPLAY BETWEEN PERTURBATIVE QCD
AND POWER CORRECTIONS: THE DESCRIPTION
OF SCALING OR AUTOMODELLING LIMIT
VIOLATION IN DEEP-INELASTIC SCATTERING

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The summary of the results of our next-to-next-to-leading fits of the Tevatron experimental data for xF_3 structure function of the νN deep-inelastic scattering is given. The special attention is paid to the extraction of twist-4 contributions and demonstration of the interplay between these effects and higher order perturbative QCD corrections. The factorization and renormalization scale uncertainties of the results obtained are analyzed.

1. The study of deep-inelastic scattering (DIS) processes has a rather long and inspiring history. One of the first realizations that the analysis of νN DIS could play an important role in investigations of the properties of the nucleon came in Ref. 1. The fundamental concept of scaling of DIS structure functions (SFs) [2] has lead to many subsequent investigations. Other important stages in the development of both theoretical and experimental studies of various characteristics of DIS processes in this productive period were reviewed in detail recently [3].

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In particular, it was stressed that after the experimental confirmation of scaling and indications of the existence of point-like constituents of the nucleon, the more rigorous theoretical explanation of the behaviour of DIS form factors came onto the agenda. A series of works by N.N.Bogoliubov and coauthors [4], were devoted to the development of the new method, which made it possible to analyse the asymptotics of the form factors of eN DIS using the Jost–Lehmann–Dyson integral representation, and explain the property of scaling (or as called by the authors of Ref.4 «automodelling») behaviour of the corresponding SFs in the framework of general principles of local quantum field theory [5].

We now know that this property is true only in the asymptotic regime and that it is violated within the framework of QCD (see, e.g., the extensive discussions in a number of books on the subject [6]). Indeed, the theory of QCD predicts that scaling or automodelling behaviour of SFs is violated by the logarithmically decreasing perturbative QCD contributions to the leading twist operators. However, in the intermediate and low Q^2 regime the higher twist operators, which give rise to scaling violations of the form $1/Q^2$, $1/Q^4$, etc., might also be important [7,8]. Indeed, the NLO DGLAP fits [9] of the BCDMS data of DIS of charged leptons on nucleons [10] and reanalysed SLAC eN data [11] resulted in the detection of the signals from the twist-4 contributions.

During the last few years there has been considerable progress in modelling these effects with the help of the infrared renormalon (IRR) approach (for the review see Ref. 12) and the dispersive method [13] (see also Ref. 14). Using these methods the authors of Ref. 15 explained the behaviour of the twist-4 contributions to the F_2 SF observed in Ref. 9 and constructed a model for the similar power-suppressed corrections to xF_3 SF. In view of this it became important to check the predictions of Ref. 15 and to study the possibility of extracting higher-twist contributions from the new more precise experimental data for νN DIS, obtained by the CCFR collaboration at Fermilab Tevatron [16], and also to exploit the considerable progress in calculations of the perturbative QCD corrections to characteristics of DIS, achieved in the last decade.

Indeed, the analytic expressions for the next-to-next-to-leading order (NNLO) perturbative QCD corrections to the coefficient functions of SFs F_2 [17] and xF_3 [18] are now known. Moreover, the expressions for the NNLO corrections to the anomalous dimensions of nonsinglet (NS) even Mellin moments of F_2 SF with $n = 2, 4, 6, 8, 10$ and for the N³LO corrections to the coefficient functions of these moments are also available [19]. In this report we will summarize the results of the series of the works of Refs. 20–22, devoted to the analysis of the CCFR data at NNLO, which has the aim to determine the NNLO value of the QCD coupling constant $\alpha_s(M_Z)$ and to extract the effects of the twist-4 contributions to SF xF_3 [21, 22]. In particular, we will concentrate on the discussion of the factorization and renormalization scale uncertainties of the results obtained.

2. Our analysis of Refs. 20–22 is based on reconstruction of the SF $x F_3$ from its Mellin moments $M_n(Q^2) = \int_0^1 x^{n-1} F_3(x, Q^2) dx$ using the Jacobi polynomials method, proposed in Ref. 23 and further developed in the works of Ref. 24. Within this framework one has:

$$x F_3(x, Q^2) = x^\alpha (1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha, \beta}(x) \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) M_{j+2}(Q^2), \quad (1)$$

where $\Theta_n^{\alpha, \beta}$ are the Jacobi polynomials, $c_j^{(n)}(\alpha, \beta)$ are combinatorial coefficients given in terms of Euler Γ -functions of the α and β weight parameters. In view of the reasons discussed in Ref. 22, they were fixed to 0.7 and 3, respectively. The QCD evolution of the moments is defined by the solution of the corresponding renormalization group equation:

$$\frac{M_n(Q^2)}{M_n(Q_0^2)} = \exp \left[- \int_{A_s(Q_0^2)}^{A_s(Q^2)} \frac{\gamma_{NS}^{(n)}(x)}{\beta(x)} dx \right] \frac{C_{NS}^{(n)}(A_s(Q^2))}{C_{NS}^{(n)}(A_s(Q_0^2))}. \quad (2)$$

The QCD running coupling constant enters this equation through $A_s(Q^2) = \frac{\alpha_s(Q^2)}{(4\pi)}$ and is defined as the expansion in terms of inverse powers of $\ln(Q^2/\Lambda_{\overline{MS}}^{(4)2})$. For the initial scale Q_0^2 , from which the evolution is started, the moments in Eq.(2) were parametrized as $M_n(Q_0^2) = \int_0^1 x^{n-2} A(Q_0^2) x^{b(Q_0^2)} (1-x)^{c(Q_0^2)} (1 + \gamma(Q_0^2)x) dx$. In the process of our analysis we took into account both target mass corrections and twist-4 contributions. The latter were modeled using the IRR approach as $M_n^{IRR} = C(n) M_n(Q^2) A_2'/Q^2$ [15] and by adding into the r.h.s. of Eq.(1) the term $h(x)/Q^2$ with $h(x)$ considered as a free parameter for each x -bin of the experimental data.

For arbitrary factorization and renormalization scales the NNLO expression for the NS Mellin moments reads:

$$M_n(Q^2) \sim (A_s(Q^2 k_F))^a \times \overline{AD}(n, A_s(Q^2 k_F)) \times C_{NS}^{(n)}(A_s(Q^2 k_R)), \quad (3)$$

where $a = \gamma_{NS}^{(0)}/(2\beta_0)$, $\overline{AD} = 1 + [p(n) + a k_1^F] A_s(Q^2 k_F) + [q(n) + p(n)(a+1) k_1^F + (\beta_1/\beta_0) a k_1^F + a(a+1)(k_1^F)^2/2] A_s^2(Q^2 k_F)$ and $C_{NS}^{(n)} = 1 + C^{(1)}(n) A_s(Q^2 k_R) + [C^{(2)}(n) + C^{(1)}(n) k_1^R] A_s^2(Q^2 k_R)$. Here $\gamma_{NS}^{(0)}$, β_0 , and β_1 are the scheme-independent coefficients of the anomalous dimension function $\gamma_{NS}(x)$ and QCD β -function $\beta(x)$, $p(n)$ and $q(n)$ -terms are expressed through the NLO and NNLO coefficients of $\gamma_{NS}(x)$ and $\beta(x)$ via equations, given in Refs. 20, 22. Within the \overline{MS} -like schemes the factorization and renormalization scale ambiguities are parameterized by the terms $k_1^F = \beta_0 \ln(k_F)$ and $k_1^R = \beta_0 \ln(k_R)$, where k_F (k_R) is the ratio of the factorization (renormalization) scale and the scale of the \overline{MS} -scheme. Following the analysis of Ref. 25 we take $k_R = k_F = k$, fixing

identically the factorization scale and the renormalization scale. We performed our fits for the case of $k = 1$ (namely, in the pure \overline{MS} -scheme) and then determine the scale uncertainties of $\Lambda_{\overline{MS}}^{(4)}$, the twist-4 parameter A_2' and the x -shape of $h(x)$ by choosing $k = 1/4$ and $k = 4$ and repeating the fits for these two cases.

3. In the process of our analysis of CCFR'97 data we applied the same kinematic cuts as in Ref. 16, namely $Q^2 > 5 \text{ GeV}^2$, $x < 0.7$ and $W^2 > 10 \text{ GeV}^2$. We started the QCD evolution from the initial scale $Q_0^2 = 20 \text{ GeV}^2$, which we consider as more appropriate from the point of view of stability of the NLO and NNLO results for $\Lambda_{\overline{MS}}^{(4)}$ due to variation of the initial scale [22]. In order to estimate the uncertainties of the NNLO results, we also performed the N³LO fits with the help of the expanded Padé approximations technique (for the detailed discussions see Ref. 22). The results are presented in Table 1.

Table 1. The results of the fits of CCFR'97 data with the cut $Q^2 > 5 \text{ GeV}^2$

	$\Lambda_{\overline{MS}}^{(4)}$ (MeV)	A_2' (GeV^2)	χ^2/points
LO	264±37	–	113.1/86
	433±53	-0.33±0.06	83.1/86
	331±162	h(x) in Fig.1	66.3/86
NLO	339±42	–	87.6/86
	369±39	-0.12±0.06	82.3/86
	440±183	h(x) in Fig.1	65.8/86
NNLO	326±35	–	77.0/86
	327±35	-0.01±0.05	76.9/86
	372±133	h(x) in Fig.1	65.0/86
N ³ LO Padé	332±28	–	76.9/86
	333±27	-0.04±0.05	76.3/86
	371±127	h(x) in Fig.1	64.8/86

At NLO the value for $\Lambda_{\overline{MS}}^{(4)}$ is in good agreement with the NLO result $\Lambda_{\overline{MS}}^{(4)} = 337 \pm 28 \text{ MeV}$, obtained by the CCFR collaboration with the help of DGLAP NLO analysis of both F_2 and xF_3 SFs data in the case when HT-corrections were neglected [16]. The obtained NLO value of the IRR-model parameter A_2' is in agreement with the estimates of Ref. 15 and of Ref. 26 especially. However, at NNLO a significant decrease of the magnitude of the parameter A_2' is observed. In view of this the results for $\Lambda_{\overline{MS}}^{(4)}$ obtained at the NNLO without HT corrections and with IRR-model of twist-4 term almost coincide. A similar tendency was observed in the process of the N³LO Padé fits. To study this feature in more detail we extracted the x -shape of the model-independent function $h(x)$ (see Fig. 1) and analyzed the factorization/renormalization scale uncertainties of the outcomes of our fits [22]. The corresponding results are presented in Table 2,

where Δ_k is defined as $\Delta_k = \Lambda_{\overline{MS}}^{(4)}(k) - \Lambda_{\overline{MS}}^{(4)}(k=1)$. The related x -shapes of $h(x)$ are presented in Fig. 2.

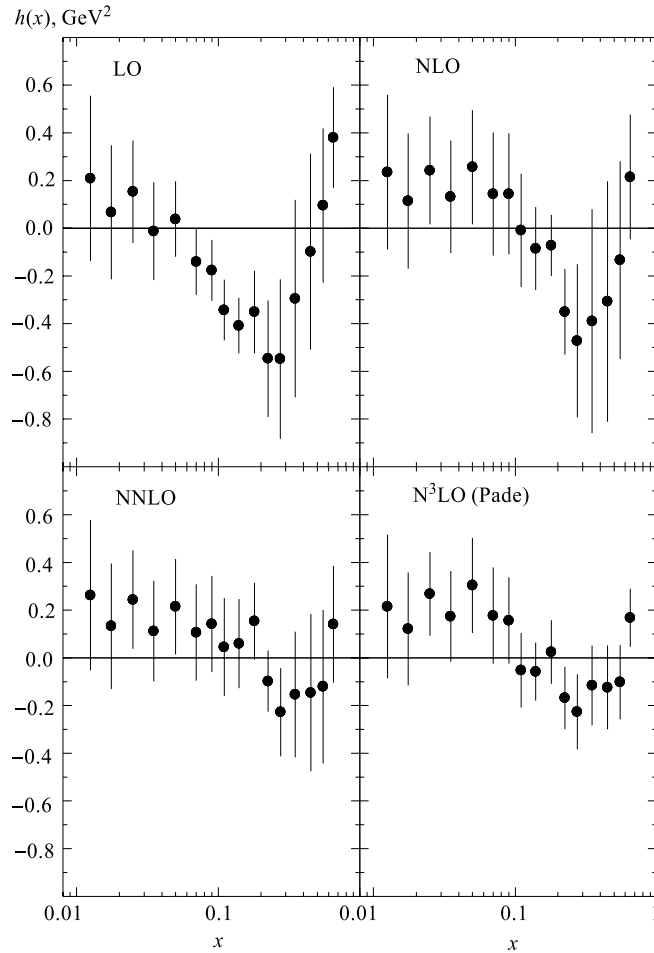
Table 2. The results of NLO and NNLO fits of CCFR'97 data for different values of factorization/renormalization scales

Order	k	Δ_k (MeV)	A'_2 (GeV ²)	χ^2/points
NLO	4	116	–	99.1/86
	4	213	-0.22±0.006	84.2/86
	1/4	-61	–	80.4/86
	1/4	-99	+0.02±0.005	80.2/86
NNLO	4	35	–	83.5/86
	4	66	-0.11±0.06	83.5/86
	1/4	-51	–	87.3/86
	1/4	-45	+0.09±0.05	84.5/86

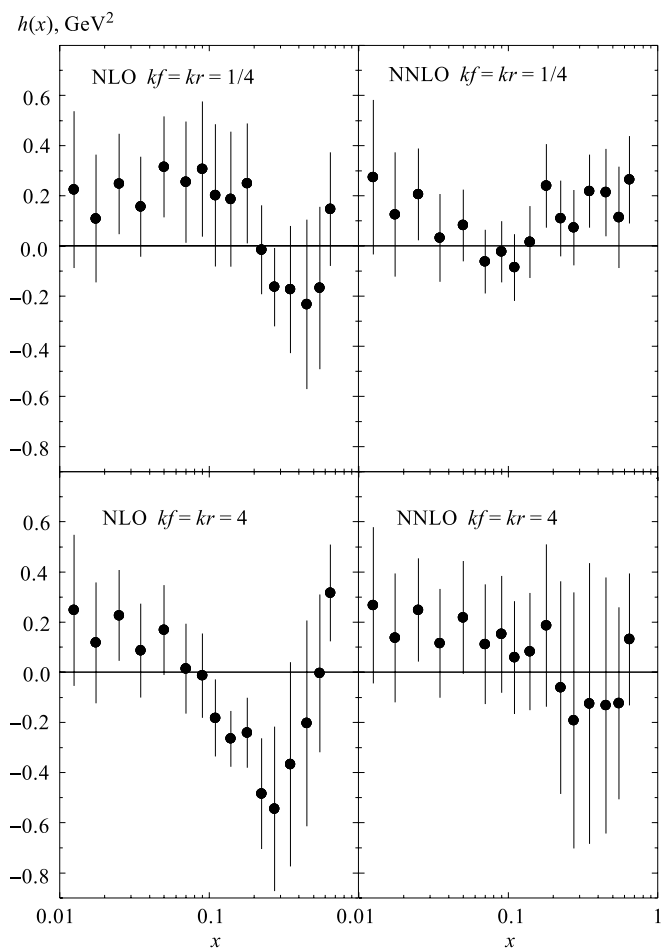
4. We will concentrate first on discussing the presented behaviour of the twist-4 parameter $h(x)$ of xF_3 SF, presented in Figs. 1,2. In the case of $k=1$, namely in the pure \overline{MS} -scheme, x -shape of $h(x)$ obtained from the LO and NLO analysis of Refs. 21,22 is in agreement with the IRR-model predictions of Ref. 15. Note also that the combination of the quark counting rules [27] with the results of Ref. 7 predict the following x -form of $h(x)$: $h(x) \sim A'_2(1-x)^2$. Taking into account the negative values of A'_2 , obtained in the process of our LO and NLO fits (see Table 1), we conclude that the related behaviour of $h(x)$ is in qualitative agreement with these predictions.

At the NNLO the situation is more intriguing. Indeed, though a certain indication of the twist-4 term survives even at this level, the NNLO part of Fig. 1 demonstrates that its extracted x -shape starts to deviate both from the IRR prediction of Ref. 15 and from the quark-parton model picture, mentioned above. Notice also that within the statistical error bars the NNLO value of A'_2 is indistinguishable from zero. These conclusions are confirmed by the studies of the factorization/renormalization scale dependence of the NLO and NNLO outcomes of the fits [22].

Indeed, it is known that the variation of the related scales is simulating in part the effects of the higher-order perturbative QCD corrections. In view of this the NLO (NNLO) results, obtained in the case of $k=1/4$ (see Table 2 and Fig. 2 in particular), are almost identical to the NNLO (Padé motivated N³LO) extractions of $h(x)$ and of the IRR model parameter A'_2 from the fits with $k=1$ (see Fig. 1 and Table 1). Thus, we conclude, that as the result of analysis of the CCFR'97 data, the NNLO and beyond we observe the minimization of the twist-4 contributions to xF_3 SF. This feature is related to the interplay between NNLO

Fig. 1. $h(x)$ extracted from the CCFR'97 data

perturbative QCD and twist-4 $1/Q^2$ corrections. The recent studies of the scale-dependence of the NLO DGLAP extraction of the twist-4 terms from different recent DIS experimental data [28] are supporting the foundations of Refs. 21,22. This means that the higher-twist parameters cannot be defined independently of the effects of perturbation theory and that the NNLO corrections can mimic the contributions of higher twists [29] provided the experimental data are not precise enough for the clear separation of the nonperturbative from perturbative effects. Thus, it is highly desirable to have new experimental data for xF_3 SF, which are more precise than the ones given by the CCFR collaboration.

Fig. 2. Scale dependence of $h(x)$

In conclusion we present also the NLO and NNLO values of $\alpha_s(M_Z)$, obtained by us in Ref. 22 from the fits of CCFR'97 data for xF_3 SF with twist-4 terms modelled through the IRR approach:

$$\begin{aligned}
 NLO \quad \alpha_s(M_Z) &= 0.120 \pm 0.003(\text{stat}) \pm 0.005(\text{syst})_{-0.007}^{+0.009} & (4) \\
 NNLO \quad \alpha_s(M_Z) &= 0.118 \pm 0.003(\text{stat}) \pm 0.005(\text{syst}) \pm 0.003.
 \end{aligned}$$

The systematical uncertainties in these results are determined by the systematical uncertainties of the CCFR'97 data, and the theoretical errors are fixed from the numbers for Δ_k (see Table 2), which reflect the factorization/renormalization scale

uncertainties of the values of $\Lambda_{\overline{MS}}^{(4)}$. The incorporation into the \overline{MS} -matching formula [30] of the proposals of Ref. 31 for estimates of the ambiguities due to smooth transition to the world with $f = 5$ numbers of active flavours was also taken into account. The theoretical uncertainties presented are in agreement with the ones, obtained in Ref. 25, while the NNLO value of $\alpha_s(M_Z)$ is in agreement with another NNLO result $\alpha_s(M_Z) = 0.1172 \pm 0.0024$, which was obtained from the analysis of SLAC, BCDMS, E665 and HERA data for F_2 SF with the help of the Bernstein polynomial technique [32]. It might be of interest to verify the theoretical errors of these two available phenomenological NNLO analysis using different variants of fixing scheme-dependence ambiguities. The first steps towards the analysis of this problem are already made [33].

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