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RESIPE — RENORMALIZATION SCHEME INDEPENDENT PERTURBATION THEORY

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A new approach to perturbation theory for renormalizable quantum field theories, developed in the last few years, is briefly reviewed. Our method gives finite perturbative predictions, which are free from renormalization scheme ambiguities, for any quantity of interest (like a cross section or Green's function) starting from the bare regularized Lagrangian.

1. INTRODUCTION

I will present a new approach to perturbation theory [1–4] for renormalizable quantum field theories (QFTs) which gives renormalization scheme (RS) independent predictions for observable and other quantities of interest (e.g., Green's functions). The resulting <u>RE</u>normalization <u>S</u>cheme <u>I</u>ndependent <u>PE</u>rturbation theory will be called RESIPE for short.

In the time available I will illustrate how RESIPE works for a renormalizable QFT with one dimensionless coupling constant (see Ref. 2). Applications of 2nd order RESIPE to some specific physical measurables, for massless QCD, are to be found in Ref. 3. Generalization of the RESIPE formalism to QFT's with masses and more than one coupling constant and its connection with the renormalization group (RG) formalism is given in Ref. 4. Here, in addition, a new scheme-independent perturbation expansion, without reference to RG techniques, is given which is valid for the general case with masses, several kinematic variables and more than one coupling constant. These references may be consulted for more detail.

2. RESIPE FORMALISM FOR A RENORMALIZABLE QFT WITH ONE COUPLING CONSTANT

Consider a QFT which is renormalizable and has one dimensionless bare coupling constant g_0 (e.g., QCD). For simplicity, consider a physical quantity which depends on only one external energy scale Q. Corresponding to it, one can always construct a dimensionless measurable quantity R (see sec. 2.2) such that its regularized unrenomalized perturbation expansion is of the form

$$R = a_0 + r_{10}\, a_0^2 + r_{20}\, a_0^3 + ... \tag{1}$$

Here the bare couplant $a_0 \equiv g_0^2/4\pi^2$ and the subscript '0' denotes bare or unrenormalized quantities. The bare perturbation series is not well defined since the coefficients of the expansion are infinite. In a renormalizable theory finite results are extracted by absorbing the infinities in the bare parameters (coupling constant, masses, etc.) and the fields present in the Lagrangian. The definitions of the renormalized fields and parameters in terms of the corresponding bare quantities are, however, not unique because of the possibility of finite renormalizations. After renormalization, since the measurable R has no anomalous dimensions, Eq.(1) becomes

$$R = a + r_1 a^2 + r_2 a^3 + ... \tag{2}$$

where the renormalized couplant $a \equiv g^2/4\pi^2$ and g = renormalized coupling constant. The coefficients r_n are finite but their values depend on the RS used to define g. Consequently, finite-order predictions for R in the renormalized theory will depend on the RS used. Thus the conventional renormalization procedure gives predictions for R which, although finite, are still ambiguous. Can this problem of RS-dependent perturbative predictions (present for all QFT's) be solved? Does the fact that the perturbative predictions based on Eq. (1) or Eq. (2) are not well defined mean that R itself is not directly computable in the theory, but instead the theory predicts some function f(R) uniqueley? How, in what form does the theory determine f(R)? RESIPE provides the answers. We will see that for a renormalizable QFT with a single dimensionless coupling constant g_0 the theory, at best, determines the Q dependence of R through the differential equation

$$\begin{aligned} Q\frac{dR}{dQ} &\equiv R'(Q) = f(R(Q)) \\ &= -f_0 R^2 (1 + f_1 R + f_2 R^2 + ...). \end{aligned} \tag{3}$$

The second line expresses f(R) as a series in R with finite RS-invariant coefficients $f_0, f_1, ...$. Each term in this series is RS-invariant and therefore so is any finite order truncation. The convergence of perturbative approximations to f(R) is now controlled by the magnitude of R itself. For practical application, one may approximate the r.h.s. by the first 2 or 3 terms if $|f_n R^n| \ll 1$ for $n \geq 2$ or 3. These would give the second or third order RESIPE prediction. Since these finite order predictions are RS-independent, their confrontation with experiment provides an *unambiguous* probe for higher-order corrections.

2.1. Determination of the RS-Invariants $f_{\underline{n}}$'s. Since the coefficients r_{no} depend on Q through the regularization scale (e.g., an ultraviolet cut-off), Eq. (1) gives

$$R' = r'_{10} a_0^2 + r'_{20} a_0^3 + ... \tag{4}$$

where $$r'_{no} \equiv Q \frac{\partial r_{no}}{\partial Q}.$$
Eliminate a_0 between Eqs. (1) and (4) to express R' as a series in R and compare with Eq. (3)., or equivalently, substitute Eq. (1) into Eq. (3) and compare the resulting series in a_0 for R' with Eq. (4). The resulting expressions for f_n's in terms of r_{no} and r'_{no} are given in Eq. (6) below. Since the theory is renormalizable, one can start with Eq. (2) to obtain

$$R' = r'_1 a^2 + r'_2 a^3 + ..., \tag{5}$$

where $$r'_n \equiv Q \frac{\partial r_n}{\partial Q}.$$
Manipulating Eqs. (2), (3) and (5) as indicated above yields expressions for f_n's in terms of r_n and r'_n. Note the algebra is the same whether one starts with Eq. (1) or Eq. (2). Thus, we find:

$$-f_o = r'_{10} = r'_1$$

$$-f_o f_1 = r'_{20} - 2r'_{10} r_{10} = r'_2 - 2r'_1 r_1$$

$$-f_o f_2 = r'_{30} - 3r'_{20} r_{10} - 2r'_{10} r_{20} + 5r'_{10} r_{10}^2 = r'_3 - 3r'_2 r_1 - 2r'_1 r_2 + 5r'_1 r_1^2, \tag{6}$$

etc. Since r_{n0} and r'_{no} are RS-independent, while r_n and r'_n are finite (by definition) Eq. (6) proves that f_n's are both finite and RS-invariant. These properties for the f_n's are, in a sense, obvious from Eq. (3), since both R and R' possess these two properties being measurables. Note that $f_0, f_1 ...$, etc., can be directly calculated from the combinations of the bare series coefficients (in Eq. (6)) without having to renormalize them. The finiteness of f_n's is guaranteed by the renormalizability of the theory. Note that f_0 and f_1 are universal in the sense that they are independent of the process under consideration. Of course, f_n, $n \geq 2$, do depend on the process, that is R, though this has not been explicitly indicated in Eq. (3) for notational simplicity.

2.2. Testing RESIPE. Eq. (3) requires the knowledge of R at some $Q = Q_0$ (which has to be obtained from experiment) to predict it at any other Q. This boundary condition on Eq. (3) provides the process dependent scale Λ_R for R to have a nontrivial dependence on Q. Dependence of R on the RS-independent scale Λ_R (undetermined by the theory) is consistent with the fact that the starting

Lagrangian contained the undetermined parameter g_0. The dependence of R on the dimensionless g_0 has now appeared, by «dimensional transmutation» [5] trough Λ_R. In the present approach, different physical quantities $R, \tilde{R}$,..., will automatically have scales $\Lambda_R, \Lambda_{\tilde{R}}$, which are specific to them. Does that mean the theory has many independent scales? The answer is no [2]. For the massless case, one can integrate Eq. (3) for the process R and the corresponding equation

$$\tilde{R}' = -f_0 \tilde{R}^2 (1 + f_1 \tilde{R} + \tilde{f}_2 \tilde{R}^2 + ...) \tag{7}$$

for the process $\tilde{R} = a + \tilde{r}_1 a^2 + ...$, since the RS-invariants f_n's and $\tilde{f}_n$'s are constants independent of Q. One can show [2] that the two scales Λ_R and $\Lambda_{\tilde{R}}$ are related:

$$\Lambda_{\tilde{R}} = \Lambda_R \exp [f_0^{-1}(\tilde{r}_{10} - r_{10})]. \tag{8}$$

Also,

$$\Lambda_R = \Lambda \exp [f_0^{-1}(r_1)_{\mu=Q}], \tag{9}$$

where Λ is the usual RS-dependent scale parameter and μ is the renormalization point. Note r_n's and $\tilde{r}_n$'s are functions of Q/μ only and $\tilde{r}_{10} - r_{10} = \tilde{r}_1 - r_1$.

To test the theory using RESIPE one can extract Λ_R and $\Lambda_{\tilde{R}}$ to a given order and see how well Eq. (8) is satisfied. Alternatively, one can use Eq. (9) and compare the value of Λ obtained in the two cases.

Some examples of processes from (massless) QCD to which 2nd order RESIPE has been applied [3] are presented. These examples also show how to construct the appropriate dimensionless R which has the perturbation expansion of the required form (viz. Eq. (1) or (2))) and which will obey Eq. (3).

2.2.1. $e^+e^- \rightarrow$ Hadrons. Experiment gives the dimensionless ratio

$$\Re \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \Re_0 \, (1 + s_1 a + s_2 a^2 + ...).$$

The theorical prediction with QCD corrections is given by the second term where $\Re_0 = 3 \sum e_q^2$ is the parton model value and **a** is the QCD couplant. The one-loop coefficient s_1 is finite and RS-independent, so in this case RESIPE is to be applied to

$$R \equiv \frac{1}{s_1}(\frac{\Re}{\Re_0} - 1) = a + \frac{s_2}{s_1} a^2 + ...$$

2.2.2. Deep Inelastic Scattering. The moments of a nonsinglet structure function $M^{(n)}(Q^2)$ with QCD perturbative corrections has the form:

$$M^{(n)}(Q^2) = A_n(a)^{d^{(n)}} [1 + s_1^{(n)} a + s_2^{(n)} a^2 + ...].$$

Since the unknown nonperturbative matrix element A_n is independent of Q^2, the appropriate quantity here is

$$R^{(n)}(q^2) \equiv -\frac{2}{b_0 d^{(n)}} \frac{d \ln M^{(n)}(Q^2)}{d \ln Q^2} = a[1 + \alpha_1^{(n)} a + ...],$$

where b_0 is defined in Eq. (13) below. It can be seen that corresponding to different processes the appropriate quantities which satisfy an equation like Eq. (3) are quite different. As shown earlier [3], second order RESIPE gives novel tests which involve only measurable quantities.

2.3. Extension of RESIPE to Quantities with Anomalous Dimensions. This is necessary if RESIPE is to apply to all quantities of interest in QFT. For application to Green's function, which has anomalous dimensions, one must first construct an object out of G (analogous to R) which does not get explicitly renormalized and thus is independent of any RS. For example, let $G(p^2)$ be a renormalized propagator (in a massless theory) corresponding to the bare propagator $G_0(p^2)$, so that

$$G(p^2) = Z_G G_0(p^2). \tag{10}$$

Since the infinite constant Z_G is independent of p^2, one has

$$R_G(p^2) = p^2 \frac{d}{dp^2}[\ln G(p^2)] = p^2 \frac{d}{dp^2}[\ln G_0(p^2)]. \tag{11}$$

Thus, R_G in this case is the analogue of a physical quantity. We may now construct from it a quantity which has a perturbation expansion of the form Eq. (1) or Eq. (2) to be able to apply the RESIPE formalism. The second-order RESIPE prediction for the gluon propagator in the Landau gauge is given in Ref. 2.

3. CONNECTION OF RESIPE WITH THE RENORMALIZATION GROUP (RG)

In showing this connection for a renormalizable QFT with one dimensionless renormalized couplant a we will obtain alternative expressions for the RS-invariants f_n's.

3.1. QFT with no Masses. Dimensionless R can depend on Q only through the ratio Q/μ, where μ is the renormalization scale. Since R is a physical quantity, we have the RG equation

$$\mu \frac{d}{d\mu} R(Q/\mu, a(\mu)) = 0 = \mu \frac{\partial R}{\partial \mu} + b(a) \frac{\partial R}{\partial a}, \tag{12}$$

where $b(a)$ is the beta-function for a, defined as

$$\mu\frac{\partial a}{\partial \mu} = b(a) \equiv -b_0 a^2[1 + b_1 a + b_2 a^2 + ...]. \tag{13}$$

Since, $\mu\frac{\partial R}{\partial \mu} = -R'$, we obtain, using Eq. (2)

$$R' = b(a)\frac{\partial R}{\partial a} = b(a)[1 + r_1 a + 2r_2 a^2 + ...]. \tag{14}$$

Now, eliminating the couplant **a** in favour of R by inverting Eq. (2) we obtain

$$R' = b(a)\frac{\partial R}{\partial a} = -b_0 R^2[1 + \rho R_1 + \rho_2 R^2 + ...]. \tag{15}$$

Comparing this with Eq. (3), gives

$$f_0 = b_0, f_1 = \rho_1 = b_1, f_2 = \rho_2 = b_2 + r_2 - b_1 r_1 - r_1^2, \text{ etc.} \tag{16}$$

These relations give an additional proof of the finiteness and regularization independence of the f_n's since, for a renormalizable QFT, the coefficients b_n and r_n are by definition finite and independent of the regularization procedure. The first two relations in Eq. (16), tell us that f_0 and f_1 are process independent and b_0 and b_1 are RS-invariant. The latter is well known to be true for massless QCD.

3.2. QFT with Masses. For our purpose, we choose to define the physical mass of a particle as the pole in its propagator. Let the masses in the theory be m_i, $i = 1, 2, \ldots$. Now R can be taken to be a function of Q/μ, m_i/μ and the couplant a. Since $\mu dR/d\mu = 0$, the RG equation reads

$$Q\frac{\partial R}{\partial Q} + \sum_i m_i \frac{\partial R}{\partial m_i} = b(a)\frac{\partial R}{\partial a}. \tag{17}$$

The f_n's in the expansion of R' in Eq. (3) are RS-invariant, as argued earlier. But the coefficients in the expansion of r.h.s. of Eq. (17) (see Eq. (15) viz. b_0, b_1 and $\rho_n (n \geq 2)$ are no longer RS-independent as they depend on m_i/μ. Using Eq. (2) one can expand $m_i\frac{\partial R}{\partial m_i}$ as a series in R:

$$m_i\frac{\partial R}{\partial m_i} = h_{0i} R^2[1 + h_{1i} R + ...] \tag{18}$$

with

$$h_{0i} = m_i\frac{\partial r_1}{\partial m_i},\ h_{0i}h_{1i} = m_i\frac{\partial}{\partial m_i}(r_2 - r_1^2), \text{ etc.} \tag{19}$$

Eq. (18) and Eq. (19) will hold for each m_i and so no sum over i is implied. From Eqs. (3), (15), (17) and (18), one obtains

$$f_0 = b_0 + \sum_i h_{0i};\ f_0 f_n = b_0 \rho_n + \sum_i h_{0i} h_{ni},\ n \geq 1. \tag{20}$$

Due to the presence of masses the h_{ni}'s and r_n's will depend on Q/μ and m_i/μ while b_n's will depend on m_i/μ so that all these will be RS-dependent. However, their combinations on the r.h.s. of Eqs. (19), which give the f_n's are RS-independent an will be functions of m_i/Q. The above formulation of the RESIPE program is equivalent to the RG formalism developed by Bogoliubov and Shirkov [6]. Recently, higher order corrections, to the total decay width $\Gamma(H^0 \to hadrons)$ of the Higgs boson H^0 have been calculated [7] keeping quark masses. Their calculations of QCD corrections in three different schemes provide an explicit example of the emergence of RS-invariants in theories with masses.

4. CONCLUDING REMARKS

The central idea of RESIPE is to use some observable quantity as the perturbation expansion parameter instead of the usual RS-dependent coupling constant, as is normally done in conventional renormalized perturbation theory (CRPT). It is because of this key ingredient, namely expanding a physical quantity as series in an RS-independent quantity, that the RESIPE formalism yields RS-indendent perturbative predictions at finite order. This central idea can be implemented in different ways depending on the technique used [4, 8]. I have presented one of these in the context of a renormalizable QFT with a single dimensionless coupling constant and shown that it can be applied to any quantity of interest, may it be a measurable or Green's function. RESIPE can be considered as a full-fledged RS-independent substitute for CRPT.

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