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DETERMINATION OF $\Delta\Gamma_s$ FROM ANALYSIS OF UNTAGGED DECAYS $B_s^0 \to J/\psi\,\phi$ BY USING THE METHOD OF ANGULAR MOMENTS

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The performance of the method of angular moments on the $\Delta\Gamma_s$ determination from analysis of untagged decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ is examined. The results of Monte Carlo studies with evaluation of measurement errors are presented. The method of angular moments gives stable results for the estimate of $\Delta\Gamma_s$ and is found to be an efficient and flexible tool for the quantitative investigation of the $B_s^0 \to J/\psi\,\phi$ decay. The statistical error of the ratio $\Delta\Gamma_s/\Gamma_s$ for values of this ratio in the interval [0.03, 0.3] was found to be independent of this value, being 0.015 for 10^5 events.

Исследована применимость метода угловых моментов в случае извлечения параметра $\Delta\Gamma_s$ из анализа немеченых распадов $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$. Представлены результаты, полученные с помощью моделирования указанных распадов методом Монте-Карло. Детально обсуждается получение оценок для статистических и систематических ошибок. Метод угловых моментов обеспечивает стабильность результатов при оценке параметра $\Delta\Gamma_s$ и является эффективным и гибким инструментом количественного исследования распада $B_s^0 \to J/\psi\,\phi$. Показано, что статистическая ошибка для отношения $\Delta\Gamma_s/\Gamma_s$ не зависит от его значения в интервале [0,03, 0,3] и составляет величину 0,015 при статистике 10^5 распадов.

INTRODUCTION

The study of decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$, which is one of the gold-plated channels for B-physics studies at the LHC, looks very interesting from the physics point of view. It presents several advantages related to the dynamics of these decays, characterized by proper-time-dependent angular distributions, which can be described in terms of bilinear combinations of transversity amplitudes. Their time evolution involves, besides the values of two transversity amplitudes at the proper time $t = 0$ and their relative strong phases, the following fundamental parameters: the difference and average value of decay rates of heavy and light mass eigenstates of B_s^0 meson, $\Delta\Gamma_s$ and Γ_s, respectively, their mass difference ΔM_s, and the CP-violating weak phase $\phi_c^{(s)}$. The angular analysis of the decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ provides complete determination of the transversity amplitudes and, in principle, gives the access to all these parameters.

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In the present paper, we examine the performance of the angular-moments method [1] applied to the angular analysis of untagged decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ for the determination of $\Delta\Gamma_s$. After giving the physics motivation in Section 1, we describe in the next section the method of angular moments based on weighting functions introduced in [1]. For the case of $\Delta\Gamma_s$ determination, this method is properly modified in Section 3. The SIMUB package [2] for physics simulation of B-meson production and decays has been used for Monte Carlo studies with two sets of weighting functions. In Section 4 we present the results of these studies and concentrate on the evaluation of measurement errors and their dependence on statistics.

1. PHENOMENOLOGICAL DESCRIPTION OF THE DECAYS $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$

The angular distributions for decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ are governed by spin-angular correlations [3–6] and involve three physically determined angles. In case of the so-called helicity frame [5], which is used in the present paper, these angles are defined as follows (see Fig. 1):

- The z axis is defined to be the direction of ϕ particle in the rest frame of the B_s^0. The x axis is defined as any arbitrary fixed direction in the plane normal to the z axis. The y axis is then fixed uniquely via $y = z \times x$ (right-handed coordinate system).
- The angles $(\Theta_{l^+},\ \chi_{l^+})$ specify the direction of the l^+ in the J/ψ rest frame, while $(\Theta_{K^+},\ \chi_{K^+})$ give the direction of K^+ in the ϕ rest frame. Since the orientation of the x axis is a matter of convention, only the difference $\chi = \chi_{l^+} - \chi_{K^+}$ of the two azimuthal angles is physically meaningful.

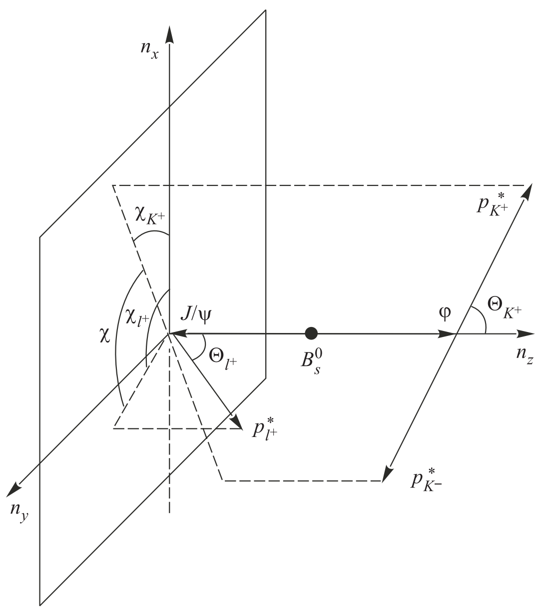

Fig. 1. Definition of physical angles for description of decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in the helicity frame

In the most general form, the angular distribution for the decay $B_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in case of a tagged B_s^0 sample can be expressed as

$$\frac{d^4 N^{\rm tag}(B_s^0)}{d\,\cos\,\Theta_{l^+}\,d\,\cos\,\Theta_{K^+} d\chi dt} = \frac{9}{32\pi}\sum_{i=1}^{6}\mathcal{O}_i(t) g_i(\Theta_{l^+}, \Theta_{K^+}, \chi). \qquad (1)$$

Here, $\mathcal{O}_i$ $(i = 1, \ldots, 6)$ are time-dependent bilinear combinations of the transversity amplitudes $A_0(t)$, $A_{||}(t)$ and $A_\perp(t)$ for the weak transition $B_s^0(t) \to J/\psi\,\phi$ [7] (we treat these combinations as observables)

$$\begin{gathered}\mathcal{O}_1 = |A_0(t)|^2, \quad \mathcal{O}_2 = |A_{||}(t)|^2, \quad \mathcal{O}_3 = |A_\perp(t)|^2, \\ \mathcal{O}_4 = \mathrm{Im}\,(A_{||}^*(t)A_\perp(t)), \quad \mathcal{O}_5 = \mathrm{Re}\,(A_0^*(t)A_{||}(t)), \quad \mathcal{O}_6 = \mathrm{Im}\,(A_0^*(t)A_\perp(t)),\end{gathered} \tag{2}$$

and the g_i are functions of the angles Θ_{l^+}, Θ_{K^+}, χ only [5]:

$$\begin{aligned} g_1 &= 2\cos^2\Theta_{K^+}\sin^2\Theta_{l^+}, \\ g_2 &= \sin^2\Theta_{K^+}(1-\sin^2\Theta_{l^+}\cos^2\chi), \\ g_3 &= \sin^2\Theta_{K^+}(1-\sin^2\Theta_{l^+}\sin^2\chi)\,, \\ g_4 &= -\sin^2\Theta_{K^+}\sin^2\Theta_{l^+}\sin 2\,\chi, \\ g_5 &= \frac{1}{\sqrt{2}}\sin 2\Theta_{l^+}\sin 2\,\Theta_{K^+}\cos\chi, \\ g_6 &= \frac{1}{\sqrt{2}}\sin 2\Theta_{l^+}\sin 2\Theta_{K^+}\sin\chi. \end{aligned} \tag{3}$$

For the decay $\overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in case of a tagged $\overline{B}_s^0$ sample, the angular distribution is given by

$$\frac{d^4N^{\mathrm{tag}}(\overline{B}_s^0)}{d\cos\Theta_{l^+}\,d\cos\Theta_{K^+}\,d\chi dt} = \frac{9}{32\pi}\sum_{i=1}^{6}\overline{\mathcal{O}}_i(t)g_i(\Theta_{l^+},\Theta_{K^+},\chi) \tag{4}$$

with the same angular functions g_i and

$$\begin{gathered}\overline{\mathcal{O}}_1 = |\bar{A}(t)|^2, \quad \overline{\mathcal{O}}_2 = |\bar{A}_{||}(t)|^2, \quad \overline{\mathcal{O}}_3 = |\bar{A}_\perp(t)|^2, \\ \overline{\mathcal{O}}_4 = \mathrm{Im}\,(\bar{A}_{||}^*(t)\bar{A}_\perp(t)), \quad \overline{\mathcal{O}}_5 = \mathrm{Re}\,(\bar{A}_0^*(t)\bar{A}_{||}(t)), \quad \overline{\mathcal{O}}_6 = \mathrm{Im}\,(\bar{A}_0^*(t)\bar{A}_\perp(t)),\end{gathered} \tag{5}$$

where $\bar{A}_0(t)$, $\bar{A}_{||}(t)$ and $\bar{A}_\perp(t)$ are the transversity amplitudes for the transition $\overline{B}_s^0(t) \to J/\psi\,\phi$.

The time dependence of the transversity amplitudes for the transitions $B_s^0(t)\,,\overline{B}_s^0(t) \to J/\psi\,\phi$ is not of purely exponential form due to the presence of $B_s^0 - \overline{B}_s^0$ mixing. This mixing arises due to either a mass difference or a decay-width difference between the mass eigenstates of the $(B_s^0 - \overline{B}_s^0)$ system. The time evolution of the state $|B_s^0(t)\rangle$ of an initially, i. e., at time $t = 0$, present B_s^0 meson can be described in general form as follows:

$$|B_s^0(t)\rangle = g_+(t)|B_s^0\rangle + g_-(t)|\overline{B}_s^0\rangle\,, \qquad g_+(t=0)=1\,, \quad g_-(t=0)=0\,,$$

i. e., the state $|B_s^0(t)\rangle$ at time t is a mixture of the flavour states $|B_s^0\rangle$ and $|\overline{B}_s^0\rangle$ with probabilities defined by the functions $g_+(t)$ and $g_-(t)$. Analogously, the time evolution of the state $|\overline{B}_s^0(t)\rangle$ of an initially present $\overline{B}_s^0$ meson is described by the relation

$$|\overline{B}_s^0(t)\rangle = \bar{g}_+(t)|B_s^0\rangle + \bar{g}_-(t)|\overline{B}_s^0\rangle, \qquad \bar{g}_+(t=0)=0, \quad \bar{g}_-(t=0)=1.$$

Diagonalization of the full Hamiltonian [8] gives

$$\begin{aligned} g_+(t) &= \frac{1}{2}(\mathrm{e}^{-i\mu_L t} + \mathrm{e}^{-i\mu_H t}), \qquad & g_-(t) &= \frac{\alpha}{2}(\mathrm{e}^{-i\mu_L t} - \mathrm{e}^{-i\mu_H t}), \\ \bar{g}_+(t) &= g_-(t)/\alpha^2, & \bar{g}_-(t) &= g_+(t). \end{aligned} \tag{6}$$

Here, $\mu_{L/H} \equiv M_{L/H} - (i/2)\Gamma_{L/H}$ are eigenvalues of the full Hamiltonian corresponding to the masses and total widths of «light» and «heavy» eigenstates $|B_{L/H}\rangle$, and α is a phase factor defining the CP transformation of flavour eigenstates of the neutral B_s-meson system: $CP|B_s^0\rangle = \alpha|\overline{B}_s^0\rangle$. In the case $|\alpha| \neq 1$ the probability for B_s^0 to oscillate to a $\overline{B}_s^0$ is not equal to the probability of a $\overline{B}_s^0$ to oscillate to a B_s^0. Such an asymmetry in mixing is often referred to as indirect CP violation, which is negligibly small in case of the neutral B-meson system.

The time evolution of the transversity amplitudes $A_f(t)$ $(f = 0, ||, \perp)$ is given by the equations

$$\begin{aligned} A_f(t) &= A_f(0)\left[g_+(t) + g_-(t)\frac{1}{\eta_{CP}^f \alpha}\xi_f^{(s)}\right], \\ \bar{A}_f(t) &= A_f(0)\left[\bar{g}_+(t) + \bar{g}_-(t)\frac{1}{\eta_{CP}^f \alpha}\xi_f^{(s)}\right]. \end{aligned} \tag{7}$$

Here, η_{CP}^f are eigenvalues of CP operator acting on the transversity components of the final state which are eigenstates of CP operator

$$CP|J/\psi\,\phi\rangle_f = \eta_{CP}^f |J/\psi\,\phi\rangle_f, \quad (f = 0, ||, \perp),$$
$$\eta_{CP}^0 = 1, \quad \eta_{CP}^{||} = 1, \quad \eta_{CP}^{\perp} = -1,$$

and $\xi_f^{(s)}$ is the CP-violating weak phase [9]:

$$\xi_f^{(s)} = \mathrm{e}^{-i\phi_c^{(s)}}, \quad \phi_c^{(s)} = 2[\arg(V_{ts}^* V_{tb}) - \arg(V_{cq}^* V_{cb})] = -2\delta\gamma,$$

where δ is the complex phase in the standard parameterization of the CKM matrix elements V_{ij} $(i \in \{u, c, t\}, \, j \in \{d, s, b\})$, and γ is the third angle of the unitarity triangle.

The phase $\phi_c^{(s)}$ is very small and vanishes at leading order in the Wolfenstein expansion. Taking into account higher-order terms in the Wolfenstein parameter $\lambda = \sin\theta_C = 0.22$ gives a nonvanishing result [10]:

$$\phi_c^{(s)} = -2\lambda^2\eta = -2\lambda^2 R_b \sin\gamma.$$

Here,

$$R_b \equiv \frac{1}{\lambda}\frac{|V_{ub}|}{|V_{cb}|}$$

is constrained by present experimental data as $R_b = 0.36 \pm 0.08$ [11]. Using the experimental estimate $\gamma = (59 \pm 13)^\circ$ [12], the following constrain can be obtained for the phase $\phi_c^{(s)}$:

$$\phi_c^{(s)} = -0.03 \pm 0.01. \tag{8}$$

According to Eq. (7), at time $t = 0$, the transversity amplitudes of $B_s^0, \overline{B^0}_s \to J/\psi\,\phi$ decays depend on the same observables $|A_0(0)|$, $|A_{||}(0)|$, $|A_\perp(0)|$ and on the two CP-conserving strong phases, $\delta_1 \equiv \arg[A_{||}^*(0)A_\perp(0)]$ and $\delta_2 \equiv \arg[A_0^*(0)A_\perp(0)]$. Time-reversal invariance of strong interactions forces the form factors parameterizing quark currents to be all relatively real and, consequently, naive factorization leads to the following common properties of the observables:

$$\mathrm{Im}\,[A_0^*(0)A_\perp(0)] = 0, \quad \mathrm{Im}\,[A_{||}^*(0)A_\perp(0)] = 0,$$

$$\mathrm{Re}\,[A_0^*(0)A_{||}(0)] = \pm|A_0(0)A_{||}(0)|.$$

Moreover, in the absence of strong final-state interactions, $\delta_1 = \pi$ and $\delta_2 = 0$.

In the framework of the effective Hamiltonian approach the two-body decays, both $B_s^0 \to J/\psi\,\phi$ and $B_d^0 \to J/\psi\,K^\star$, correspond to the transitions $\bar{b} \to \bar{s}\bar{c}c$ with topologies of colour-suppressed spectator diagrams shown in Fig. 2. Factorizing the hadronic matrix elements of the four-quark operators of the effective Hamiltonian into hadronic matrix elements of quark currents, the transversity amplitudes $|A_0(0)|$, $|A_{||}(0)|$, $|A_\perp(0)|$ of decays $B_q^0, \overline{B}_q^0 \to J/\psi V((q,V) \in \{(s,\phi),(d,K^\star)\})$ can be expressed in terms of effective Wilson coefficient functions, constants of J/ψ decay and form factors of transitions $B_q \to V$ induced by quark currents [1]. In Table 1 we collect the predictions of Ref. [1] for the transversity amplitudes of $B_s^0 \to J/\psi\,\phi$ ($B_d^0 \to J/\psi K^\star$) calculated with $B \to K^\star$ form factors given by different models [13–15]. The $B \to K^\star$ form factors can be related to the $B \to \phi$ case using $SU(3)$ flavour symmetry. The most precise polarization measurements performed recently in decays $B \to J/\psi\,K^\star$,

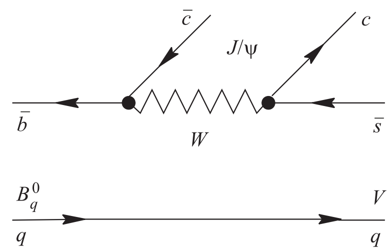

Fig. 2. Colour-suppressed diagrams for decays $B_q^0 \to J/\psi V$ $((q,V) \in \{(s,\phi),(d,K^\star)\})$

$$|A_0(0)|^2 = 0.60 \pm 0.04, \quad |A_\perp(0)|^2 = 0.16 \pm 0.03 \quad \text{(BaBar [16])},$$

$$|A_0(0)|^2 = 0.62 \pm 0.04, \quad |A_\perp(0)|^2 = 0.19 \pm 0.04 \quad \text{(Belle [17])},$$

confirm the predictions based on the model [15].

Table 1. **Predictions for $B_s^0 \to J/\psi\,\phi$ (in brackets — for $B_d^0 \to J/\psi\,K^\star$) observables obtained in Ref. [1] for various model estimates of the $B \to K^\star$ form factors [13–15] (the normalization condition $|A_0(0)|^2 + |A_{||}(0)|^2 + |A_\perp(0)|^2 = 1$ is implied)**

Observable	BSW [13]	Soares [14]	Cheng [15]
$\|A_0(0)\|^2$	0.55 (0.57)	0.41 (0.42)	0.54 (0.56)
$\|A_\perp(0)\|^2$	0.09 (0.09)	0.32 (0.33)	0.16 (0.16)

2. ANGULAR-MOMENTS METHOD

The angular distributions for decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in case of tagged B_s^0 and $\overline{B}_s^0(t)$ samples (see Eqs. (1) and (4), respectively) as well as in case of the untagged sample can be expressed in the most general form in terms of observables $b_i(t)$:

$$f(\Theta_{l^+}, \Theta_{K^+}, \chi; t) = \frac{9}{32\pi} \sum_{i=1}^{6} b_i(t) g_i(\Theta_{l^+}, \Theta_{K^+}, \chi). \tag{9}$$

The explicit time dependence of observables is given by the following relations:

$$\begin{aligned}
b_1(t) &= |A_0(0)|^2 G_L(t),\\
b_2(t) &= |A_{||}(0)|^2 G_L(t),\\
b_3(t) &= |A_\perp(0)|^2 G_H(t),\\
b_4(t) &= |A_{||}(0)|\,|A_\perp(0)|\,Z_1(t),\\
b_5(t) &= |A_0(0)|\,|A_{||}(0)\,|G_L(t)\cos(\delta_2 - \delta_1),\\
b_6(t) &= |A_0(0)|\,|A_\perp(0)|\,Z_2(t),
\end{aligned} \tag{10}$$

where we have used the general compact notations:

$$G_{L/H}(t) = \frac{1}{2}[(1 \pm \cos\phi_c^{(s)})\,\mathrm{e}^{-\Gamma_L t} + (1 \mp \cos\phi_c^{(s)})\,\mathrm{e}^{-\Gamma_H t}],$$
$$Z_{1,2}(t) = \frac{1}{2}(\mathrm{e}^{-\Gamma_H t} - \mathrm{e}^{-\Gamma_L t})\cos\delta_{1,2}\sin\phi_c^{(s)}$$

— for observables $b_i \equiv (\mathcal{O}_i + \overline{\mathcal{O}}_i)/2$ in case of the untagged sample with equal initial numbers of B_s^0 and $\overline{B}_s^0$, while

$$G_{L/H}^{(B_s^0)/(\overline{B}_s^0)}(t) = G_{L/H}(t) \pm \mathrm{e}^{-\Gamma_s t}\sin(\Delta M t)\sin\phi_c^{(s)},$$
$$Z_{1,2}^{(B_s^0)/(\overline{B}_s^0)}(t) = Z_{1,2}(t) \pm\, \mathrm{e}^{-\Gamma_s t}\,[\sin\delta_{1,2}\cos(\Delta M t) - \cos\delta_{1,2}\sin(\Delta M t)\sin\phi_c^{(s)}]$$

— for observables $b_i^{(B_s^0)} \equiv \mathcal{O}_i$ and $b_i^{(\overline{B}_s^0)} \equiv \overline{\mathcal{O}}_i$ in case of tagged B_s^0 and $\overline{B}_s^0(t)$ samples, respectively, with $\Gamma_s \equiv (\Gamma_L + \Gamma_H)/2$. It is easy to see that in both the tagged and untagged cases we have

$$G_{L/H}(t)|_{\phi_c^{(s)}=0} = \mathrm{e}^{-\Gamma_{L/H} t}.$$

According to [1], the observables $b_i(t)$ can be extracted from distribution function (9) by means of weighting functions $w_i(\Theta_{l^+},\, \Theta_{K^+},\, \chi)$ for each i such that

$$\frac{9}{32\pi}\int d\cos\Theta_{l^+}\, d\cos\Theta_{K^+} d\chi\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\, g_j(\Theta_{l^+}, \Theta_{K^+}, \chi) = \delta_{ij}, \tag{11}$$

projecting out the desired observable alone:

$$b_i(t) = \int d\cos\Theta_{l^+}\, d\cos\Theta_{K^+} d\chi\, f(\Theta_{l^+}, \Theta_{K^+}, \chi; t)\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi). \tag{12}$$

The angular-distribution function (9) obeys the condition

$$L(t) \equiv \int d\cos\Theta_{l^+}\, d\cos\Theta_{K^+}\, d\chi\, f(\Theta_{l^+}, \Theta_{K^+}, \chi; t) = b_1(t) + b_2(t) + b_3(t). \qquad (13)$$

For decays $B \to J/\psi(\to\, l^+l^-)\,\phi(\to\, K^+K^-)$, the explicit expressions of weighting functions, given in Table 5 of Ref. [1] for physically meaningful angles in the transversity frame, get the following form (set A) after transformation into the helicity frame:

$$\begin{aligned}
w_1^{(A)} &= 2 - 5\cos^2\Theta_{l^+},\\
w_2^{(A)} &= 2 - 5\sin^2\Theta_{l^+}\cos^2\chi,\\
w_3^{(A)} &= 2 - 5\sin^2\Theta_{l^+}\sin^2\chi,\\
w_4^{(A)} &= -\frac{5}{2}\sin^2\Theta_{K^+}\sin\,2\chi,\\
w_5^{(A)} &= \frac{25}{4\sqrt{2}}\sin\,2\Theta_{K^+}\sin\,2\Theta_{l^+}\cos\chi,\\
w_6^{(A)} &= \frac{25}{4\sqrt{2}}\sin\,2\Theta_{K^+}\sin\,2\Theta_{l^+}\sin\,\chi.
\end{aligned} \qquad (14)$$

The expressions of Eq. (14) are not unique and there are many legitimate choices of weighting functions. A particular set can be derived by linear combination of angular functions g_i (see [1] for more discussions):

$$w_i(\Theta_{l^+}, \Theta_{K^+}, \chi) = \sum_{j=1}^{6} \lambda_{ij}\, g_j(\Theta_{l^+}, \Theta_{K^+}, \chi), \qquad (15)$$

where the 36 unknown coefficients λ_{ij} are solutions for 36 equations

$$\frac{9}{32\pi}\sum_{j=1}^{6}\lambda_{ij}\int d\cos\Theta_{l^+}\, d\cos\Theta_{K^+}\, d\chi g_j(\Theta_{l^+}, \Theta_{K^+}, \chi) g_k(\Theta_{l^+}, \Theta_{K^+}, \chi) = \delta_{ik}. \qquad (16)$$

The weighting functions (set B) corresponding to the linear combination of the angular functions (3) are given by

$$\begin{aligned}
w_1^{(B)} &= \frac{1}{12}[28\cos^2\Theta_{K^+}\sin^2\Theta_{l^+} - 3\sin^2\Theta_{K^+}(1+\cos^2\Theta_{l^+})],\\
w_2^{(B)} &= -\frac{1}{8}[4\cos^2\Theta_{K^+}\sin^2\Theta_{l^+} - 29\sin^2\Theta_{K^+}(1-\sin^2\Theta_{l^+}\cos^2\chi)+\\
&\qquad +21\sin^2\Theta_{K^+}(1-\sin^2\Theta_{l^+}\sin^2\chi)],\\
w_3^{(B)} &= -\frac{1}{8}[4\cos^2\Theta_{K^+}\sin^2\Theta_{l^+} + 21\sin^2\Theta_{K^+}(1-\sin^2\Theta_{l^+}\cos^2\chi)-\\
&\qquad -29\sin^2\Theta_{K^+}(1-\sin^2\Theta_{l^+}\sin^2\chi)],\\
w_4^{(B)} &= -\frac{25}{8}\sin^2\Theta_{K^+}\sin^2\Theta_{l^+}\sin\,2\chi,\\
w_5^{(B)} &= w_5^{(A)},\\
w_6^{(B)} &= w_6^{(A)}.
\end{aligned} \qquad (17)$$

For a limited number of experimental events N in the time bin around the fixed value of the proper time t, distributed according to the angular function (9), it is convenient to introduce the normalized observables

$$\bar{b}_i(t) \equiv b_i(t)/L(t), \tag{18}$$

with normalization factor $L(t)$ given by Eq. (13). Then, as it follows from Eq. (12), the observables $\bar{b}_i(t)$ (18) are measured experimentally by

$$\bar{b}_i^{(\exp)} = \frac{1}{N}\sum_{j=1}^{N} w_i^j, \tag{19}$$

with summation over events in a time bin around t. Here $w_i^j \equiv w_i(\Theta_{l^+}^j, \Theta_{K^+}^j, \chi^j)$, where $\Theta_{l^+}^j$, $\Theta_{K^+}^j$ and χ^j are angles measured in the jth event. The statistical measurement error of the observable (19) can be estimated as

$$\delta\bar{b}_i^{(\exp)} = \frac{1}{N}\sqrt{\sum_{j=1}^{N}(\bar{b}_i^{(\exp)} - w_i^j)^2},$$

with summation over all events in the same time bin.

3. TIME-INTEGRATED OBSERVABLES

For data analysis it is rather convenient to use the time-integrated observables defined as

$$\tilde{b}_i(T_0) = \frac{1}{\tilde{L}(T)}\int_0^{T_0} dt \int d\cos\Theta_{l^+}\, d\cos\Theta_{K^+}\, d\chi\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\times \\ \times f(\Theta_{l^+}, \Theta_{K^+}, \chi; t) \tag{20}$$

with argument $T_0 \leq T$, where T is the maximal value of the B-meson proper time measured for the sample of events being used, and $\tilde{L}(T)$ is a new normalization factor, which has the form:

$$\tilde{L}(T) \equiv \int_0^T L(t) = \int_0^T dt \int d\cos\Theta_{l^+}\, d\cos\Theta_{K^+}\, d\chi\;\, f(\cos\Theta_{l^+}, \cos\Theta_{K^+}, \chi; t) \;= \\ = (|A_0(0)|^2 + |A_{||}(0)|^2)\, \tilde{G}_L(T) + |A_\perp(0)|^2\, \tilde{G}_H(T), \tag{21}$$

where, in the compact notations used in Eq. (10),

$$\tilde{G}_{L/H}(T) \equiv \int_0^T dt\, G_{L/H}(t).$$

The following normalization condition is valid for the observables (20): $\tilde{b}_1 + \tilde{b}_2 + \tilde{b}_3 = 1$. For a limited number of experimental events $N(T)$, measured in the proper time region $t \in [0, T]$, Eq. (20) reduces to

$$\tilde{b}_i^{(\exp)}(T_0) = \frac{1}{N(T)}\sum_{j=1}^{N(T_0)} w_i^j \tag{22}$$

with summation over all events $N(T_0)$ in the time interval $t \in [0, T_0]$ for $T_0 \leq T$. In case of the untagged sample, we have

$$\tilde{G}_{L/H}(T) = -\frac{1}{2}\left[(1 \pm \cos\phi_c^{(s)})\frac{\mathrm{e}^{-\Gamma_L T} - 1}{\Gamma_L} + (1 \mp \cos\phi_c^{(s)})\frac{\mathrm{e}^{-\Gamma_H T} - 1}{\Gamma_H}\right] \tag{23}$$

and

$$\tilde{Z}(T) \equiv \frac{1}{\cos\delta_{1,2}\sin\phi_c^{(s)}}\int_0^T dt\, Z_{1,2}(T) = -\frac{1}{2}\left[(\mathrm{e}^{-\Gamma_H T} - 1)/\Gamma_H - (\mathrm{e}^{-\Gamma_L T} - 1)/\Gamma_L\right].$$

For the untagged sample the explicit form of time-integrated normalized observables (20) in terms of the functions $\tilde{G}_{L/H}(T)$ and $\tilde{Z}(T)$ is given by

$$\begin{aligned}
\tilde{b}_1(T_0) &= |A_0(0)|^2\, \tilde{G}_L(T_0)/\tilde{L}(T),\\
\tilde{b}_2(T_0) &= |A_{||}(0)|^2\, \tilde{G}_L(T_0)/\tilde{L}(T),\\
\tilde{b}_3(T_0) &= |A_\perp(0)|^2\, \tilde{G}_H(T_0)/\tilde{L}(T),\\
\tilde{b}_4(T_0) &= |A_{||}(0)|\, |A_\perp(0)|\, \tilde{Z}(T_0)\cos\delta_1 \sin\phi_c^{(s)}/\tilde{L}(T),\\
\tilde{b}_5(T_0) &= |A_0(0)|\, |A_{||}(0)|\, \tilde{G}_L(T_0)\cos(\delta_2 - \delta_1)/\tilde{L}(T),\\
\tilde{b}_6(T_0) &= |A_0(0)|\, |A_\perp(0)|\, \tilde{Z}(T_0)\cos\delta_2 \sin\phi_c^{(s)}/\tilde{L}(T).
\end{aligned} \tag{24}$$

In the Standard Model (SM) $\sin\phi_c^{(s)} \approx 0$ and the observables $\tilde{b}_{4,5}(T_0)$ are vanishing. In case of a new physics signal, the values of $\sin\phi_c^{(s)}$ and $\tilde{b}_{4,5}(T_0)$ can be sizable, however.

The following relations are valid for the observables (24):

$$\begin{aligned}
\tilde{b}_4(T_0) &= \cos\delta_1 \sin\phi_c^{(s)}\, \tilde{Z}(T_0)\sqrt{\frac{\tilde{b}_2(T_0)\,\tilde{b}_3(T_0)}{\tilde{G}_L(T_0)\,\tilde{G}_H(T_0)}}\,,\\
\tilde{b}_5(T_0) &= \cos(\delta_2 - \delta_1)\sqrt{\tilde{b}_1(T_0)\,\tilde{b}_2(T_0)},\\
\tilde{b}_6(T_0) &= \cos\delta_2 \sin\phi_c^{(s)}\, \tilde{Z}(T_0)\sqrt{\frac{\tilde{b}_1(T_0)\,\tilde{b}_3(T_0)}{\tilde{G}_L(T_0)\,\tilde{G}_H(T_0)}}\,.
\end{aligned}$$

If we introduce the function

$$\tilde{\gamma}(T) \equiv \tilde{G}_H(T)/\tilde{G}_L(T), \tag{25}$$

then the values of initial transversity amplitudes at $t = 0$ and the strong-phase difference

$(\delta_2 - \delta_1)$ are determined from the observables $\tilde{b}_i(T) \equiv \tilde{b}_i(T = T_0)$ by

$$
\begin{aligned}
|A_0(0)|^2 &= \frac{\tilde{b}_1(T)}{\tilde{b}_1(T) + \tilde{b}_2(T) + \tilde{b}_3(T)/\tilde{\gamma}(T)},\\
|A_{||}(0)|^2 &= \frac{\tilde{b}_2(T)}{\tilde{b}_1(T) + \tilde{b}_2(T) + \tilde{b}_3(T)/\tilde{\gamma}(T)},\\
|A_{\perp}(0)|^2 &= \frac{\tilde{b}_3(T)/\tilde{\gamma}(T)}{\tilde{b}_1(T) + \tilde{b}_2(T) + \tilde{b}_3(T)/\tilde{\gamma}(T)},\\
\cos(\delta_2 - \delta_1) &= \frac{\tilde{b}_5(T)}{\sqrt{\tilde{b}_1(T)\,\tilde{b}_2(T)}},
\end{aligned}
\tag{26}
$$

where we consider the initial amplitudes normalized as $|A_0(0)|^2 + |A_{||}(0)|^2 + |A_{\perp}(0)|^2 = 1$. We have also

$$
\sin\phi_c^{(s)} \cos\delta_{1,2} = \frac{\tilde{b}_{4,6}(T)}{\sqrt{\tilde{b}_{2,1}(T)\tilde{b}_3(T)}} \, \frac{\sqrt{\tilde{G}_L(T)\,\tilde{G}_H(T)}}{\tilde{Z}(T)}. \tag{27}
$$

For extraction of the B_s^0 width difference $\Delta\Gamma_s \equiv \Gamma_H - \Gamma_L$ from experimental data, it is convenient to use a special set of the time-integrated normalized observables:

$$
\hat{b}_i(T_0) = \frac{1}{\tilde{L}(T)} \int_0^{T_0} dt \int d\cos\Theta_{l^+} d\cos\Theta_{K^+} d\chi\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi) \times \\
\times \mathrm{e}^{\Gamma' t} f(\Theta_{l^+}, \Theta_{K^+}, \chi; t)\,, \tag{28}
$$

where Γ' is some arbitrary initial approximation of the B_s^0-meson total decay width. These observables can be extracted from the experimental events $N(T)$, measured in the proper time region $t \in [0, T]$, by using the formula

$$
\hat{b}_i^{(\mathrm{exp})}(T_0) = \frac{1}{N(T)} \sum_{j=1}^{N(T_0)} W_i^j, \tag{29}
$$

where $W_i^j \equiv \mathrm{e}^{\Gamma' t^j}\, w_i^j$, and summation is performed over all events $N(T_0)$ in the time interval $t^j \in [0, T_0]$.

For the untagged sample, the explicit expressions for the time-integrated observables (28) can be easily obtained by replacing $\tilde{b}_i$, $\tilde{G}_{L/H}$ and $\tilde{Z}$ in the expressions of Eq. (24) by $\hat{b}_i$, $\hat{G}_{L/H}$ and $\hat{Z}$, respectively, (with the same normalization factor (21)) after introducing the following notations

$$
\hat{G}_{L/H}(T) \equiv \int_0^T dt\, \mathrm{e}^{\Gamma' t}\, G_{L/H}(t) = \\
= (1 \pm \cos\phi_c^{(s)})\, \frac{\mathrm{e}^{\Delta\Gamma_L T/2} - 1}{\Delta\Gamma_L} - (1 \mp \cos\phi_c^{(s)})\, \frac{\mathrm{e}^{-\Delta\Gamma_H T/2} - 1}{\Delta\Gamma_H}, \tag{30}
$$

$$\hat{Z}(T) \equiv \frac{1}{\cos\delta_{1,2}\sin\phi_c^{(s)}}\int_0^T dt\,\mathrm{e}^{\Gamma' t}\,Z_{1,2}(T) = \frac{1-\mathrm{e}^{\Delta\Gamma_L T/2}}{\Delta\Gamma_L} + \frac{1-\mathrm{e}^{-\Delta\Gamma_H T/2}}{\Delta\Gamma_H},$$

where $\Delta\Gamma_{L/H}$ are auxiliary parameters given by

$$\Delta\Gamma_L = 2(\Gamma' - \Gamma_L), \qquad \Delta\Gamma_H = -2(\Gamma' - \Gamma_H). \tag{31}$$

Eq. (26) is also valid after such a replacement.

4. MONTE CARLO STUDIES

For Monte Carlo studies of the estimation of physical parameters by applying the angular-moments method, untagged samples of events of $B_s^0(t) \to J/\psi\,\phi$ decays have been generated by using the package SIMUB [2] with various sets of the input values of initial amplitudes $|A_0(0)|$ and $|A_\perp(0)|$ and $\Delta\Gamma_s$. Other parameters are fixed as follows:

$$\delta_1 = \pi,\ \delta_2 = 0,\ \Gamma_s = 1/\tau_s = 2.278\ (\text{mm/c})^{-1},\ \phi_c^{(s)} = 0.04.$$

The value of Γ_s used corresponds to the lifetime $\tau_s = 1.464$ ps [12], while the CP-violating weak phase $\phi_c^{(s)}$ was fixed as the upper limit of the constrain (8). The value of $\Delta\Gamma_s$ is expected to be negative in the SM. The combined experimental result for $|\Delta\Gamma_s|/\Gamma_s$ is not precise: $|\Delta\Gamma_s|/\Gamma_s < 0.52$ at 95 % C.L. [12]. In the approximation of the equal B_s^0 and B_d^0 lifetimes, the $|\Delta\Gamma_s|$ extraction can be improved [12]: $|\Delta\Gamma_s|/\Gamma_s < 0.31$ at 95 % C.L. A set of the untagged-event samples has been generated with $\Delta\Gamma_s/\Gamma_s \in [-0.3, -0.01]$ to study the influence of $\Delta\Gamma_s$ value on the estimation of $B_s^0(t) \to J/\psi\,\phi$ decay parameters from data analysis.

The values of the time-integrated observables $\tilde{b}_i^{(\text{exp})}(T_0)$, defined by Eq. (20), can be extracted from data according to Eq. (22) by summation of weighting functions for each event. The statistical error of $\tilde{b}_i(T_0)$ is defined by

$$(\delta\tilde{b}_i)^{(\text{stat})} = \frac{1}{N(T)}\sqrt{\sum_{j=1}^{N(T_0)}(\tilde{b}_i^{(\text{exp})} - w_i^j)^2}\,, \tag{32}$$

while a systematic error due to limited precision of angular measurements can be estimated as

$$(\delta\tilde{b}_i)^{(\text{syst})} = \sqrt{\frac{\sum_{j=1}^{N(T_0)}\Delta_i^j}{N(T)}}\,. \tag{33}$$

Here,

$$\Delta_i^j = \left[\frac{\partial w_i^j}{\partial\cos\Theta_{l^+}}\Delta(\cos\Theta_{l^+})\right]^2 + \left[\frac{\partial w_i^j}{\partial\cos\Theta_{K^+}}\Delta(\cos\Theta_{K^+})\right]^2 + \left[\frac{\partial w_i^j}{\partial\chi}\Delta(\chi)\right]^2.$$

In a similar way, the values of the observables $\hat{b}_i^{(\text{exp})}(T_0)$, defined by Eq. (28), can be extracted from the data according to Eq. (29). The formulae for statistical and systematic errors for $\hat{b}_i^{(\mathrm{exp})}(T_0)$ can be obtained by replacement of w_i^j to W_i^j in Eq. (32) and the following redefinition of Δ_i^j in Eq. (33):

$$\Delta_i^j = \left[\frac{\partial W_i^j}{\partial \cos\Theta_{l+}}\Delta(\cos\Theta_{l+})\right]^2 + \left[\frac{\partial W_i^j}{\partial \cos\Theta_{K+}}\Delta(\cos\Theta_{K+})\right]^2 + \\ + \left[\frac{\partial W_i^j}{\partial \chi}\Delta(\chi)\right]^2 + \left[\frac{\partial W_i^j}{\partial t}\Delta(t)\right]^2.$$

Equation (33) can be applied to estimate the systematic errors related both to the measurement precision of the detector and to the limited resolution of the Monte Carlo generator. In the SIMUB generator, for each variable $V \in \{\cos\Theta_{l+}, \cos\Theta_{K+}, \chi, t\}$ randomly generated for decays $B_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$, the number of bins in the region $[V_{\min}, V_{\max}]$ was set as $N^{\mathrm{gen}} = 50\,000$. The generation precision for the variable V is defined as $\Delta(V) = (V_{\max} - V_{\min})/N^{\mathrm{gen}}$ and systematic errors (33) are proportional to $(N^{\mathrm{gen}})^{-1/2}$. The B-meson proper time was generated within the interval $t \in [0, T = 2\,\mathrm{mm/c}]$ which includes 99.3 % of all B decays. We have used samples with a maximum of 100 000 events of the decay $B_s^0 \to J/\psi\,\phi$ because a statistics of about 83800 events is expected to be obtained per year at the CMS detector at the LHC low luminosity under realistic triggering conditions [19].

Table 2 shows the values of the observables $\tilde{b}_i^{(\mathrm{exp})}(T) \equiv \tilde{b}_i^{(\mathrm{exp})}(T_0 = T)$ extracted from the Monte Carlo data by applying the sets A and B of weighting functions, given in Eqs. (14) and (17), respectively. Various theoretical models for estimation of the transversity amplitudes $|A_0(0)|$ and $|A_\perp(0)|$ (see Table 1) have been considered to fix these parameters in the SIMUB generator. It can be seen from Table 2 that the choice of $N^{\mathrm{gen}} = 50\,000$ provides negligibly small systematic errors for the observables as compared with the statistical ones. Moreover, the both errors slightly depend on the values of the observables. For observables obtained by using the set-B weighting functions, the statistical errors are significantly smaller than those in case of the set-A weighting functions. We should also note that even with the statistics of 100 000 events, the values of observables $\tilde{b}_{4,6}^{(\mathrm{exp})}(T)$ and — as a consequence of Eq. (27) — the combination $\cos\delta_{1,2}\sin\phi$ cannot be extracted from the data if the CP-violating weak phase $\phi_c^{(s)}$ is small, according to the SM expectation (8). In this case, these parameters can be estimated only by using a statistics which is not less than $3 \cdot 10^9$ $B_s^0(t) \to J/\psi\,\phi$ decays.

Analysis of the same Monte Carlo data leads to similar conclusions concerning the behaviour of statistical and systematic errors for the observables $\hat{b}_i^{(\mathrm{exp})}(T) \equiv \hat{b}_i^{(\mathrm{exp})}(T_0 = T)$. To illustrate the performance of our method in this case, only the results obtained for transversity amplitudes, corresponding to Cheng's model [15], are shown in Table 3.

Figure 3 shows the dependence of the observables $\tilde{b}_i(T)$ and

$$\hat{b}_i'(T) \equiv \frac{1 - \mathrm{e}^{-\Gamma_s T}}{\Gamma_s T}\hat{b}_i(T) \quad (i = 1, 2, 3)$$

on the value of the ratio $\Delta\Gamma_s/\Gamma_s$. For $\Delta\Gamma_s = 0$, we have

$$\tilde{b}_{1,2,3}(T)|_{\Delta\Gamma_s=0} = \hat{b}'_{1,2,3}(T)|_{\Delta\Gamma_s=0} = |A_{0,||,\perp}(0)|^2.$$

The observables $\tilde{b}_i(T)$ slightly depend on $\Delta\Gamma_s$. A rather strong dependence of the observables $\hat{b}_i(T)$ on the decay width difference $\Delta\Gamma_s$, shown in Fig. 3, can be used for extraction of this parameter from the data analysis as will be discussed below.

Table 2. **Comparison of the observables $\tilde{b}_i^{(\mathrm{exp})}(T)$, extracted from the Monte Carlo data, with their values $\tilde{b}_i^{(\mathrm{th})}(T)$ corresponding to various theoretical models for $|A_0(0)|$ and $|A_\perp(0)|$. A sample of 100 000 decay events generated with $\Delta\Gamma_s/\Gamma_s = -0.15$ was used. The first errors are statistical while the second errors correspond to the systematic uncertainties**

BSW model [13]			
i	$\tilde{b}_i^{(\mathrm{th})}(T)$	$\tilde{b}_i^{(\mathrm{exp})}(T)$ (set A)	$\tilde{b}_i^{(\mathrm{exp})}(T)$ (set B)
1	0.5425	$0.5409 \pm 0.0044 \pm 0.0003$	$0.5432 \pm 0.0024 \pm 0.0002$
2	0.3551	$0.3619 \pm 0.0047 \pm 0.0004$	$0.3579 \pm 0.0036 \pm 0.0004$
3	0.1024	$0.0972 \pm 0.0049 \pm 0.0004$	$0.0991 \pm 0.0034 \pm 0.0004$
4	-0.00055	$-0.0017 \pm 0.0037 \pm 0.0004$	$-0.0021 \pm 0.0033 \pm 0.0003$
5	-0.4389	$-0.4344 \pm 0.0050 \pm 0.0003$	$-0.4344 \pm 0.0050 \pm 0.0003$
6	0.00067	$0.0037 \pm 0.0055 \pm 0.0003$	$0.0037 \pm 0.0055 \pm 0.0003$
Model by Soares [14]			
1	0.3908	$0.3900 \pm 0.0046 \pm 0.0003$	$0.3955 \pm 0.0023 \pm 0.0002$
2	0.2574	$0.2617 \pm 0.0049 \pm 0.0004$	$0.2551 \pm 0.0037 \pm 0.0004$
3	0.3518	$0.3483 \pm 0.0047 \pm 0.0004$	$0.3509 \pm 0.0037 \pm 0.0004$
4	-0.00086	$-0.0083 \pm 0.0040 \pm 0.0004$	$-0.0017 \pm 0.0035 \pm 0.0003$
5	-0.3171	$-0.3156 \pm 0.0052 \pm 0.0003$	$-0.3156 \pm 0.0052 \pm 0.0003$
6	0.0011	$0.0008 \pm 0.0052 \pm 0.0003$	$0.0008 \pm 0.0052 \pm 0.0003$
Model by Cheng [15]			
1	0.5271	$0.5228 \pm 0.0045 \pm 0.0003$	$0.5267 \pm 0.0024 \pm 0.0001$
2	0.2928	$0.2980 \pm 0.0048 \pm 0.0004$	$0.2950 \pm 0.0036 \pm 0.0004$
3	0.1801	$0.1791 \pm 0.0048 \pm 0.0004$	$0.1778 \pm 0.0035 \pm 0.0004$
4	-0.00066	$-0.0030 \pm 0.0037 \pm 0.0003$	$-0.0034 \pm 0.0034 \pm 0.0003$
5	-0.3928	$-0.3927 \pm 0.0051 \pm 0.0003$	$-0.3927 \pm 0.0051 \pm 0.0003$
6	0.00088	$-0.0019 \pm 0.0054 \pm 0.0003$	$-0.0019 \pm 0.0054 \pm 0.0003$

Table 3. **Comparison of the values of observables $\hat{b}_i^{(\mathrm{exp})}(T)$, with $\Gamma' = \Gamma_s$, extracted from the Monte Carlo data, with their values $\hat{b}_i^{(\mathrm{th})}(T)$ corresponding to the model of Cheng [15] for initial transversity amplitudes**

i	$\hat{b}_i^{(\mathrm{th})}(T)$	$\hat{b}_i^{(\mathrm{exp})}(T)$ (set A)	$\hat{b}_i^{(\mathrm{exp})}(T)$ (set B)
1	2.2036	$2.176 \pm 0.044 \pm 0.003$	$2.206 \pm 0.026 \pm 0.001$
2	1.2242	$1.282 \pm 0.045 \pm 0.004$	$1.245 \pm 0.034 \pm 0.004$
3	0.9187	$0.917 \pm 0.045 \pm 0.004$	$0.930 \pm 0.034 \pm 0.004$
4	-0.0073	$-0.099 \pm 0.036 \pm 0.003$	$-0.094 \pm 0.032 \pm 0.003$
5	-1.6425	$-1.618 \pm 0.048 \pm 0.003$	$-1.618 \pm 0.048 \pm 0.003$
6	0.0098	$0.067 \pm 0.050 \pm 0.003$	$0.067 \pm 0.050 \pm 0.003$

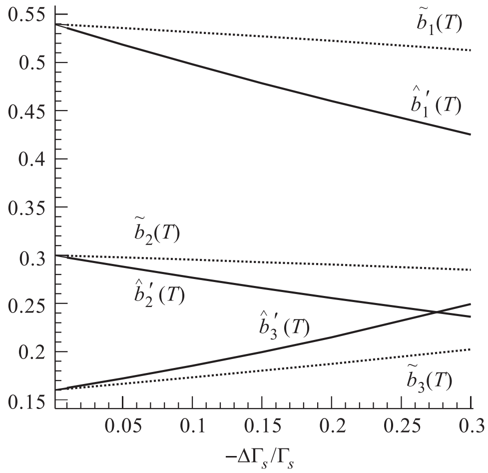

Fig. 3. Dependence of the observables $\tilde{b}_i(T)$ and $\hat{b}_i'(T) \equiv \hat{b}_i(T)[1-\exp(-\Gamma T)]/(\Gamma T)$ ($i = 1,2,3$) on the value of $\Delta\Gamma_s/\Gamma_s$. The observables have been calculated for the case of Cheng's model for transversity amplitudes

Under the assumption $\phi_c^{(s)} = 0$, we have from Eq. (30)

$$\hat{G}^{(0)}_{L/H}(T) = \pm 2\frac{\mathrm{e}^{\pm\Delta\Gamma_{L/H}T/2} - 1}{\Delta\Gamma_{L/H}}.$$

Therefore, the values of the auxiliary parameters $\Delta\Gamma_{L/H}$, defined by Eq. (31), can be determined separately by using the ratios of observables $\hat{b}_i^{(\exp)}(T)/\hat{b}_i^{(\exp)}(T_0)$, extracted from the data analysis, and by solving numerically the equations which arise from one of the following relations:

$$\hat{b}_i(T)/\hat{b}_i(T_0) = \hat{G}_L(T)/\hat{G}_L(T_0) \qquad (i = 1,2,5) \quad (34)$$

— to determine $\Delta\Gamma_L$, and the relation

$$\hat{b}_3(T)/\hat{b}_3(T_0) = \hat{G}_H(T)/\hat{G}_H(T_0) \qquad (35)$$

— to determine $\Delta\Gamma_H$. Then the decay-width parameters Γ_s, $\Delta\Gamma_s$ and $\Gamma_{L/H}$ can be determined via Γ' and $\Delta\Gamma_{L/H}$ as

$$\Gamma_s = \Gamma' - \frac{\Delta\Gamma_L - \Delta\Gamma_H}{4}, \qquad \Delta\Gamma_s = \frac{\Delta\Gamma_L + \Delta\Gamma_H}{2}, \qquad \Gamma_{L/H} = \Gamma' \mp \frac{\Delta\Gamma_L}{2}. \qquad (36)$$

So, using some reasonable approximation for Γ' as a starting point for the data analysis, the experimental value of Γ_s can be essentially improved simultaneously with determination of $\Delta\Gamma_s$. The statistical error of Γ_s determination is expected to be two times smaller than for $\Delta\Gamma_s$ determination.

The direct numerical calculations have shown that the difference between the values of observables $\hat{b}_i(T)$ ($i = 1,2,3,5$), calculated with $\phi_c^{(s)} = 0$ and $\phi_c^{(s)} = 0.04$, does not exceed 0.01 %. Even in case of statistics of 100 000 events this difference is negligibly small as compared with statistical errors for these observables (see Table 3). Therefore, the assumption $\phi_c^{(s)} = 0$ is a good approximation for Γ_s, $\Delta\Gamma_s$ and $\Gamma_{L/H}$ determination by the method described above.

Table 4 shows the results of determination of the decay-width parameters after applying the described procedure to the Monte Carlo data. The sample of 100 000 events generated in case of Cheng's model with $\Delta\Gamma_s/\Gamma_s = -0.15$ has been used. Both sets A and B of weighting functions have been applied to extract the observables $\hat{b}_i^{(\exp)}$. The value of Γ', which is treated as some arbitrary initial approximation for the total decay width of B_s^0 meson, was fixed as $\Gamma' = 1.05\,\Gamma_s$; i. e., it was shifted by 5 % relative to the «true» value of Γ_s fixed in the Monte Carlo generator SIMUB. The value of $T_0 = 0.1\,T$ was chosen as it provides the minimal statistical errors to determine the ratios $\hat{b}_i^{(\exp)}(T)/\hat{b}_i^{(\exp)}(T_0)$. In Table 4 we present the result for $\Delta\Gamma_L$ obtained from the ratio $\hat{b}_1^{(\exp)}(T)/\hat{b}_1^{(\exp)}(T_0)$ only, which gives the best precision.

Table 4. **Results of determination of the decay-width parameters (in units (mm/c)$^{-1}$) based on extraction of the observables $\hat{b}_i^{(\rm exp)}$ from analysis of 100 000 Monte Carlo events. The input value of $\Delta\Gamma_s$ corresponds to $\Delta\Gamma_s/\Gamma_s = -0.15$**

Parameter	Input value	Measurement (set A)	Measurement (set B)
$\Delta\Gamma_L$	-0.1139	$-0.103 \pm 0.058 \pm 0.003$	$-0.110 \pm 0.034 \pm 0.002$
$\Delta\Gamma_H$	-0.5696	$-0.478 \pm 0.137 \pm 0.012$	$-0.554 \pm 0.101 \pm 0.012$
Γ_L	2.4493	$2.444 \pm 0.029 \pm 0.002$	$2.447 \pm 0.017 \pm 0.001$
Γ_H	2.1076	$2.154 \pm 0.068 \pm 0.006$	$2.115 \pm 0.050 \pm 0.006$
Γ_s	2.2784	$2.299 \pm 0.037 \pm 0.003$	$2.281 \pm 0.027 \pm 0.003$
$\Delta\Gamma_s$	-0.3418	$-0.290 \pm 0.074 \pm 0.006$	$-0.332 \pm 0.053 \pm 0.006$

Table 4 shows that the set-B weighting functions give more precise and stable results than the set-A functions.

To improve the precision of $\Delta\Gamma_s$ determination, the same procedure should be repeated with Γ' fixed to be equal to the value of Γ_s determined at the first step. Because of $\Delta\Gamma_s = \Delta\Gamma_L = \Delta\Gamma_H$ in case of $\Gamma' = \Gamma_s$, the value of $\Delta\Gamma_s$ is defined at the second step to be equal to the value of $\Delta\Gamma_L$ determined from the ratio $\hat{b}_1^{(\rm exp)}(T)/\hat{b}_1^{(\rm exp)}(T_0)$ by using Eq. (34). Using the values of $\Delta\Gamma_s$ from Table 4 as an input value of Γ' at the second step, we have obtained finally the following results (to be compared with the input value $\Delta\Gamma_s = -0.3418$ set in the SIMUB generator):

$$\Delta\Gamma_s^{\rm exp} = -0.330 \pm 0.057 \pm 0.004 \qquad \text{(set A)},$$
$$\Delta\Gamma_s^{\rm exp} = -0.338 \pm 0.034 \pm 0.002 \qquad \text{(set B)}.$$

In this way one can reduce not only the statistical error but also essentially improve the stability of the $\Delta\Gamma_s$ result even in case of using the set-A weighting functions.

Table 5 shows the statistical errors of $\Delta\Gamma_s/\Gamma_s$ determination by the described approach applied to different statistics of Monte Carlo events generated with various «true» values of $\Delta\Gamma_s$. The lack of numbers in the table corresponds to cases when the approach is not able to give a certain result for $\Delta\Gamma_s$. The use of set-B weighting functions gives more stable results even in case of too small statistics and values of $\Delta\Gamma_s$, for which the same approach does

Table 5. **Statistical errors of $\Delta\Gamma_s$ extraction (in units (mm/c)$^{-1}$) obtained by applying the angular-moments method with set-B (set-A) weighting functions to the Monte Carlo data samples with different numbers of events**

$\Delta\Gamma_s/\Gamma_s$	200 events	500 events	10^3 events	10^4 events	10^5 events
-0.03	—	—	—	0.035 (–)	0.014 (0.023)
-0.05	—	—	—	0.046 (–)	0.014 (0.022)
-0.1	—	—	0.11 (–)	0.046 (0.079)	0.014 (0.024)
-0.15	—	—	0.13 (0.19)	0.045 (0.078)	0.014 (0.024)
-0.2	—	0.23 (–)	0.12 (0.18)	0.048 (0.072)	0.015 (0.026)
-0.3	0.21 (–)	0.23 (–)	0.18 (0.20)	0.050 (0.083)	0.016 (0.028)

Table 6. **Determination of initial transversity amplitudes and strong-phase difference by using the values of observables $\tilde{b}_i^{(\exp)}(T)$ extracted from Monte Carlo data. The events sample has been generated for the case of Cheng's model [15] for transversity amplitudes and with $\Delta\Gamma_s = 0.15\,\Gamma_s$. The first errors are statistical while the second errors are caused by uncertainties of $\Delta\Gamma_s$ determination**

Parameter	Input value	10 000 events	100 000 events
$\lvert A_0(0)\rvert^2$	0.54	$0.527 \pm 0.007 \pm 0.012$	$0.5398 \pm 0.0023 \pm 0.0011$
$\lvert A_{\vert\vert}\rvert^2$	0.30	$0.337 \pm 0.011 \pm 0.008$	$0.3023 \pm 0.0036 \pm 0.0006$
$\lvert A_\perp\rvert^2$	0.16	$0.136 \pm 0.010 \pm 0.020$	$0.1579 \pm 0.0032 \pm 0.0018$
$\cos(\delta_2 - \delta_1)$	-1	-1.021 ± 0.044	-0.9962 ± 0.015

not work with set-A functions. The sensitivity of the method is measured by the statistical error of $\Delta\Gamma_s/\Gamma_s$, which only slightly depends on the value of this ratio and is proportional to $1/\sqrt{N}$, where N is the number of events. In particular, for a statistics of 100 000 events, the statistical error is about 0.015, while for 1000 events — about 0.15.

In principle, the value of $\Delta\Gamma_s$ can be determined similarly by using the ratios $\tilde{b}_i^{(\exp)}(T)/\tilde{b}_i^{(\exp)}(T_0)$ or $\hat{b}_i^{(\exp)}(T)/\tilde{b}_i^{(\exp)}(T)$, extracted from the data analysis with $\Gamma' = \Gamma_s$, and by solving the equations arising from the relations

$$\tilde{b}_i(T)/\tilde{b}_i(T_0) = \tilde{G}_L(T)/\tilde{G}_L(T_0) \quad (i = 1, 2, 5), \qquad \tilde{b}_3(T)/\tilde{b}_3(T_0) = \tilde{G}_H(T)/\tilde{G}_H(T_0)$$

or

$$\hat{b}_i(T)/\tilde{b}_i(T) = \hat{G}_L(T)/\tilde{G}_L(T) \quad (i = 1, 2, 5), \qquad \hat{b}_3(T)/\tilde{b}_3(T) = \hat{G}_H(T)/\tilde{G}_H(T).$$

But in both cases the precision of $\Delta\Gamma_s$ determination turns out to be worse than that in the approach based on the ratios $\hat{b}_i^{(\exp)}(T)/\hat{b}_i^{(\exp)}(T_0)$ because of the weak $\Delta\Gamma_s$-dependence of the $\tilde{b}_i(T)$ observables.

The initial transversity amplitudes and strong-phase difference can be recalculated from the values of observables $\tilde{b}_i^{(\exp)}(T)$ according to Eq. (26). The results of such determination of the parameters $|A_f(0)|^2$ $(f = 0, ||, \perp)$ and $\cos(\delta_2 - \delta_1)$ are shown in Table 6 for different statistics. We have used the Monte Carlo sample generated with the theoretical values of the amplitudes $|A_0(0)|$ and $|A_\perp(0)|$ corresponding to Cheng's model [15]. To extract the observables $\tilde{b}_i^{(\exp)}(T)$, the set B of the weighting function has been applied to Monte Carlo data. To estimate the statistical errors for parameters $|A_f(0)|^2$ $(f = 0, ||, \perp)$ and $\cos(\delta_2 - \delta_1)$, the standard error-propagation method has been applied to the statistical errors of the observables $\tilde{b}_i^{(\exp)}(T)$, taking into account the correlation between pairs of different observables. The systematic errors of the observables related to the limited generator resolution are neglected. The total errors for parameters $|A_f(0)|^2$ $(f = 0, ||, \perp)$ should also include the additional uncertainty related to the error of calculation of $\tilde{\gamma}(T)$ caused by the error of $\Delta\Gamma_s$ (see definition of $\tilde{\gamma}(T)$ in Eqs. (25) and (23)). In Table 6 we also show these errors calculated by assuming $\Delta\Gamma_s = -0.15\,\Gamma_s$ (see Table 5):

$$\frac{\delta(\Delta\Gamma_s)}{|\Delta\Gamma_s|} = \begin{cases} 30\,\% \text{ for } 10\,000 \text{ events}, \\ 9.3\,\% \text{ for } 100\,000 \text{ events}. \end{cases} \tag{37}$$

CONCLUSION

For the decay $B_s^0 \to J/\psi\,\phi$ in the framework of the method of angular moments a nonfit scheme for separate estimation of the parameters $\Delta\Gamma_s$, Γ_s and $|A_f(0)|^2$ $(f = 0, ||, \perp)$ has been proposed, based on analysis of an untagged sample, and studied by the Monte Carlo method. A strong dependence of statistical measurement errors on the choice of the weighting functions has been demonstrated. The statistical error of the ratio $\Delta\Gamma_s/\Gamma_s$ for values of this ratio in the interval [0.03, 0.3] was found to be independent of this value and about 0.015 for 10^5 events. The method of angular moments gives stable results for the estimate of $\Delta\Gamma_s$ and is found to be an efficient and flexible tool for the quantitative investigation of the $B_s^0 \to J/\psi\,\phi$ decay.

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