УДК 629.384.634.5

COMPARISON BETWEEN SCHEMES FOR HEAVY ION INJECTION INTO NUCLOTRON BOOSTER

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In order to increase the intensity of the Nuclotron beams, a project of circular injector, the so-called booster, is under consideration for several years. This paper is especially devoted to the problem of heavy ion injection into the Nuclotron booster. Several injection methods have been studied, namely: stacking in the horizontal phase space, RF stacking, stripping injection and injection by means of electron cooling of the injected ions. A comparison between all these methods is given.

Для увеличения интенсивности пучков нуклотрона рассматривается сооружение циклического инжектора, так называемого бустера. Статья посвящена проблеме инжекции тяжелых ионов в бустер. Рассмотрены следующие методы инжекции: накопление в горизонтальном фазовом пространстве, высокочастотное накопление, перезарядная инжекция и инжекция с применением электронного охлаждения. Приводится сравнение этих методов инжекции.

INTRODUCTION

In order to increase the intensity of the Nuclotron beams, a project of circular injector, the so-called booster, is under consideration for several years.

The first proposal [1] was for a warm booster with circumference equal to one fifth of those in the Nuclotron and which would accelerate ions with $Z/A = 0.5$ up to 200 MeV/u.

The new variant of the booster [2] is based on superconducting magnets of Dubna type. It has circumference equal to one third of Nuclotron circumference and would accelerate ions with $Z/A = 0.5$ up to 250 MeV/u. This will be a rapid cycling synchrotron with a frequency 1 Hz.

The circular injector (booster) will allow one to increase the injection energy to a great extent and thus to solve two basic problems that accelerators of synchrotron type face.

1. In synchrotrons large acceptance is necessary at injection energy when the beam size is large. On the other hand, at maximum energy the beam size is small due to the adiabatic damping of the betatron oscillations. The transverse emittance is inverse proportional to the longitudinal momentum. So synchrotrons face the discrepancy of having at maximum field $B_{\max}$ small beam size but the energy of the magnetic field and the consumer power to be large due to the large acceptance.

2. The smaller injection energy the higher adverse space charge effects. The incoherent betatron tune shift due to the space charge forces is proportional to $\beta^{-2}\gamma^{-3}$, β and γ being

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the relativistic factors, and at the same time to the full number of stored particles N. The boundary value $\Delta Q = 0.2$ is usually adopted for the incoherent tune shift in synchrotrons. Although this limit could be increased to $\Delta Q = 0.5$ together with a careful correction of the crossed resonances, as a rule in synchrotrons higher intensity requires higher injection energy.

Another important advantage of boosters is the possibility for a large acceptance to be used. This will not lead to enormous energy of the magnetic field and consumes power because the size of the booster is much smaller than this of the main accelerator. The large booster acceptance allows for a multiturn injection to be applied and hence to increase the beam intensity many times.

As the booster is relatively small machine, it could have high repetition frequency. It is also much easier to have high vacuum in the booster.

As a rule the beam extracted from a booster is injected into the main accelerator by means of fast bunch to bucket injection. If as usual the harmonic number in the booster is equal to one ($h = 1$), we could inject six successive booster pulses into Nuclotron.

This paper is especially devoted to the problem of heavy ion injection into the Nuclotron booster. Several injection methods have been studied, namely: stacking in the horizontal phase space, RF stacking, stripping injection and injection by means of electron cooling of the injected ions. A comparison between all these methods is given.

1. ION SOURCES AND INJECTOR

Two types of heavy ion sources are now in use in JINR–LHE: cryogenic electron beam ion source (EBIS)–CRION and laser ion source.

The former delivers 25 μs pulses with $4 \cdot 10^8$ Ar_{40}^{18+}, $1 \cdot 10^8$ Kr_{84}^{35+}, $1 \cdot 10^6$ Xe_{131}^{59+}, etc., ions. EBIS has the highest charge-state performance.

The laser ion source with 10J CO_2 laser produces 5–10 μs pulses with $5 \cdot 10^{10}$ Li_7^{3+}, $1.5 \cdot 10^{10}$ C_{12}^{6+}, $1.0 \cdot 10^9$ Mg_{24}^{12+}, etc., ions.

Both ion sources generate short pulses and are well suited for single turn injection into Nuclotron.

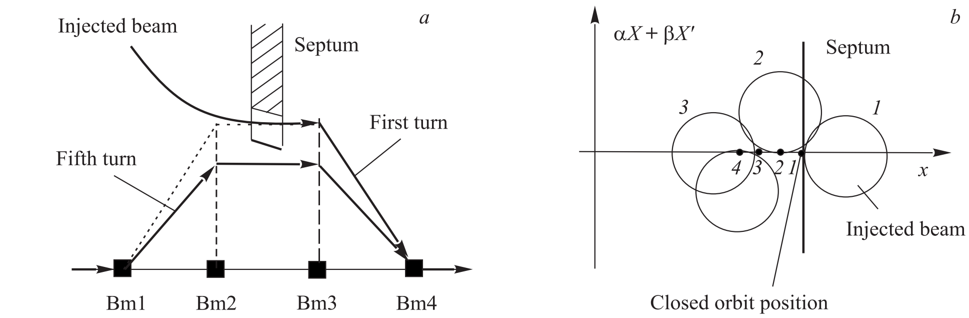
On the other hand CRION can be used also for up to ten turns multiturn injection in the Nuclotron booster.

Another type of ion source which revolutionized the heavy ion accelerator field and which could be of interest is the electron-cyclotron resonance ion source (ECR). It has cw nature of the ion beams, produces ions in high charge states with good intensities, has high reliability.

In the Synchrophasotron-Nuclotron accelearating complex an Alvarez-type linac LU–20 is used as injector. It could accelerate ions with Z/A from 0.33 to 0.5 up to 5 MeV/u.

2. MULTITURN INJECTION INTO NUCLOTRON BOOSTER

The superconducting variant of the Nuclotron booster, which is now under consideration, has large acceptances: $A_x = 400\pi$ mm $\cdot$ mrad, $A_z = 225\pi$ mm $\cdot$ mrad. Nevertheless, due to the large emittance of the injected beams $\varepsilon_x = 50\pi$ mm $\cdot$ mrad, $\varepsilon_z = 32\pi$ mm $\cdot$ mrad the ratio acceptance to emittance is rather small $A_x/\varepsilon_x = 8$, $A_z/\varepsilon_z = 7$. This is the main limitation factor for the multiturn injection into the booster.



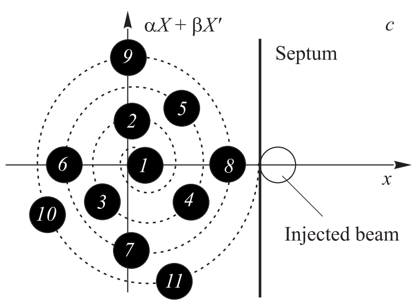


Fig. 1. *a*) Four magnets orbit bump; *b*) positions of the successive injected beam slices in the horizontal phase space; the fractional part of the betatron tune Q is equal to 0.25; *c*) successive positions of the injected beam slices

We have studied betatron stacking in the horizontal phase plane [3]. The principle of this injection method is shown on Fig. 1,*a*. The closed orbit is locally distorted by means of four bump magnets. From the very beginning the bump is as large as to pass close to the septum and then it is gradually reduced to zero following a linear, exponential or other law.

On the second turn the particles will avoid the septum due to the betatron oscillations around the instantaneous closed orbit, Fig. 1,*b*. Meanwhile a new portion of particles is injected. These particles will have larger amplitudes of the betatron oscillations as the orbit bump is reduced.

It could be shown that the successive slices of the injected beam lie on a spiral in the horizontal phase space (Fig. 1,*c*). The origin of the spiral is on the simultaneous orbit.

Two definitions of the injection efficiency will be used in this paper. The first is:

$$\varepsilon_1 = \frac{N_{\text{st}}}{n_\Sigma N_{\text{st}}} = \frac{n_{\text{eff}}}{n_\Sigma}, \tag{1}$$

where N_{st} is the full number of stored particles; N_{inj} — the number of particles injected per turn; n_Σ — the total number of injected turns and n_{eff} — the number of effective turns.

The second definition of the injection efficiency is especially suited for the stacking in the transverse phase plane. It is related with the definition of the accumulation factor (AF):

$$\text{AF} = \frac{I_{\text{accel}}}{I_{\text{inj}}}. \tag{2}$$

Now we will define the efficiency of the multiturn injection as:

$$\varepsilon_2 \frac{\text{AF}}{A_x/\varepsilon_x}. \tag{3}$$

The numerical simulations [3] show that an efficiency $\varepsilon_2 = 50\,\%$ could be expected.

Figure 2 shows the dependence of the accumulation factor on the number of injected turns for exponential orbit fall.

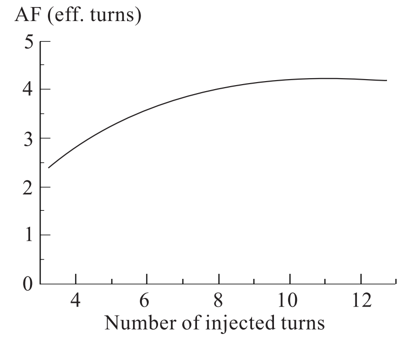


Fig. 2. Accumulation factor for muti-turn injection versus the number of injected turns

3. RF STACKING

The principle of the RF stacking is explained in Fig. 3. [4]. The beam is injected by means of a septum magnet. After the injection of the first portion of particles is completed, the stacking RF cavity is switched on and the particles are accelerated (or usually decelerated) to anouter (inner) orbit following the relation:

$$\frac{E}{R}\frac{dR}{dE} = \frac{\alpha}{\beta^2}, \tag{4}$$

where R is the physical radius of the machine; β is the relativistic factor and α is the momentum compaction factor.

When the top of the stack is reached, the RF voltage is switched off and the particles are released from the RF buckets.

In the repetitive stacking mode of operation (stacking at the top) the new portion is moved again to the same position, i.e., to the top of the stack. According to the Liouville theorem the particles already accumulated in the stack will be displaced toward lower (higher) energies. Due to the very small value of the momentum compaction factor the portions of particles with different energies largely overlap in the physical and transverse phase spaces.

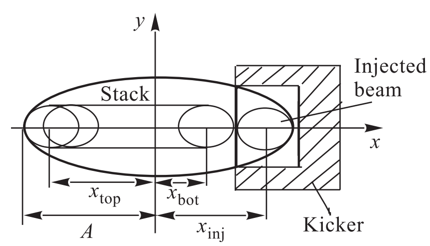


Fig. 3. Principle of RF stacking

The stacking takes place in the longitudinal phase space, while the density in the six-dimensional μ-phase space is conserved in agreement with the Liouville's theorem. A beam slice with large intensity is built up.

In the nonrepetitive stacking mode (stacking at the bottom) each successive portion of particles is moved to a slightly different energy than the previous one. The energy difference is equal to the final bucket area A_b divided by 2π. So new particles will be added to the bottom of the stack.

According to [4] the following relation must be satisfied at any point along the ring:

$$E_{\mathrm{inj}} - E_{\mathrm{top}} \leqslant 2E\beta^2 \left(\frac{a - \sqrt{\varepsilon\beta(s)}}{D(s)} \right), \tag{5}$$

where $\beta(s)$ is the Twiss amplitude function and $D(s)$ is the dispersion.

At the injection point:

$$E_{\rm inj} - E_{\rm bot} = E\beta^2 \left(\frac{2\sqrt{\varepsilon\beta_{\rm inj} + \Delta}}{D_{\rm inj}} \right), \tag{6}$$

Δ being the distance between the stack bottom and injected beam edges.

The number of RF cycles is:

$$n_{\rm rf} = \varepsilon_{\rm rf} \frac{E_{\rm bot} - E_{\rm top}}{\Delta E}, \tag{7}$$

where ΔE is the phase displacement of the stack during single crossing by the buckets and $\varepsilon_{\rm rf}$ is the stacking efficiency defined as the ratio of the ideal stack width to the width of the real stack.

It follows from (5–7) that:

$$n_{\rm rf} = \varepsilon_{\rm rf} \frac{E\beta^2}{\Delta E} \left(\frac{2(a - \sqrt{\varepsilon\beta^*})}{D^*} - \frac{2\sqrt{\varepsilon\beta_{\rm inj} + \Delta}}{D_{\rm inj}} \right). \tag{8}$$

For the Nuclotron booster a combination of single turn injection and RF stacking is a good choice.

An estimation based on the formula (8) shows that the stored intensity in the booster could be increased by a factor of eight.

4. CHARGE EXCHANGE INJECTION

Proposed by G.I.Dimov in Novosibirsk in 1969 [5], nowadays the charge exchange or stripping injection is a preferred injection method for proton machines due to its relative simplicity and a very high intensity of the stored beams. Recently this injection method has been successfully applied for light ion storage in CELSIUS [6].

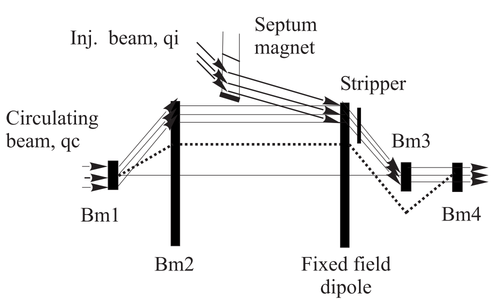


Fig. 4. Stripping injection of heavy ions into Nuclotron booster

The principle of charge exchange injection consists in letting the injected beam pass through a thin carbon (or other appropriate material) foil. Having passed, the foil ions change their charge while energy is practically unaltered and beam rigidity $B\rho$ jumps to a new value according to the relation:

$$B\rho = \frac{A}{300Z} \sqrt{T_n^2 + 2E_{0n}T_n}, \tag{9}$$

where $B\rho$ is in Tm, the kinetic energy per nucleon T_n is in MeV, and the rest energy per nucleon is also in MeV.

This provides a spatial separation for the trajectories of the injected and circulating beam.

Heavy ions which change their charge from 1.3 to 1.7 times in stripping foil crossings could be injected into the booster with the help of a four magnets closed orbit bump (Fig. 4).

A consistent analytical description of the charge exchange injection of heavy ions was developed in [7] on the base of a kinetic treatment.

Light ions with Z up to 14 could be successfully stored in the booster.

Figure 5 shows the process of ion storage in the booster without any painting (with fixed closed orbit).

As the ratio of the booster acceptance to the beam emittance is rather small, $A_x/\varepsilon_x = 8$ and $A_z/\varepsilon_z = 7$, a simplest painting scheme making use of the fact that the fractional part of the betatron tune is equal to 0.75 is proposed for the booster. In this scheme the closed orbit is moved inward the machine centre and remains fixed. The number of foil crossings will be reduced due to the betatron oscillations and correspondingly will increase the intensity gain by a factor of two (taking into account the emittance growth).

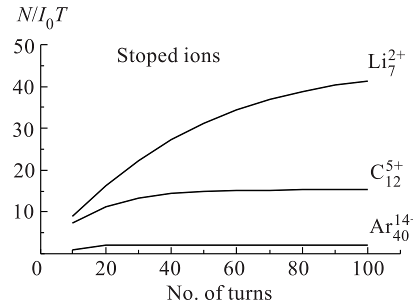


Fig. 5. Ion storage during charge exchange injection of heavy ions into Nuclotron booster (no painting or fixed closed orbit mode)

5. INJECTION BY MEANS OF ELECTRON COOLING

An effective way of particle storage recently applied in TSR, SIS, and CELSIUS is the use of electron cooling of the ion beam. This approach has several varieties. One may cool the phase space area filled by multiturn injection or the particle stack created by RF storage method. In CELSIUS particles stored by means of ion stripping are cooled. In all three methods the cooling shrinks the phase space area occupied by particles thus releasing space necessary for injection of new portion of particles.

A different approach suggested in [8] uses standard single turn injection. During the cooling particles are captured in a stationary bucket. A steady bunch is formed releasing the biggest part of the orbit for injection of new particles.

In electron cooling, the cooling time is proportional to $\beta^4\gamma^5$, β and γ being the relativistic factors, i.e., the method is well suited for injection energies.

On the other hand, the cooling time in transverse direction is proportional to $\varepsilon^{3/2}$, ε being the emittance, and the cooling time in longitudinal direction is proportional to $(\Delta p/p)^3$. Hence it is more effective to use single turn than multiturn injection and cool the beam afterwards.

We will propose here the following injection scheme. At the injection point the dispersion is nonzero while the electron cooler is disposed in a dispersion free area. The particles are put on an off-momentum orbit by means of a fast kicker. Then ions are cooled but the mean velocity of the electrons is set a bit smaller than the mean ion velocity ($\Delta v/v \sim -0.5\,\%$). Due to the dispersion in the injection point the ions are pushed toward the machine centre. After that a new portion of particles is injected.

The cooling time in transverse direction is given by [9]:

$$\tau_{\perp\mathrm{cool}} = 2\cdot 10^7\,\frac{\beta^4\gamma^5\theta_\perp^3}{\eta j_e}\,\frac{A}{Z^2}. \tag{10}$$

where: $\theta_\perp = \sqrt{\varepsilon/\beta_x}$ is the maximum transverse angle; η is the ratio of the cooler length and accelerator circumference; j_e is the density of the electron beam in A/cm^2.

The cooling time in longitudinal direction is:

$$\tau_{\parallel\mathrm{cool}} = 2\cdot 10^7 \frac{\beta^4\gamma^5(\Delta p/p)^3}{\eta j_e}\frac{A}{Z^2}, \tag{11}$$

For the case of Nuclotron booster we will assume that the length of the electron cooler is 3 m and that the electron density is $n_e = 3\cdot 10^{13}$ m^{-3}.

From (10), (11) one can receive that:

$$\tau_{\perp\mathrm{cool}} = \frac{0.18}{Z}, \qquad \tau_{\parallel\mathrm{cool}} = \frac{0.96}{Z}.$$

The diameter of the electron beam must be equal to the diameter of the ion beam, i.e., 3.5 cm.

6. INTENSITY LIMITS

The beam intensities that could be reached are limited not only by the injection process but also by the different kind of beam instabilities. Which phenomenon will prevail depends on both machine and beam parameters.

Generally speaking a charged particle in an accelerator interacts not only with the external guided, confining and accelerating electromagnetic fields but also with all the other beam particles. Provided the electromagnetic field of these neighbouring particles is strong enough, the actual particle motion will be noticeable modified. In principle there are both coherent and incoherent effects.

Also one should distinguished between the intensity limits set on a hot beam and those set on a beam after electron cooling.

The incoherent shifts of the betatron tunes caused by the beam space charge forces are between the major limiting factors at injection. If these shifts are large enough the working point will cross low-order resonance lines and the beam would be unstable.

One could write for the space charge limit due to the incoherent shift of the betatron tunes (the so-called Laslet tune shift):

$$\Delta Q = \frac{1}{FB_f}\frac{Rr_i}{\pi b(a+b)}\frac{N}{\beta^2\gamma^3 Q}, \tag{12}$$

where β and γ are the relativistic factors; a and b are the horizontal and vertical beam radii; $R = C/2\pi$ is the mean machine radius; B_f is the bunching factor (B_f is equal or smaller than one); r_i is the classical radius of the ion ($r_i = (q^2/A)1.544\cdot 10^{-18}$ m); F is a geometrical factor describing the image forces from the metallic wall of the vacuum pipe.

$$F = \left\{1 + b\frac{a+b}{h^2}\left[E_1(1+B_f(\gamma^2-1)) + E_2 C_m B_f(\gamma^2-1)\left(\frac{h}{g}\right)^2\right]\right\}^{-1} \tag{13}$$

with C_m the fraction of the machine circumference occupied by magnets; h is the half aperture of the vacuum chamber in vertical direction; g is the half gap between magnet poles; E_1 and E_2 are coefficients with values $E_2 \approx 0.206$ and E_1 according to Table 1.

Table 1.

w/h	1 (circle)	1.25	1.33	1.50	2.00	∞
E_1	0	0.090	0.107	0.134	0.172	0.206

For synchrotrons a value $\Delta Q_{\text{inc}} = 0.1{-}0.2$ is accepted while for storage rings $\Delta Q_{\text{inc}} = 0.05$ is used.

For the Nuclotron booster assuming $\Delta Q = 0.2$ one can calculate from (12), (13):

$$N = \frac{1.1}{Z} 10^{12}.$$

When the beam undergoes small coherent deformations, an electromagnetic field is induced in the enclosed vacuum chamber, which in turn acts back on the beam. Above a threshold intensity, the interaction leads to growing coherent oscillations. The particle motion becomes unstable.

In the longitudinal direction the coherent deformation consists in beam density fluctuations. A sufficient momentum spread in the beam can stabilize the longitudinal instability through the mechanism of the Landau damping.

The stability limit is given by [11–13]:

$$\left| \frac{Z_{\|}}{p} \right| \leqslant \frac{m_0 c^2}{e} \frac{\gamma \beta^2 \eta}{(q/A) I} \left(\frac{\Delta p}{p} \right)^2_{\text{FWHM}}, \tag{14}$$

where $\eta = \Delta\omega/w/\Delta p/p$; I is the beam current; q and A are the charge and mass of the ion; m_0 and e are the proton rest mass and charge; $Z_{\|}$ is the longitudinal coupling impedance; p is the harmonic number ($\omega = p\omega_0$) and FWHM stands for «full wide at half maximum».

We will limit ourselves to the rough model of a cylindrical beam with radius «a» travelling in a cylindrical vacuum chamber with radius «b» with perfectly conducting walls. For this approximation the longitudinal coupling impedance, the so-called «longitudinal space charge impedance» is given by:

$$\frac{Z_{\|\text{SC}}}{p} = -j \frac{Z_0 g}{2\beta\gamma^2}, \tag{15}$$

where $g = 1 + 2\lg(b/a)$ and $Z_0 = \mu_0 c = 377\Omega$ is the impedance of the free space.

One can receive from (14), (15) that for multiturn and stripping injection (with zero dispersion at the foil) the threshold intensity is $3.5 \cdot 10^{11}/Z$ ions. The estimation of the growing rate of the instability shows that just at the threshold it is very high, while for $N = 10^{12}/Z$ ions it is of the order of several ms, i.e., is much higher than the storage time. For RF stacking (14), (15) give a threshold of $3.5 \cdot 10^{13}/Z$ ions.

In the case of transverse instability the perturbation consists of a small transverse displacement of the beam as a whole. We look for solutions of the form of travelling waves $\sim \exp(n\theta - \omega t)$, θ being the machine azimuth and ω — the coherent instability frequency.

For a beam with momentum spread the instability threshold is given by:

$$\frac{q}{A} I \leqslant \frac{4Q(m_0 c^2/e)\gamma \Delta\omega_{\text{FWHM}}}{c|Z_{\perp}|}, \tag{16}$$

where $Z_\perp$ is the transverse coupling impedance and $\Delta\omega_{\mathrm{FWHM}} = \omega_0[(p - Q)\eta + +Q\xi](\Delta p/p)_{\mathrm{FWHM}}$, ξ being the chromaticity.

For the above used rough model of the accelerator with cylindrical beam and cylindrical vacuum chamber with perfectly conducting walls the coupling impedance, the so-called «transverse space charge impedance», is given by:

$$Z_{\perp\mathrm{SC}} = -j\frac{RZ_0}{\beta^2\gamma^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right). \tag{17}$$

From formulae (16), (17) we could calculate that the threshold for multiturn injection and for stripping injection (with zero dispersion at the foil) is equal to $1.4 \cdot 10^{10}/Z$ ions while for RF stacking this threshold is $N \leqslant 1.4 \cdot 10^{11}/Z$ ions. The growing time of the transverse instability is however much longer than the storage time.

Let's turn to the instabilities of the beams after performing electron cooling.

The cooling force, which reduces the beam emittance and momentum spread during the electron cooling, also counteracts the onset of the collective oscillations. The formulae (14), (16) for the threshold intensities are no longer valid.

One can calculate the threshold from the condition that the growth time of the instability must be equal to the beam cooling time. The former time must be taken for a monochromatic beam.

The cooling times are given in chapter 5.

The growth time for monochromatic beam in longitudinal direction is given by:

$$\tau_\parallel = \frac{\omega_0}{2}\left(\frac{(q/A)I\eta Z_{\parallel\mathrm{RW}}}{\pi\dfrac{m_0c^2}{e}\gamma\beta^2}\right)^{1/2}, \tag{18}$$

where $Z_{\parallel\mathrm{RW}}$ is the longitudinal coupling impedance of a resistive vacuum chamber.

The growth time for monochromatic beam in transverse direction is given by:

$$\tau_\perp = \frac{4\pi\dfrac{m_0c^2}{e}\gamma}{(q/A)Ic\,\mathrm{Re}\,(Z_\perp)}, \tag{19}$$

where $Z_\perp$ is the transverse coupling impedance.

Comparing the growth times (18), (19) with the cooling times given in chapter 5 we can calculate that the transverse instability threshold is $N = 2.8 \cdot 10^{12}$, while the longitudinal instability threshold is $N/Z = 8 \cdot 10^{12}$.

As for the Laslett tune shift of the cooled beam a shift $\Delta Q = 0.2$ will be reached with intensity $N = 1.7 \cdot 10^{10}/Z$ ions.

CONCLUSIONS

First of all let's make a brief comment of the injection methods discussed so far.

1. *Multiturn injection.* It requires small emittance of the injected beam and large acceptance of the accelerator. Typical figures are $\varepsilon \sim 5\pi$ mm · mrad and $A \sim 200\pi$ mm · mrad.

An efficiency $\varepsilon_2 \sim 0.5$ could be reached. The stored intensity could be increased by means of painting in both horizontal and vertical planes. This is done by firing a skew quadrupole in order to produce a X-Y coupling. Such a painting scheme successfully works in the AGS booster.

2. *RF stacking.* Generally speaking β_x at injection point should be small to have small beam size. Small dispersion at injection point increases the overlapping of the successive portions in the stack but simultaneously requires higher values of V sin φ_s in the stacking cavity and higher RF power to be applied. As for the equilibrium phase φ_s the smaller $\Gamma = \sin \varphi_s$ the higher the stacking efficiency. On the other hand, the speed of the stacking is proportional to Γ, so a compromise is necessary. Provided the injection emittance is small enough, it is more efficient to combine RF stacking with the multiturn injection. Between the disadvantages of the method is that it puts a large momentum spread to the beam and that it is time consuming.

3. *Stripping injection.* Only light ions can be injected by means of this method. The higher particle energy the wider the range of ions that could be stored. The adverse effects of elastic scattering and ionization energy losses in the target strongly decrease with the projectile energy. In practice fully stripped ions are stored (CELSIUS). Small emittance will allow for efficient painting to be realized.

4. *Injection with electron cooling.* The method has the disadvantage to be very slow — injection times of several seconds or even higher.

Going back to the case of Nuclotron booster we have summarized the features of the different injection methods in Table 2.

Table 2. **Comparison between methods of injection into Nuclotron booster**

	Multiturn injection	RF stacking with single turn injection	Stripping injection	Injection with electron cooling
Injected ions	all	all	$Z \leqslant 14$	all
Injected turns	8–10	8	120	100
Effective turns	3–4	8	73 for Li_7^{3+}	100
Time of injection	27 μs	80 ms	324 μs	$(0.96/Z)x$ injected turns, s

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Received on July 17, 2001.