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In the first part of the review we discuss the effective nonlocal approach in the quantum field theory. It concerns primarily the historical retrospective of this approach, and then we concentrate on the interaction of matter particles (fermions and bosons) with the (Abelian and non-Abelian) gauge fields. In the second part of the review we consider the hadronic corrections (vacuum polarization) to the anomalous magnetic moment of the muon $g-2$ factor discussed within the $SU_f(2)$ nonlocal chiral quark model. This is considered in the leading and, partially, in the next-to-leading orders (the effect of the fermion propagator dressing due to pion field) of expansion in small parameter $1/N_c$ (N_c is the number of colors in QCD).

В первой части обзора мы обсуждаем эффективный нелокальный подход в квантовой теории поля. Изложена история этого подхода, и дан вывод взаимодействия частиц материи (фермионов и бозонов) с (абелевыми и неабелевыми) калибровочными полями. Во второй части обзора мы рассматриваем адронные вклады (вакуумной поляризации) к аномальному магнитному моменту мюона $g-2$ в $SU_f(2)$ нелокальной киральной кварковой модели. Эти вклады рассмотрены в лидирующем и, частично, в следующем за лидирующим порядках (эффект одевания фермионного пропагатора пионным полем) разложения по малому параметру $1/N_c$ (N_c — число цветов в КХД).

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INTRODUCTION

The Lepton Anomalous Magnetic Moment. The quantum mechanics predicts the gyromagnetic ratio g for the charged point-like fermions with spin $1/2$ equal to 2. In relativistic quantum theory this fact is direct consequence of the

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Dirac equation. From the quantum field theory (quantum electrodynamics (QED) in that time) formulated by R. Feynman, J. Schwinger, and S. Tomonaga, there follows the existence of virtual particles, leading to the so-called vacuum polarization effects. The most famous examples of these effects are the Lamb shift in the hydrogen atom levels and the appearance of the anomalous magnetic moment of the electron. These effects were predicted and confirmed experimentally almost at the same time.

In QED the general form of the interaction vertex of fermions (with incoming and outgoing momenta p and p', correspondingly) with photon of momentum $q = p' - p$ reads as (see, e.g., [1]):

$$\Gamma^{\mu}(p,p') = \gamma_{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2), \quad p'^2 = p'^2 = m^2, \tag{1}$$

where F_1 and F_2 are the Dirac and Pauli form factors, respectively, $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})$. At tree level for the charged point-like fermions, one has $F_1 = 1$, $F_2 = 0$. In QED it is possible to get the relation between the form factors $F_1(0) = 1$, $F_2(0)$ and the gyromagnetic ratio g,

$$g = 2\,[F_1(0) + F_2(0)] = 2 + 2F_2(0). \tag{2}$$

Thus, a new quantity, the anomalous magnetic moment (AMM) $a = F_2(0) = (g-2)/2$, appears. In quantum field theory $a \neq 0$ due to internal structure of fermions emergent from the virtual radiative corrections.

The AMM of the leptons (electron or muon) is one of the most accurately measured and theoretically studied quantities in the elementary particle physics. The interest to this problem is motivated by our wish to understand the most delicate features of our microworld at its boundary and extension beyond the modern knowledge. The simple rule [2, 3] is that the effect of the second-order contribution to AMM of the lepton a_l with mass m_l due to a possible particle exchange of mass M is proportional to

$$a_l \propto (m_l/M)^2. \tag{3}$$

Thus, sensitivity of the muon to hypothetical interaction with the scale M is 40000 times higher than that of the electron. This fact compensates a less experimental accuracy of the measurements of the muon AMM and makes this study more perspective from the point of view of search of new physics.

Recent experiment E821 at the Brookhaven National Laboratory (BNL, USA) got the muon AMM with very high precision [4]:

$$a_{\mu}^{\exp} = 659208.0(6.3) \cdot 10^{-10}. \tag{4}$$

In the near future it is planed to increase the experimental accuracy by a factor of 4 in new experiments at FermiLab (USA) [5] and JPARC (Japan) [6]. The standard model predicts the number

$$a_\mu^{\text{theory}} = 659179.0(6.5)\cdot 10^{-10} \tag{5}$$

that differs from the experiment by 3–4 standard deviations (depending on theoretical estimates of different groups).

Hadronic Contribution. The theoretical error in (5) is dominated by the contribution of the strong interaction to the muon AMM (Fig. 1). In the leading order (LO) in the fine coupling constant α this is the hadronic vacuum polarization (HVP) contribution, and in the next-to-leading order (NLO) this is the iterations of the HVP and the contribution from the light-by-light (LbL) process.

The HVP contribution to a_μ is given by the expression

$$a_\mu^{\text{HVP}} = \frac{\alpha}{2\pi}\int\limits_0^1 dx\frac{(1-x)(2-x)}{x}D\left(\frac{m_\mu^2 x^2}{1-x}\right), \tag{6}$$

where m_μ is the muon mass, $D(Q^2)$ is the Adler function defined as the logarithmic derivative of the photon polarization operator

$$D(Q^2) = \frac{\partial\Pi(Q^2)}{\partial\ln(Q^2)}. \tag{7}$$

From Eq. (6) it is clear that a_μ^{HVP} is determined by the behavior of the Adler function in the low-momentum region, of order of the muon mass m_μ. The simplest expression for the relation between the AMM and the Adler function at zero momentum obtained in [7] is

$$a_\mu^{\text{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 m_\mu^2\frac{4\pi^2}{3}\left[\frac{D(Q^2)}{Q^2}\right]_{Q^2\to 0}. \tag{8}$$

Later on, new estimates were made in different model approaches: the nonlocal constituent quark models [8, 9], the Dyson–Schwinger approach [10], the local constituent chiral quark model [11], and some others.

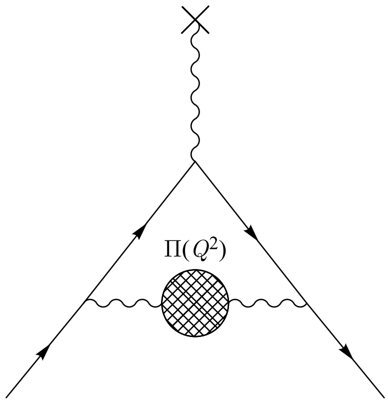


Fig. 1. The diagram for the hadronic vacuum polarization contribution to the muon AMM. The muon is the solid line, the photons are the wavy lines, and the external wavy line is the interaction of the muon with the magnetic photon with momentum $q\to 0$. The hatched circle is for the HVP

In this work, we consider some details of the nonlocal chiral quark model that allows one to interpolate the chiral physics from low to large momenta.

The Nonlocal Chiral Quark Model (NχQM). The partially bosonized action of the $SU(2) \times SU(2)$ nonlocal chiral quark model (NχQM) is written as [12]:

$$S = \int d^4x \Bigg\{ \bar{q}(x) \left(i\hat{\partial} - m_c + \hat{V}(x) + \hat{A}(x)\gamma_5 \right) q(x) - \\ - \frac{1}{2G_1} \left(\pi^a(x)^2 + \sigma(x)^2 \right) + \frac{1}{2G_2} \left(\rho^{\mu a}(x)^2 + a_1^{\mu a}(x)^2 \right) + \\ + \sum_{\Phi_i = \sigma,\pi,\rho,\omega,a_1} \Phi_i(x) \int d^4x_1\, d^4x_2\, f(x_1)\, f(x_2) \bar{Q}(x - x_1, x)\, \Gamma_i Q(x, x + x_2) \Bigg\}, \quad (9)$$

with $q(x) = \{u(x), d(x)\}$ being the fermion fields of the u and d quarks, $Q(x,y)$ — the corresponding guaged quark fields

$$Q(x,y) = P \exp \left\{ -i \int\limits_x^y dz^\mu (V_\mu^a(z) + A_\mu^a(z)\gamma_5) T^a \right\} q(y), \\ \bar{Q}(x,y) = \bar{q}(x) P \exp \left\{ -i \int\limits_x^y dz^\mu (V_\mu^a(z) - A_\mu^a(z)\gamma_5) T^a \right\}, \quad (10)$$

$V_\mu^a(z)$ and $A_\mu^a(z)$ are external vector and axial-vector fields, respectively (the notation is $\hat{V} = V^\mu \gamma_\mu = V_\mu^a \gamma^\mu T^a$); T^a represents the generators of the flavor group; P is the ordering operator of T^a along the integration path in every term of the Taylor series of the exponent; m_c is the current quark mass; σ, π, ρ, ω, a_1 are the boson fields for the mesons. G_1 and G_2 are the coupling constants determined by experimental input for the masses and other low-energy properties of the light mesons. $f(x)$ is the form factor of the nonlocal interaction with the characteristic nonlocality parameter Λ. The spin-flavor matrices Γ_i for mesons are given by

$$\Gamma_\sigma = 1, \quad \Gamma_\pi^a = i\gamma_5\tau^a, \quad \Gamma_\rho^{\mu a} = \gamma^\mu \tau^a, \quad \Gamma_\omega^\mu = \gamma^\mu, \quad \Gamma_{a_1}^{\mu a} = \gamma_5 \gamma^\mu \tau^a, \quad (11)$$

where τ^a stands for the Pauli matrices.

The action (9) contains the gauge-invariant interaction of quarks and mesons with external fields. At small momenta the action takes into account the nonperturbative structure of the strong interaction, as it follows from the instanton liquid model or the Dyson–Schwinger approach. The nonlocal action interpolates the physics of low momenta to the region of large momenta, where the nonlocality disappears and one has only free current quarks. Note that due to the Schwinger path-ordered exponent factor (10), the action (9) generates the contact interaction of quarks and mesons with any number of photons (see discussion in the next Section).

The scalar field σ has nonzero vacuum average σ_0. Thus, if the external fields are switched off (A, $V=0$), the effective action (9) may be integrated in quark fields. The action obtained has a nontrivial extremum derived by variation of the action in the σ field and equating the variation to zero. Such a condition is the so-called gap equation that has the nontrivial solution. In momentum space the gap equation reads

$$m(p) = m_c + 4iG_1 N_f N_c f^2(p) \int \frac{d^4k}{(2\pi)^4} \frac{f^2(k)m(k)}{k^2 - m^2(k)}, \tag{12}$$

where N_c and N_f are the number of colors and flavors, respectively. The nontrivial solution of the gap equation (12) means spontaneous violation of the chiral symmetry. The pion becomes massless in the chiral limit $m_c = 0$, and the current quarks become the dynamical quarks with the (inverse) propagator given by

$$S^{-1}(p) = \hat{p} - M(p). \tag{13}$$

The solution of the gap equation (12) is

$$m(p) = m_c - \sigma_0 f^2(p) = m_c + (m(0) - m_c) f^2(p) = m_c + m_d f^2(p), \tag{14}$$

where m_d is the dynamical (constituent) quark mass.

The meson propagators are the solutions of the Bethe–Salpeter equations [13]:

$$\begin{gathered} D_{\sigma,\pi}(p) = \frac{1}{J_{\sigma,\pi}(p) - G_1^{-1}}, \\ J_{\sigma,\pi}(p) = i \int \frac{d^4k}{(2\pi)^4} f^2(k_-) f^2(k_+) \,\mathrm{Tr}\left[S(k_-)\Gamma_{\sigma\pi} S(k_+)\Gamma_{\sigma,\pi}\right], \end{gathered} \tag{15}$$

where $k_- = k - p/2$, $k_+ = k + p/2$. The meson mass with particular quantum numbers corresponds to the pole of the meson propagator $J_{\sigma,\pi}(M_{\sigma,\pi}) = G_1^{-1}$.

The vector meson propagators have the transverse and longitudinal parts

$$D^{\mu\nu}_{\omega,\rho,a_1}(p) = T^{\mu\nu} D^T_{\omega,\rho,a_1}(p) + L^{\mu\nu} D_{\omega,\rho,a_1}(p), \tag{16}$$

where the transverse and longitudinal projectors are introduced by

$$T^{\mu\nu} = g^{\mu\nu} - p^\mu p^\nu/p^2, \quad L^{\mu\nu} = p^\mu p^\nu/p^2, \tag{17}$$

$$D^T_{\omega,\rho,a_1}(p) = \frac{1}{G_2^{-1} + J^T_{\omega,\rho,a_1}(p)}, \quad D^L_{\omega,\rho,a_1} = \frac{1}{G_2^{-1} + J^L_{\omega,\rho,a_1}(p)}, \tag{18}$$

with $J^T_{\omega,\rho,a_1}(p)$ and $J^L_{\omega,\rho,a_1}(p)$ being the transverse and longitudinal parts of the quark loop

$$J^{\mu\nu}_{\omega,\rho,a_1}(p) = i\int \frac{d^4k}{(2\pi)^4} f(k_-)^2 f(k_+)^2 \,\mathrm{Tr}\left[S(k_-)\Gamma^{\mu}_{\omega,\rho,a_1} S(k_+)\Gamma^{\nu}_{\omega,\rho,a_1}\right]. \quad (19)$$

At physical values of masses for the vector mesons, one has the pole condition

$$J^T_{\omega,\rho,a_1}(M_{\omega,\rho,a_1}) = -G_2^{-1}. \quad (20)$$

Also, note that within the nonlocal model, one has the mixing of the pseudoscalar and axial-vector fields due to the additional nondiagonal quark loop transition

$$J^{\mu}_{\pi,a_1}(p) = i\int \frac{d^4k}{(2\pi)^4} f^2(k_-) f^2(k_+) \,\mathrm{Tr}\left[S(k_-)\Gamma_{\pi} S(k_+)\Gamma^{\mu}_{a_1}\right]. \quad (21)$$

Formally, the models have two small parameters: the chiral parameter ratio $M(0)/4\pi f_\pi$ and the inverse of color number $1/N_c$. The color counting rules reduce to the following: the quark loop gives the factor N_c, and each meson propagator is suppressed by $1/N_c$ factor.

The $SU(2)$ model contains five parameters: the current quark mass m_c, the dynamical quark mass m_d, the nonlocality parameter Λ, and the scalar G_1 and vector G_2 coupling constants. The gap equation (12) relates the parameters m_c, m_d, Λ, and G_1 with each other. The couplings G_1 and G_2 are fitted by physical masses of the pion and ρ meson, respectively. One more parameter is fixed, e.g., by the pion decay $\pi^0 \to \gamma\gamma$. Thus, one parameter remains free and may be varied.

1. THE MODEL OF MINIMAL ELECTROMAGNETIC INTERACTION IN THE NONLOCAL MODELS

1.1. The Kroll Construction and the Kazes–Chang–Mani Identities. The Feynman rules for interaction of photons with quarks and the Goldstone bosons are known since 1991 from the work by J. Terning [14]. To derive these rules, he used the effective gauge-invariant action similar to that written above (9). The properties of the Schwinger exponent were crucial for this derivation, and specific path-independent formalism was used. The similar results were obtained earlier by K. Ohta for the photon interaction with extended nucleon [15]. The Terning rules were also rederived, e.g., in [16] for the nonlocal quark model and in [17] for unparticle model, basing on the nonlocal action and using similar to Terning's methods. The aim of this Section is to give alternative method of derivation based on the work by N. Kroll [18] published in 1966. The value of this derivation is the fact that only the form of the quark propagator (or quark–pion vertex) within the nonlocal model is needed. The Kroll's work is based on the question: if one modifies the fermion propagator, then how to get the gauged fermion–photon vertex? This question was raised in some previous works [19–21]. Other important results on gauging of nonlocal models were obtained in [10, 22, 23] and some other works.

It is important that all nonlocal models have formally the same general structure of the fermion propagator

$$S_G^{-1}(q) = A(q)\hat{q} - B(q). \tag{22}$$

In order to get the minimal interaction, we shall use the operator formalism based on the Kroll construction [18]. First, we replace the momentum variable q_μ by four noncommuting operator variables z_μ and introduce the shift operator $U_k = \mathrm{e}^{-k_\mu \partial^\mu}$, acting as

$$U_k^{-1} z_\mu U_k = z_\mu + k_\mu. \tag{23}$$

Next, we define formally the operator d_μ^k on the functions F of variable z

$$n_\mu d_\mu^k F(z) = \lim_{\varepsilon \to 0} U_k^{-1} \left[\frac{F(z + \varepsilon n U_k) - F(z)}{\varepsilon} \right], \tag{24}$$

where n_μ is a unit four-vector. Note that after all d_μ^k operations are performed, one needs to return the operator-valued quantity z to the momentum variable q.

The important properties following from the definition (24) are

$$d_\mu^k F(z)\, G(z) = [d_\mu^k F(z)]\, G(z) + F(z+k)\, d_\mu^k G(z), \tag{25}$$
$$d_\mu^k F^{-1}(z) = -F^{-1}(z+k)[d_\mu^k F(z)]\, F^{-1}(z), \tag{26}$$
$$d_\mu^k F(z)\, G(z) \neq d_\mu^k G(z)\, F(z). \tag{27}$$

The last property is due to noncommutativity of variables involved. Let us prove the first relation. By definition one has

$$n_\mu\, d_\mu^k [F(z)\, G(z)] = \lim_{\varepsilon \to 0} U_k^{-1} \left[\frac{F(z + \varepsilon n U_k)\, G(z + \varepsilon n U_k) - F(z)\, G(z)}{i\varepsilon} \right].$$

It is clear for infinitesimal shifts that

$$F(z + \varepsilon n U_k)\, G(z + \varepsilon n U_k) = \\ = F(z + \varepsilon n U_k)\, G(z) + F(z) G(z + \varepsilon n U_k) - F(z)\, G(z).$$

Then, the numerator transforms as

$$U_k^{-1}\left[F(z+\varepsilon nU_k)\,G(z)+F(z)\,G(z+\varepsilon nU_k)\;-2F(z)\,G(z)\right]=$$
$$=U_k^{-1}\left[F(z+\varepsilon nU_k)\,G(z)-F(z)\,G(z)+F(z)\,G(z+\varepsilon nU_k)-F(z)\,G(z)\right]=$$
$$=U_k^{-1}\left[F(z+\varepsilon nU_k)\;-F(z)\right]G(z)+U_k^{-1}F(z)\left[G(z+\varepsilon nU_k)\;-G(z)\right]=$$
$$=\left\{U_k^{-1}\left[F(z+\varepsilon nU_k)\;-F(z)\right]\right\}G(z)+$$
$$+\,F(z+k)\left\{U_k^{-1}\left[G(z+\varepsilon nU_k)\;-G(z)\right]\right\},$$

and Eq. (25) follows. The second property is proved analogously.

The minimal electromagnetic interaction of the fermion with propagator (22) with n photons is defined recursively

$$V_{\mu_1,\ldots,\mu_n}(z;k_1,\ldots,k_n)=d_{\mu_n}^{k_n}V_{\mu_1,\ldots,\mu_{n-1}}(z;k_1,\ldots,k_{n-1}),\qquad V_\mu(z,k)=d_\mu^k S_G^{-1}(z),\tag{28}$$

where $k_1,\ldots,k_n$ are the momenta of photons, S_G is the particle propagator. The vertices defined in (28) satisfy the Kazes–Chang–Mani identities (KCMI) (generalized Ward–Takahashi identities) [20, 21]:

$$k^{\mu_n}V_{\mu_1,\ldots,\mu_n}(q;k_1,\ldots,k_n)=$$
$$=V_{\mu_1,\ldots,\mu_{n-1}}(q+k_n;k_1,\ldots,k_{n-1})-V_{\mu_1,\ldots,\mu_{n-1}}(q;k_1,\ldots,k_{n-1})\tag{29}$$

or

$$V_{\mu_1,\ldots,\mu_{n-1},\lambda}(q;k_1,\ldots,k_{n-1},0)=\frac{\partial}{\partial q_\lambda}V_{\mu_1,\ldots,\mu_{n-1}}(q;k_1,\ldots,k_{n-1}).\tag{30}$$

This construction reproduces the standard rules for the local interaction of n photons with charged particle (fermion or scalar). Note also that very similar procedure was used later by H. Haberzettl in [24], where a “gauge derivative” was introduced with exactly the same properties as they followed from the properties of the operator d_μ^k.

1.2. The Application of the Kroll Construction to Derivation of n-Photon Vertices. Let us consider the action of the operator d_μ^k to the simplest but important cases, the powers of variable z. One gets

$$\begin{aligned}
&d_\mu^k z_\nu=U_k^{-1}\partial_\mu z_\nu U_k=g_{\mu\nu},\\
&d_\mu^k z^2=d_\mu^k z_\nu z^\nu=z_\mu+(z+k)_\mu=(2z+k)_\mu,\\
&d_\mu^k z^4=(2z+k)_\mu\left[(z+k)^2+z^2\right]=(2z+k)_\mu\frac{(z+k)^4-z^4}{(z+k)^2-z^2},\\
&d_\mu^k z^{2n}=(2z+k)_\mu\frac{(z+k)^{2n}-z^{2n}}{(z+k)^2-z^2}.
\end{aligned}\tag{31}$$

Note that at this point we do not fully coincide with the original Kroll's work. He considered the variables z as the Dirac matrix value $2z_\mu = \hat{z}\gamma_\mu + \gamma_\mu\hat{z}$. With this prescription we get, e.g., $d_\mu^k z^2 = d_\mu^k z_\nu\gamma_\nu z^\rho\gamma_\rho = 2z_\mu + \hat{k}\gamma_\mu$. (As we see below, another important prescription that we add to the Kroll construction is symmetrization of the factors depending on z. This prescription leads to commutativity of factors in Eq. (26).) Moreover, in [18] (see also [25]) only local propagators were considered as examples.

From (31) it follows that the action of d_μ^k on the functions that have the expansion $F(z) = \sum c_n z^{2n}$ reduces to

$$d_\mu^k F(z) = (2z+k)_\mu \frac{F(z+k) - F(z)}{(z+k)^2 - z^2}. \qquad (32)$$

The important example of this relation is the exponential function

$$\begin{aligned} d_\mu^k\, \mathrm{e}^{-\alpha z^2} &= (2z+k)_\mu \frac{\mathrm{e}^{-\alpha(z+k)^2} - \mathrm{e}^{-\alpha z^2}}{(z+k)^2 - z^2} = \\ &= (2z+k)_\mu(-\alpha)\int\limits_0^1 dt \exp\left[-\alpha\left(t(z+k)^2 + \bar{t}z^2\right)\right], \qquad (33) \end{aligned}$$

where $\bar{t} = 1 - t$.

The one-photon vertex is obtained immediately from (32). The only subtle point is that the action of $d_\mu(k)$ on $\hat{z}A(z)$ and $A(z)\hat{z}$ produces different results, both satisfying the KCMI and thus being gauge-invariant. To obtain physically correct result, we need to consider the symmetrized expression:

$$\begin{aligned} d_\mu^k \frac{A(z)\hat{z} + \hat{z}A(z)}{2} &= \frac{(2z+k)_\mu}{2}\frac{A(z+k) - A(z)}{(z+k)^2 - z^2}\hat{z} + \frac{A(z+k)}{2}\gamma_\mu + \\ &+ \frac{A(z)}{2}\gamma_\mu + \frac{\hat{z}+\hat{k}}{2}(2z+k)_\mu \frac{A(z+k) - A(z)}{(z+k)^2 - z^2} = \\ &= \frac{A(z) + A(z+k)}{2}\gamma_\mu + \frac{2\hat{z}+\hat{k}}{2}(2z+k)_\mu \frac{A(z+k) - A(z)}{(z+k)^2 - z^2}. \qquad (34) \end{aligned}$$

This corresponds to the kinetic part of the propagator. The scalar part generates the vertex by simple substitution $F \to B$ in (32).

The final result for the one-photon vertex in the minimal approach becomes

$$\begin{aligned} \Gamma_\mu^G(q,k) &= \frac{A(q) + A(q+k)}{2}\gamma_\mu + \\ &+ \frac{2\hat{q}+\hat{k}}{2}(2q+k)_\mu \frac{A(q+k) - A(q)}{(q+k)^2 - q^2} - (2q+k)_\mu \frac{B(q+k) - B(q)}{(q+k)^2 - q^2}, \qquad (35) \end{aligned}$$

where k is the photon momentum, q is the momentum of incoming quark. This result agrees with [14, 22]. It is easy to check that this vertex satisfies the Ward–Takahashi identity

$$k^\mu \Gamma_\mu^G(q,k) = (\hat{k}+\hat{q})A(k+q) - M(k+q) - \hat{k}A(k) + M(k) = S_G^{-1}(k+q) - S_G^{-1}(k). \tag{36}$$

1.3. Derivation of Two-Photon Vertex. To obtain the two-photon vertex in accordance with the Kroll construction, we need to calculate $d_\nu^{k'} d_\mu^k F(z)$. At this step the ordering of factors becomes important (see Fig. 2). This is one more prescription that one should add to the Kroll construction in order to treat the non-Abelian models. We find that the most convenient way is to use the α representation

$$F(z) = \int\limits_0^\infty d\alpha\, F(\alpha)\, \mathrm{e}^{-\alpha z^2}. \tag{37}$$

By using (33) we get the correct ordering

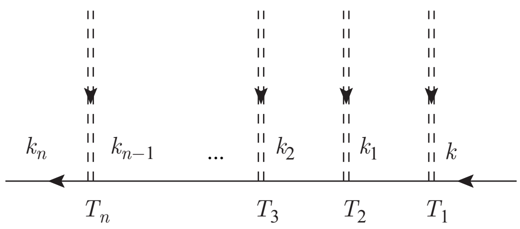
$$d_\mu^k F(z) = \int\limits_0^1 dt \int\limits_0^\infty d\alpha\, (-\alpha)\, F(\alpha)\, \mathrm{e}^{-\alpha t (z+k)^2} (2z+k)_\mu \mathrm{e}^{-\alpha \bar{t} z^2}. \tag{38}$$

This expression written in the α representation is equivalent to (32).

Acting on the integrand (38) by the second operator $d_\nu^{k'}$, we get

$$\begin{aligned}
&\left[d_\nu^{k'}\, \mathrm{e}^{-\alpha t (z+k)^2}\right] (2z+k)_\mu\, \mathrm{e}^{-\alpha \bar{t} z^2} + \mathrm{e}^{-\alpha t (z+k+k')^2} \left[d_\nu^{k'} (2z+k)_\mu\, \mathrm{e}^{-\alpha \bar{t} z^2}\right] = \\
&= (2z+2k+k')_\nu (-\alpha t) \int\limits_0^1 dt' \exp\left[-\alpha t \left(t'(z+k+k')^2 + \bar{t}'(z+k)^2\right)\right] \times \\
&\times (2z+k)_\mu\, \mathrm{e}^{-\alpha \bar{t} z^2} + \mathrm{e}^{-\alpha t (z+k+k')^2} \Big\{ \left[d_\nu^{k'} (2z+k)_\mu\right] \mathrm{e}^{-\alpha \bar{t} z^2} + \\
&+ (2z+2k'+k)_\mu d_\nu^{k'}\, \mathrm{e}^{-\alpha \bar{t} z^2} \Big\} = (2z+2k+k')_\nu (-\alpha t) \times \\
&\times \int\limits_0^1 dt' \exp\left[-\alpha t (t'(z+k+k')^2 + \bar{t}'(z+k)^2)\right] (2z+k)_\mu\, \mathrm{e}^{-\alpha \bar{t} z^2} + \\
&+ \mathrm{e}^{-\alpha t (z+k+k')^2} (2z+2k'+k)_\mu (2z+k')_\nu \times \\
&\times \int\limits_0^1 dt' (-\alpha \bar{t})\, \mathrm{e}^{-\alpha \bar{t}\left[t'(z+k')^2 + \bar{t}' z^2\right]} + 2 g_{\mu\nu}\, \mathrm{e}^{-\alpha t (z+k+k')^2 - \alpha \bar{t} z^2}.
\end{aligned} \tag{39}$$

Fig. 2. Figure shows that one needs to order the α_i parameters and the charge matrices T_i from the right (the initial state) to the left (the final state) in accordance with momentum flow. The double dashed lines are for photons or mesons, the solid line is for a quark



Now, integrating this expression with the weight (38), we obtain for the first term

$$\int_0^1 dt \int_0^\infty d\alpha\,(-\alpha)F(\alpha)(-\alpha t)\times$$
$$\times \int_0^1 dt' \exp\left[-\alpha t\left(t'(z+k+k')^2 + \bar{t}'(z+k)^2\right) - \alpha\bar{t}z^2\right] = \int_0^1 dt \times$$
$$\times \int_0^\infty d\alpha\,(-\alpha)F(\alpha) \int_0^1 dt' \frac{d}{dt'} \frac{\exp\left[-\alpha t\left(t'(z+k+k')^2 + \bar{t}'(z+k)^2\right) - \alpha\bar{t}z^2\right]}{(z+k+k')^2 - (z+k)^2} =$$
$$= \int_0^1 dt \int_0^\infty d\alpha(-\alpha)F(\alpha) \frac{\mathrm{e}^{-\alpha t(z+k+k')^2 - \alpha\bar{t}z^2} - \mathrm{e}^{-\alpha t(z+k)^2 - \alpha\bar{t}z^2}}{(z+k+k')^2 - (z+k)^2} =$$
$$= \frac{1}{(z+k+k')^2 - (z+k)^2}\left[\frac{F(z+k+k') - F(z)}{(z+k+k')^2 - z^2} - \frac{F(z+k) - F(z)}{(z+k)^2 - z^2}\right], \tag{40}$$

for the second term

$$\int_0^1 dt \int_0^\infty d\alpha\,(-\alpha)\,F(\alpha) \int_0^1 d\bar{t}'\,\mathrm{e}^{-\alpha t(z+k+k')^2}(-\alpha\bar{t})\,\mathrm{e}^{-\alpha\bar{t}\left[t'(z+k')^2 + \bar{t}'z^2\right]} =$$
$$= \int_0^1 dt \int_0^\infty d\alpha\,(-\alpha)\,F(\alpha)\,\mathrm{e}^{-\alpha t(z+k+k')^2} \frac{\mathrm{e}^{-\alpha\bar{t}(z+k')^2} - \mathrm{e}^{-\alpha\bar{t}z^2}}{(z+k')^2 - z^2} =$$
$$= \frac{1}{(z+k')^2 - z^2}\left[\frac{F(z+k+k') - F(z+k')}{(z+k+k')^2 - (z+k')^2} - \frac{F(z+k+k') - F(z)}{(z+k+k')^2 - z^2}\right], \tag{41}$$

and for the third term

$$\int_0^1 dt \int_0^\infty d\alpha\, F(\alpha)\, \mathrm{e}^{-\alpha t(z+k+k')^2-\alpha\bar{t}z^2} = \frac{F(z+k+k')-F(z)}{(z+k+k')^2-z^2}.$$

By using the notation for finite differences

$$F^{(1)}(k_1,k_2) = \frac{F(k_1)-F(k_2)}{k_1^2-k_2^2},$$
$$F^{(2)}(k_1,k_2,k_3) = \frac{F^{(1)}(k_1,k_3)-F^{(1)}(k_1,k_2)}{k_3^2-k_2^2}, \qquad (42)$$

the result becomes

$$d_\nu^{k'} d_\mu^k F(z) = g_{\mu\nu} F^{(1)}(z+k+k',z) + \\ + (2z+2k+k')_\nu (2z+k)_\mu F^{(2)}(z+k+k',z+k,z) + \\ + (\mu\leftrightarrow\nu, k\leftrightarrow k'). \qquad (43)$$

Let us transform identically the second term in (35) to

$$\frac{2\hat{z}+\hat{k}}{2}(2z+k)_\mu \frac{A(z+k)-A(z)}{(z+k)^2-z^2} \to \frac{2\hat{z}+\hat{k}}{4} d_\mu^k A(z) + \left[d_\mu^k A(z)\right]\frac{2\hat{z}+\hat{k}}{4}$$

and then act by $d_\nu^{k'}$

$$d_\nu^{k'}[(2\hat{z}+\hat{k})d_\mu^k A(z)] = 2\gamma_\nu [d_\mu^k A(z)] + (2\hat{z}+2\hat{k'}+\hat{k})\, d_\nu^{k'} d_\mu^k A(z),$$
$$d_\nu^{k'}\left[d_\mu^k A(z)\right](2\hat{z}+\hat{k}) = \left[d_\nu^{k'} d_\mu^k A(z)\right](2\hat{z}+\hat{k}) + 2 d_\mu^k A(z+k')\gamma_\nu. \qquad (44)$$

As a result, we get the two-photon vertex

$$\Gamma_{\mu\nu}^G(q,k,k') = \frac{\gamma_\mu}{2}(2q+k')_\nu A^{(1)}(q+k',q) + \\ + \frac{\gamma_\mu}{2}(2q+2k+k')_\nu A^{(1)}(q+k+k',q+k) + \\ + \frac{2\hat{q}+\hat{k}+\hat{k'}}{2} g_{\mu\nu} A^{(1)}(q+k+k',q) + (2q+2k+k')_\nu (2q+k)_\mu \times \\ \times A^{(2)}(q+k+k',q+k,q) - g_{\mu\nu} B^{(1)}(q+k+k',q) - \\ - (2q+2k+k')_\nu (2q+k)_\mu B^{(2)}(q+k+k',q+k,q) + (\mu\leftrightarrow\nu, k\leftrightarrow k'). \qquad (45)$$

It satisfies the KCMI

$$k'^\nu \Gamma_{\mu\nu}^G(q,k,k') = \Gamma_\mu^G(q+k',k) - \Gamma_\mu^G(q,k). \qquad (46)$$

If we put in (45) $A = 1$, the vertex agrees with that obtained in [14].

1.4. Derivation of the Quark–Pion–Photon Vertex. Let the quark momentum be k, the pion momentum be p, and the photon momentum be q. The quark–pion vertex in the Terning model [14] is

$$F_\pi^T(k,k+p) = \frac{g_{\pi q}}{M_q} i\gamma_5 \tau^a (M_k + M_{k+p}). \quad (47)$$

Then, by the Kroll construction, the quark–pion–photon vertex is trivially reproduced [14]:

$$d_\mu^q(M_k + M_{k+p}) = \tau^a Q(2k+q)_\mu \frac{M_{k+q} - M_k}{(k+q)^2 - k^2} + \\ + Q\tau^a \left(2(k+p)+q\right)_\mu \frac{M_{k+p+q} - M_{k+p}}{(k+p+q)^2 - (k+p)^2}. \quad (48)$$

In the instanton model the quark–pion vertex is

$$F_\pi^I(k,k+p) = \frac{g_{\pi q}}{M_q} i\gamma_5 \tau^a M_q f_{k+p} f_k. \quad (49)$$

The ordering of f is important! Then, by using the Kroll construction, one has

$$d_\mu^q(f_{k+p} f_k) = (d_\mu^q f_{k+p}) f_k + f_{k+p+q}(d_\mu^q f_k) = \\ = \tau^a Q f_{k+p+q}(2k+q)_\mu \frac{f_{k+q} - f_k}{(k+q)^2 - k^2} + \\ + Q\tau^a f_k \left(2(k+p)+q\right)_\mu \frac{f_{k+p+q} - f_{k+p}}{(k+p+q)^2 - (k+p)^2} = \\ = \tau^a Q(2k+q)_\mu f_{k+p+q} f^{(1)}(k;q) + Q\tau^a \left(2(k+p)+q\right)_\mu f_k f^{(1)}(k+p;q).$$

It corresponds to the previous work [26].

After studying these simple examples, we are able to consider the general quark–pion vertex

$$F_\pi^g(k,k+p) = \frac{g_{\pi q}}{M_q} i\gamma_5 \tau^a M_q G\left(k^2, p^2, (k+p)^2\right). \quad (50)$$

First, write formally the function G as the 3-variable α integral

$$G\left(k^2, p^2, (k+p)^2\right) = \int_0^\infty d\alpha\, d\beta\, d\gamma g(\alpha,\beta,\gamma)\, \mathrm{e}^{-\gamma(k+p)^2} \left[\mathrm{e}^{-\beta p^2}\, \mathrm{e}^{-\alpha k^2}\right]. \quad (51)$$

As in the previous example, the ordering of the exponents is important. Now, apply the operator d^q_μ to exponents and use (33):

$$
\begin{aligned}
d^q_\mu\, \mathrm{e}^{-\gamma(k+p)^2}\, \mathrm{e}^{-\beta p^2}\, \mathrm{e}^{-\alpha k^2} &= Q\lambda^a \left[d^q_\mu\, \mathrm{e}^{-\gamma(k+p)^2}\right] \mathrm{e}^{-\beta p^2}\, \mathrm{e}^{-\alpha k^2} + \\
&+ \frac{1}{2}(Q\lambda^a - \lambda^a Q)\, \mathrm{e}^{-\gamma(k+p+q)^2} \left[d^q_\mu\, \mathrm{e}^{-\beta p^2}\right] \mathrm{e}^{-\alpha k^2} + \\
&+ \lambda^a Q\, \mathrm{e}^{-\gamma(k+p+q)^2}\, \mathrm{e}^{-\beta p^2} \left[d^q_\mu\, \mathrm{e}^{-\alpha k^2}\right] = \\
= Q\lambda^a &\left[(2(k+p)+q)_\mu \int\limits_0^1 dt\,(-\gamma)\, \mathrm{e}^{-\gamma[t(k+p+q)^2+\bar{t}(k+p)^2]}\right] \mathrm{e}^{-\beta p^2}\, \mathrm{e}^{-\alpha k^2} + \\
+ \frac{1}{2}\,(Q\lambda^a - \lambda^a Q)\, \mathrm{e}^{-\gamma(k+p+q)^2} &\left[(2p+q)_\mu \int\limits_0^1 dt\,(-\beta)\, \mathrm{e}^{-\beta[t(p+q)^2+\bar{t}(p)^2]}\right] \mathrm{e}^{-\alpha k^2} + \\
+ \lambda^a Q\, \mathrm{e}^{-\gamma(k+p+q)^2}\, \mathrm{e}^{-\beta p^2} &\left[(2k+q)_\mu \int\limits_0^1 dt\,(-\alpha)\, \mathrm{e}^{-\alpha[t(k+q)^2+\bar{t}k^2]}\right].
\end{aligned}
$$

Put this back to (51), use (33) as a finite difference and return to the momentum representation. Then, we get the vertex in the form

$$
\begin{aligned}
&d^q_\mu G(k^2, p^2, (k+p)^2) = \\
&= Q\lambda^a(2(k+p)+q)_\mu \frac{G(k^2, p^2, (k+p+q)^2) - G(k^2, p^2, (k+p)^2)}{(k+p+q)^2 - (k+p)^2} + \\
&+ \frac{1}{2}\,(Q\lambda^a - \lambda^a Q)(2p+q)_\mu \frac{G(k^2, (p+q)^2, k'^2) - G(k^2, p^2, k'^2)}{(p+q)^2 - p^2} + \\
&+ \lambda^a Q(2k+q)_\mu \frac{G((k+q)^2, p^2, k'^2) - G(k^2, p^2, k'^2)}{(k+q)^2 - k^2}.
\end{aligned}
$$

This result agrees with that obtained by K. Ohta in [15], where the nucleons were considered as fermions. He applied this result to the study of the pion photoproduction. Note, the first term corresponds to the u-channel exchange, and the second and the third ones to t- and s-channel exchanges, respectively.

The quark–two-pion vertex in the Terning model [14] is

$$
\begin{aligned}
&F^T_\pi\,(k, k+p) = \\
&= -\left(\frac{g_{\pi q}}{M_q}\right)^2 \frac{\tau^{\{ab\}}}{2}\,(M_k + M_{k+p1} + M_{k+p2} + M_{k+p1+p2})\,. \qquad (52)
\end{aligned}
$$

Acting by the Kroll operator, we reproduce Eq. (33) in [14] up to isospin matrices.

2. THE LEADING CONTRIBUTION TO THE MUON AMM

2.1. The Polarization of Vacuum through the Quarks. In the leading order of expansion in the small parameter $1/N_c$, the quark vacuum polarization is given by the sum of two diagrams [9] (Fig. 3):

$$\Pi_{\mu\nu}(q) = iN_c \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}\{\Gamma_\mu(k_+,k_-)S(k_+)\Gamma_\nu(k_+,k_-)S(k_-)\} + \\ + iN_c \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}\{\Gamma_{\mu\nu}(k,q,-q)S(k)\}, \quad (53)$$

where the vertices $\Gamma_\mu(k_+,k_-)$ and $\Gamma_{\mu\nu}(k,q,-q)$ are defined in (35) and (45). Contracting (53) with transverse projector (17), one gets the polarization operator

$$\Pi(q^2) = \frac{4iN_c}{q^2} \int \frac{d^4k}{(2\pi)^4} \frac{\sum Q_i^2}{D_+D_-} \Big\{ m_+m_- - k_+k_- + \frac{2}{3}k_\perp^2 + \\ + \frac{4}{3}k_\perp^2 \Big[\big(m^{(1)}(k_+,k_-)\big)^2 (m_+m_- + k_+k_-) - \big(m^2(k_+,k_-)\big)^{(1)} \Big] \Big\} + \\ + \frac{8iN_c}{q^2} \sum Q_i^2 \int \frac{d^4k}{(2\pi)^4} \frac{m_k}{D_k} \left[m_k' + \frac{4}{3} m^{(2)}(k,k,k+q) \right], \quad (54)$$

$k_\perp = k - \dfrac{(kq)}{q^2}q$, $k_\perp^2 = k^2 - \dfrac{(kq)^2}{q^2}$, $D_k = k^2 - m_k^2$ is the denominator of quark propagator $S(k)$, and we introduce the notation $F_\pm = F(k_\pm)$.

2.2. The Contribution of Intermediate Vector Mesons. In addition, there is a contribution with intermediate states with quantum numbers of ρ_0 or ω mesons. The photon–meson vertex in Fig. 4, *a* is defined by a sum of two diagrams (Fig. 4, *b* and Fig. 4, *c*):

$$\Gamma^{\mu\nu}_{\gamma\to\rho_0,\omega}(q) = iN_c \int \frac{d^4k}{(2\pi)^4} \Big\{ \operatorname{Tr}\left[S_+\Gamma^\mu(k_+,k_-)S_-\Gamma^\nu_{\rho_0,\omega}(k_+,k_-)\right] + \\ + \operatorname{Tr}\left[\Gamma^{\mu\nu}_{\rho_0,\omega}(k,q,-q)S_k\right] \Big\}, \quad (55)$$

where the vertices are

$$\Gamma^\nu_{\rho_0,\omega}(k_+,k_-) = f(k_+)f(k_-)\gamma_n u T_{\rho_0,\omega} \quad (56)$$

and

$$\Gamma^{\mu\nu}_{\rho_0,\omega}(k,q,-q) = -e\gamma_\nu[f^{(1)}(k,k+q)f(k)(2k+q)^\mu Q T_{\rho_0,\omega} + \\ + f^{(1)}(k,k-q)f(k)(2k-q)^\mu T_{\rho_0,\omega} Q]. \quad (57)$$

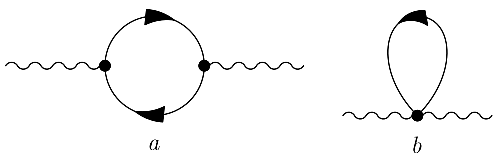


Fig. 3. Dispersion (*a*) and contact (*b*) diagrams

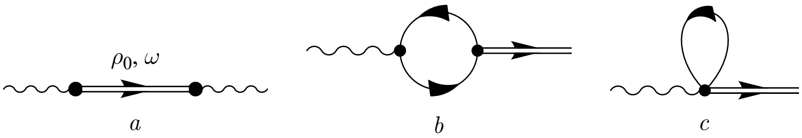


Fig. 4. The diagrams for the $\gamma - \rho$ interference

The vertex (55) has transverse structure

$$q_\mu \Gamma^{\mu\nu}_{\gamma\to\rho_0,\omega}(q) = 0, \tag{58}$$

and thus selects the transverse part of the meson propagator in the polarization operator

$$\Pi_v(q) = \frac{D^T_{\rho_0,\omega}(q)\Gamma^T_{\gamma\to\rho_0,\omega}(q^2)^2}{q^2}. \tag{59}$$

After simplifications one gets

$$\Pi_v(q) = \frac{1}{q^2}\frac{B_v^2(q)}{G_2^{-1} - J^T_{\rho_0,\omega}(q)}, \tag{60}$$

$$B_v(q) = 4iN_c C_{\rho_0,\omega}\int\frac{d^4k}{(2\pi)^4}\left[\frac{f_+f_-}{D_+D_-}\Bigg(m_+m_- - k_+k_- + \right.$$
$$\left. +\frac{2}{3}k_\perp^2\left[1-\left(m^2(k_+,k_-)\right)^{(1)}\right]\Bigg) - \frac{4}{3}k_\perp^2\frac{f_k}{D_k}f^{(1)}(k,k+q)\right], \tag{61}$$

$C_{\rho_0,\omega} = \mathrm{Tr}\,[(\tau^3+1/3)/2\cdot\tau^3] = 1$ for the ρ_0 meson and $\mathrm{Tr}\,[(\tau^3+1/3)/2\cdot 1] = 1/3$ for the ω meson. $C_{\rho_0,\omega}$ is a result of the trace in flavor. From that it is clear that the contribution to $F_2(0)$ from the ω meson is suppressed by a factor $1/9$ comparing with the ρ_0-meson contribution.

Note that $B_v(0) = 0$. After performing the Wick rotation in (61) one gets

$$B_v(0) \sim \int\limits_0^\infty dk^2 k^2 \left[\frac{f_k^2}{D_k^2} \left(m^2 + k^2 - \frac{1}{2}k^2 \left[1 + 2m_k m_k'\right] \right) + k^2 \frac{f_k}{D_k} f_k' \right]. \quad (62)$$

The third term under integral may be integrated by parts by using $D_k' = 1 + 2mm_k'$:

$$\int\limits_0^\infty dk^2 (k^2)^2 \frac{f_k f_k'}{D_k} = -\int\limits_0^\infty f_k^2 d\left(\frac{(k^2)^2}{2}\frac{1}{D_k}\right) =$$

$$= -\int\limits_0^\infty dk^2 \frac{k^2}{D_k} f_k^2 + \int\limits_0^\infty dk^2 f_k^2 \frac{(k^2)^2}{2} \frac{1+2mm_k'}{D_k^2}. \quad (63)$$

After substitution in (62) the integrand becomes identically zero.

2.3. Numerical Results. We choose the nonlocal form factors $f(p)$ in the Gaussian form

$$f(p) = \mathrm{e}^{-p^2/\Lambda^2}, \quad (64)$$

where Λ is a nonlocality parameter. The nonlocal form factors (64) not only reflect the nontrivial nature of QCD vacuum, but, at the same time, serve as regulators to make quark loop integrals finite in ultraviolet region.

The contribution to the muon AMM from the quark vacuum polarization is shown in Fig. 5, with the dynamical quark mass being varied in the interval from 150 to 400 MeV.

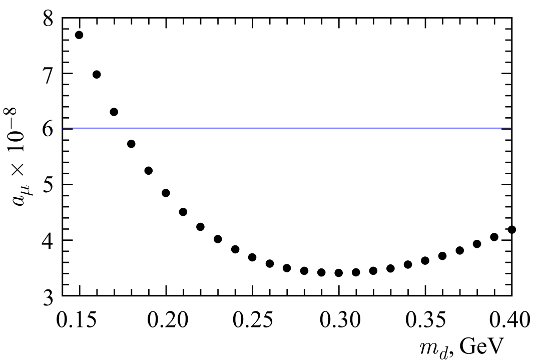


Fig. 5. The solid line is the contribution to a_μ of the quark polarization vacuum, calculated in terms of the spectral function obtained in the experiment [27, 28]

Table 1. **The model parameters and the values of contribution to the muon AMM of vector mesons in the leading in $1/N_c$ order**

m_c, MeV	m_d, MeV	Λ, GeV	G_1, GeV^{-2}	G_2, GeV^{-2}	$a_{\rho_0} \cdot 10^9$	$a_\omega \cdot 10^{10}$
7.6	300	1.04	32.7	−4.27	3.1	3.44
8.2	310	0.99	37.1	−5.23	3.43	3.81
8.8	320	0.95	41.8	−5.81	3.44	3.82
9.4	330	0.91	46.9	−6.14	3.27	3.63
10	340	0.87	52.4	−6.19	2.98	3.31
11	350	0.84	58.2	−5.87	2.55	2.84

The contribution of the vector mesons depends on the coupling G_2 (Table 1), the value of which is determined from (20). The corresponding integral becomes divergent in time-like region when the effective quark mass becomes $m_q \lesssim m_{\rho_0,\omega}/2$. It means unphysical creation of quark–antiquark pair. This problem may be solved by several ways. One way is to introduce the infrared cut-off for the loop integral. In this work, we use another method when the quark propagator is replaced by the so-called "confining" propagator, which does not have poles in physical time-like region. In this case, the dynamical quark mass $m(p)$ is defined by

$$m(p) = \sqrt{\frac{m_c^2 + p^2 \exp\left(-(p^2+m_c^2)/\Lambda^2\right)}{1-\exp\left(-(p^2+m_c^2)/\Lambda^2\right)}} \quad (p^2 \text{ is in the Euclidean metric}), \tag{65}$$

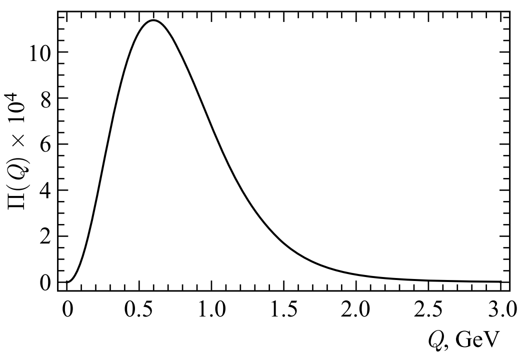


Fig. 6. The polarization operator (60) for the ρ_0 meson with $m_c = 8.8$ MeV, $m_d = 20$ MeV, $\Lambda = 0.95$ MeV, $G_2 = -5.81$ GeV^{-2}

and we have condition $m_d = \Lambda$. For some set of parameters it is possible to get the finite result in the more simple case

$$m(p) = m_c + m_d f^2(p),$$
$$m_d = 4G_1 N_c N_f \int \frac{d^4k}{(2\pi)^4} \frac{m(k) f^2(k)}{k^2 + m(k)^2}. \tag{66}$$

The muon AMM is calculated from the expression

$$a_\mu = 4\alpha^2 \sum Q_i^2 \int_0^1 dx(1-x)\Pi\left(\frac{m_\mu^2 x^2}{1-x}\right). \tag{67}$$

The function $B(q)$ is determined by the transition vertex $\gamma \to \rho$. Its behavior for one set of parameters is shown in Fig. 6.

3. NEXT-TO-LEADING IN $1/N_c$ CORRECTION TO THE QUARK VACUUM POLARIZATION AND TO THE MUON AMM

3.1. Next-to-Leading in $1/N_c$ Correction to the Quark Vacuum Polarization. Correction to the photon propagator in the leading in α order for the quark propagator of general form $S_G^{-1}(p) = A(p)\hat{p} - B(p)$ is defined by the sum of two diagrams shown in Fig. 7:

$$\Pi_{\mu\nu}(q) = iN_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\{\Gamma_\mu^G(k_+, k_-) S_G(k_+) \Gamma_\nu^G(k_+, k_-) S_G(k_-)\} + \\ + iN_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\{\Gamma_{\mu\nu}^G(k, q, -q) S_G(k)\}, \tag{68}$$

where the vertices $\Gamma_\mu^G(k_+, k_-)$ and $\Gamma_{\mu\nu}^G(k, q, -q)$ are defined in (35) and (45).

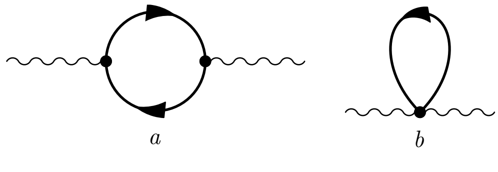


Fig. 7. Dispersion (*a*) and contact (*b*) diagrams. The thick line is for dressed quark propagator

After contraction with the transverse projector, one gets the polarization operator

$$\Pi(q^2) = i\frac{N_c}{q^2}\sum_i Q_i^2\times$$

$$\times\int\frac{d^4k}{(2\pi)^4}\frac{1}{D_+^G D_-^G}\Bigg\{(A_+ + A_-)^2(B_+B_- - (k_+k_-)A_+A_-) + \frac{8}{3}k_\perp^2 A_+^2 A_-^2 +$$

$$+\frac{8}{3}k_\perp^2\left[(A^2(k_+,k_-))^{(1)}(A_+A_-[2k^2-(k_+k_-)]-B_+B_-) - 2\frac{B_+^2A_-^2-B_-^2A_+^2}{k_+^2-k_-^2}\right]+$$

$$+\frac{16}{3}k_\perp^2\Bigg[A_+A_-(k_+k_-)\left(B^{(1)}(k_+,k_-)^2 - k^2A^{(1)}(k_+,k_-)^2\right)+$$

$$+2k^2A^{(1)}(k_+,k_-)\left(A_+A_-k^2A^{(1)}(k_+,k_-) - \frac{A_-B_+^2 - A_+B_-^2}{k_+^2-k_-^2}\right)+$$

$$+B_+B_-\left(B^{(1)}(k_+,k_-)^2 - k^2A^{(1)}(k_+,k_-)^2\right)\Bigg]\Bigg\}+$$

$$+\frac{8iN_c}{q^2}\sum_i Q_i^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{D_k^G}\Bigg[\frac{2}{3}k_\perp^2A_kA^{(1)}(k,k+q) + k^2A_kA_k'+$$

$$+\frac{4}{3}k^2k_\perp^2A_kA^{(2)}(k,k+q,k) - B_kB_k' - \frac{4}{3}k_\perp^2B_kB^{(2)}(k,k+q,k)\Bigg], \quad (69)$$

where $D_\pm^G$ are defined as

$$S_G(p) = \frac{1}{\hat{p}-m_p-\Sigma_p} = \frac{A_p\hat{p}+B_p}{A_p^2p^2-B_p^2} = \frac{A_p\hat{p}+B_p}{D_p^G}, \quad (70)$$

the functions A_p and B_p in terms of $F_v(p)$ and $F_s(p)$ read

$$A_p = 1+F_v(p), \quad B_p = m_p + F_s(p). \quad (71)$$

The expansion of (69) at small q^2 in the Euclidean metric starts from zero power in q^2. The expansion of the Adler function starts from the first power in q^2, because the coefficient at $(q^2)^{-1}$ in expansion of (69) becomes zero under condition that M_k' and A_k' decrease rather fast (exponentially for our model) at infinity. Indeed, the corresponding integral reduces to the integral of a total divergence

$$\sum_i Q_i^2\frac{N_c}{8\pi^2}\int\limits_0^\infty dk^2\,k^2A_k\frac{k^2A_k^3+2k^2B_k^2A_k'+2A_kB_k(B_k-k^2B_k')}{(k^2A_k^2+B_k^2)^2} =$$

$$= \sum_i Q_i^2\frac{N_c}{8\pi^2}\frac{k^4A_k^2}{k^2A_k^2+B_k^2}\Bigg|_0^\infty, \quad (72)$$

which is zero after subtraction of the local contribution

$$\sum_i Q_i^2 \frac{N_c}{8\pi^2} \frac{k^4}{k^2+m_c^2}\Bigg|_0^\infty . \qquad (73)$$

3.2. Quark Self-Energy. The NLO corrections to the quark self-energy were obtained in [29]. By summing diagrams of Fig. 8, one gets

$$\Sigma_p = F_s(p) - F_v(p)\hat{p}, \qquad (74)$$

where $F_v(p)$ and $F_s(p)$ depend on p^2 as

$$F_s(p) = i \sum_{M=\sigma,\pi} f_p^2 \int \frac{d^4l}{(2\pi)^4} \frac{f_{p-l}^2}{D_l^M} \frac{\pm m_{p-l}}{D_{p-l}} - \frac{f_p^2}{D_0^\sigma} \sum_{M=\sigma,\pi} i \int \frac{d^4l}{(2\pi)^4} \frac{1}{D_l^M} \times$$

$$\times 4iN_cN_f \int \frac{d^4k}{(2\pi)^4} \frac{f_k^2 f_{k+l}^4}{D_p D_{p+l}^2} \left[2k(k+l)m_{k+l} \pm m_k(m_{k+l}^2 + (k+l)^2)\right], \qquad (75)$$

$$F_v(p) = -if_p^2 \sum_{M=\sigma,\pi} \int \frac{d^4l}{(2\pi)^4} \frac{f_{p-l}^2}{D_l^M} \frac{1-(pl)/p^2}{D_{p-l}}.$$

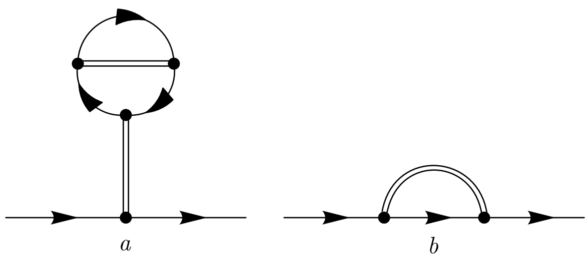


Fig. 8. $1/N_c$ corrections to the quark self-energy

3.3. Numerical Results. The quark propagator may be parameterized in terms of the wave function renormalization Z and the mass function M

$$S_G(p) = \frac{Z(p)}{\hat{p} - M(p)}, \quad Z(p) = A^{-1}(p), \quad M(p) = \frac{B(p)}{A(p)}. \qquad (76)$$

The functions Z and M are calculated by using (75), (71) and are shown in Fig. 9 for one set of parameters. For qualitative comparison, in this figure we also show the result of lattice calculations. Note, however, that both calculations are

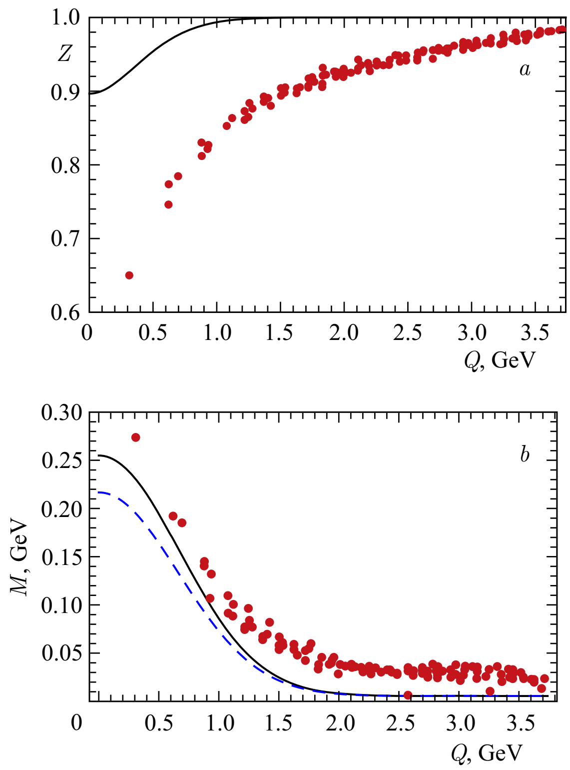


Fig. 9. The wave function renormalization Z (a) and the mass function M (b). In both figures the points are the data for lattice calculations in the Landau gauge for $m_c =$ 29 MeV [30]. In plot b, the solid line is for the mass function $M(p) = B(p)/A(p)$, the dashed line is for the dynamical quark mass in the leading in $1/N_c$ order of expansion (the solution of the gap equation). The figures are drawn for the set of parameters $m_c = 5.58$ MeV, $m_d = 211$ MeV, $\Lambda = 1.32$ GeV

performed in different gauges. The model calculations correspond to the Fock–Schwinger gauge, while the lattice calculations correspond to the Landau gauge.

From Fig. 9, a it is clear that at large momenta ($p \gtrsim \Lambda$) the wave function renormalization Z goes to unit. At small p the nonperturbative QCD effects become important and Z deviates significantly from canonical normalization of the quark field. From Fig. 9, b we see that the mass function at large p tends to the current quark mass $m_c \propto 1$ MeV, while at $p = 0$ its value is of order of several hundreds MeV, depending on model parameters.

Next, the expression (69) written in the Euclidean metric is used to calculate the contribution of the quark self-energy in NLO approximation to the muon $g-2$. Now, the model parameters are determined with taking into account the

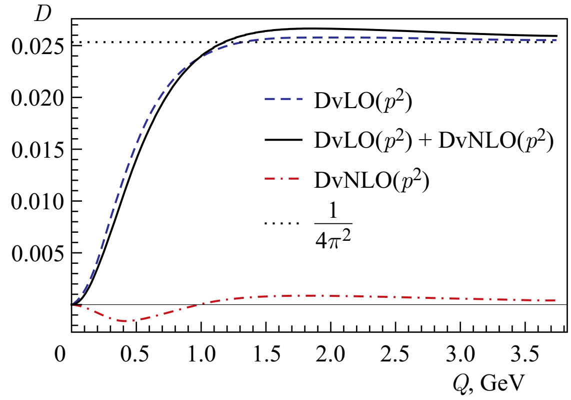


Fig. 10. The Adler function in the LO and NLO in $1/N_c$ expansion. At large p the Adler function converges to the QCD prediction $N_c/12\pi^2$

Table 2. **The model parameters and the contributions of the quark vacuum polarization to the muon AMM in the LO and NLO in $1/N_c$ expansion**

m_c, MeV	m_d, MeV	Λ, GeV	$a_{\rm LO} \cdot 10^8$	$a_{\rm LO+NLO} \cdot 10^8$
2.82	139.2	2.1	8.89	5.23
5.58	211.2	1.32	4.83	3.81
8.64	269.1	1	3.96	3.51
9.38	281.9	0.95	3.9	3.62
11.78	322.5	0.82	3.94	3.58
18.15	424	0.63	4.91	4.67

NLO corrections to the quark propagator (70). For example, G_1 is fixed by physical pion mass. The Adler function with dressing effects is shown in Fig. 10.

As is clear from Fig. 10, the NLO contribution at low Q is negative. This is due to additional terms in the NLO contact diagram (Fig. 7, *b*) in comparison with the LO contact diagram. Thus, the NLO corrections to the quark vacuum polarization lead to diminishing the muon AMM, since its value is dominated by the low momenta behavior of the Adler function. This fact is reflected numerically in the last two columns of Table 2.

4. REVIEW OF THE HVP CONTRIBUTION TO THE MUON AMM WITHIN SOME OTHER MODELS

4.1. The Maris–Tandy (MT) Model. P. Maris and P. Tandy suggested simple but phenomenologically successive approximation for the quark–gluon interaction [31]. By using this model, Ch. Fischer and coauthors [10] calculated the polarization operator and the Adler function for the photon self-energy with dressing by gluons of the quark propagator and the quark–photon vertex.

The quark propagator within the MT model is determined as a solution of the Dyson–Schwinger equation

$$S(p)^{-1} = iZ_2\hat{p} + Z_4 m(\mu) + \\ + Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p), \quad (77)$$

where $D_{\mu\nu}(k)$ is the renormalized dressed gluon propagator; $\Gamma^a_\nu(q,p)$ is the renormalized dressed quark–gluon vertex; Z_1, Z_2, Z_4 are the renormalization constants; and $\int^{\Lambda}$ is the translation invariant regularization of the integral by means of the cut-off parameter Λ.

After removing the regularization, the solution of Eq. (77) has the general form

$$S(p)^{-1} = i\hat{p}A(p^2,\mu^2) + B(p^2,\mu^2), \quad (78)$$

with the renormalization condition

$$S(p)^{-1}|_{p^2=\mu^2} = i\hat{p} + m(\mu). \quad (79)$$

The authors of [10] use the gluon propagator in the Landau gauge

$$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{Z(k^2)}{k^2} \quad \text{(Euclidean metric)} \quad (80)$$

and the dressed quark–gluon vertex $\Gamma_\mu(p,q)$ in the Dirac form $\Gamma_\mu(k^2) = \gamma_\mu \Gamma^{\rm YM}(k^2)$, where the vertex depends only on a gluon momentum squared k^2. Within this model the product of the renormalization functions of the gluon propagator and the quark–gluon vertex is chosen as

$$Z(k^2)\Gamma^{\rm YM}(k^2) = \frac{4\pi}{g^2}\left(\frac{\pi}{\omega^6} D k^4 \, {\rm e}^{-k^2/\omega^2} + \right. \\ \left. + \frac{2\pi\gamma_m}{\ln(\tau + (1 + k^2/\Lambda^2_{\rm QCD})^2)} \left[1 - {\rm e}^{-k^2/(4m_\tau^2)}\right]\right), \quad (81)$$

where $m_\tau = 0.5$ GeV, $\tau = {\rm e}^2 - 1$, $\gamma_m = 12/(33 - 2N_f)$, $\Lambda_{\rm QCD} = 0.234$ GeV, $\omega = 0.4$ GeV, and $D = 0.93\ {\rm GeV}^2$.

The dressed quark–gluon vertex is a solution of the Bethe–Salpeter equation

$$\Gamma_\mu(P,k) = Z_2\gamma_\mu + \\ + \frac{4}{3} g^2 Z_2^2 \int \frac{d^4q}{(2\pi)^4} \left[\gamma^\alpha S(q_-) \Gamma_\mu(P,q) S(q_+) \gamma^\beta\right] D_{\alpha\beta}(q-k) \Gamma^{\rm YM}(q-k). \quad (82)$$

Then, the polarization operator in the leading order is given by

$$\Pi_{\mu\nu}(P) = Z_2 \int \frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\,[S(q_-)\Gamma_\mu(P,q)S(q_+)\gamma_\nu]\,, \tag{83}$$

which is logarithmically divergent. It is regularized by the standard manner

$$\Pi_R(P^2) = \Pi(P^2) - \Pi(0).$$

With all this, the leading in $1/N_c$ contributions to the muon $g-2$ are calculated in [10] for two sets of parameters (see Table 3). The results of calculations are shown in Fig. 11.

Table 3. **Two sets of parameters at normalization point $\mu^2 = 19$ GeV^2**

$m_{u,d}$, MeV	m_s, MeV	m_π, MeV	m_K, MeV	$m_{\rho,\omega}$, MeV	m_ϕ, MeV	$a_\mu \cdot 10^{10}$
3.7	85	138	495	740	1080	744
11	72	240	477	770	1020	676

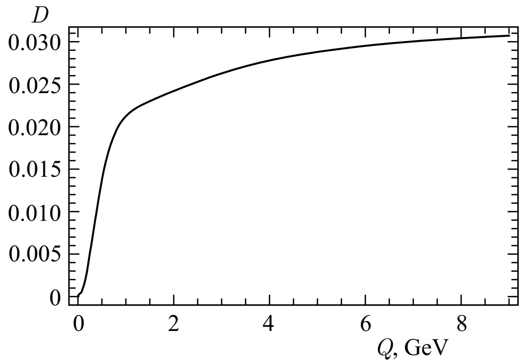


Fig. 11. The Adler function in the MT model. There is a specific for this model "hump" in vicinity of $Q = 1$ GeV

4.2. The Effective NJL Model. In 1993, E. de Rafael made an estimate of the low-energy contribution to the muon AMM and showed that within the Nambu–Jona-Lasinio model it is possible also to estimate the NLO in $1/N_c$ contribution [7].

Because the renormalized polarization operator satisfies $\Pi_R^H(0) = 0$, then in the chiral perturbation theory the leading contribution to a_μ cooresponds to the $O(p^6)$ term

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} - P_1\,\mathrm{e}^2\partial^\lambda F^{\mu\nu}\partial_\lambda F_{\mu\nu} + \ldots\right), \tag{84}$$

where

$$P_1 = -\left.\frac{\partial \Pi_R^H(Q^2)}{\partial Q^2}\right|_{Q^2=0}. \tag{85}$$

In the chiral limit, the hadron contribution to a_μ in the leading in $1/N_c$ order is given by

$$a_\mu \simeq \left(\frac{\alpha}{\pi}\right)^2 m_\mu^2 \frac{4}{3}\pi^2 P_1. \tag{86}$$

Thus, to make an estimate, one needs to know the constant P_1.

To obtain the low-momentum behavior of the two-point correlation function $\Pi_R^H(Q)_{\mu\nu}$, E. de Rafael takes as a model the extended NJL model (ENJL) [32], which is quite good for momenta in the region $Q \lesssim \Lambda_\chi$, where Λ_χ is a scale of spontaneous breaking of the chiral symmetry

$$\mathcal{L}_{\rm QCD} \to \mathcal{L}_{\rm QCD}^{\Lambda_\chi} + \mathcal{L}_{\rm NJL}^{S,\,P} + \mathcal{L}_{\rm NJL}^{V,\,A} + O\left(\frac{1}{\Lambda_\chi^4}\right), \tag{87}$$

where

$$\begin{aligned} \mathcal{L}_{\rm NJL}^{V,\,A}(x) &= \frac{8\pi^2 G_V(\Lambda_\chi)}{N_c\Lambda_\chi^2}\sum_{a,b}\left[(\bar{q}_L^a\gamma^\mu q_L^b)(\bar{q}_L^b\gamma_\mu q_L^a) + (L\to R)\right], \\ \mathcal{L}_{\rm NJL}^{S,\,P}(x) &= -\frac{8\pi^2 G_S(\Lambda_\chi)}{N_c\Lambda_\chi^2}\sum_{a,b}(\bar{q}_R^a q_L^b)(\bar{q}_L^b q_R^a). \end{aligned} \tag{88}$$

By using (88), the polarization operator in the leading in $1/N_c$ order is

$$\begin{aligned} \Pi_V^{(1)} &= \frac{\overline{\Pi}_V^{(1)}(Q^2)}{1 + Q^2(8\pi^2 G_V/N_c\Lambda_\chi^2)\overline{\Pi}_V^{(1)}(Q^2)}, \\ \overline{\Pi}_V^{(1)}(Q^2) &= \frac{N_c}{16\pi^2} 8\int_0^1 dy\, y(1-y)\Gamma\left(0, \frac{M_Q^2 + Q^2 y(1-y)}{\Lambda_\chi^2}\right), \end{aligned} \tag{89}$$

where $\Gamma(n,\varepsilon) = \int\limits_\varepsilon^\infty \frac{dz}{z}\,{\rm e}^{-z} z^n$ is incomplete gamma function. Then, the constant P_1 reads

$$P_1^{\rm ENJL} = \frac{N_c}{16\pi^2}\frac{2}{3}\frac{1}{M_Q^2}\frac{4}{15}\left[\Gamma\left(1, \frac{M_Q^2}{\Lambda_\chi^2}\right) + \frac{5}{4}\frac{1-g_A}{g_A}\right], \tag{90}$$

where

$$g_A = \frac{1}{1 + 4G_V\dfrac{M_Q^2}{\Lambda_\chi^2}\Gamma\left(0, \dfrac{M_Q^2}{\Lambda_\chi^2}\right)}$$

is the axial-vector constant.

The result for a_μ, for model parameters $M_Q = 265$ MeV, $\Lambda_\chi = 1165$ MeV, $g_A = 0.61$ obtained in [33], is

$$a_\mu = 6.7 \cdot 10^{-8}, \tag{91}$$

where Eq. (86) is used for calculation.

4.3. Nonlocal Models. The nonlocal model for two kinds of nonlocal currents with an action

$$S_E = \int d^4x \left[\overline{\psi}(x)(-i\hat{\partial} + m_c)\,\psi(x) - \frac{G_S}{2} j_a(x) j_a(x) \right] \tag{92}$$

was considered in [34]. In the first case, the effective action is considered within the instanton liquid model, and the nonlocal current reads

$$j_a(x) = \int d^4y\, d^4z r(y-x) r(x-z)\,\overline{\psi}(y)\,\Gamma_a \psi(z), \quad \Gamma_a = (1,\, i\gamma_5 \boldsymbol{\tau}). \tag{93}$$

In the second case, the nonlocal current is due to effective one-gluon exchange in the separable approximation

$$j_a(x) = \int d^4z\, g(z) \overline{\psi}\left(x + \frac{z}{2}\right) \Gamma_a \psi\left(x - \frac{z}{2}\right). \tag{94}$$

As we have shown before, considering the Kroll construction, the quark–photon vertices are the same for both cases, and thus the expression for the polarization operator (54) is valid for both cases. However, the sets of model parameters are different, in particular, because the expressions for the pion decay constant f_π do not coincide.

Let us use Eq. (54) to calculate the contribution of the quark vacuum polarization to a_μ for two sets of parameters corresponding to different currents (Table 4).

Table 4. **The model parameters (in MeV, except for dimensionless $G_S\Lambda^2$) and the quark vacuum polarization to $a_\mu \cdot 10^{10}$**

$-\langle \bar{q}q \rangle^{1/3}$	Form factor	Case 1					Case 2				
		m_c	m_d	Λ	$G_S\Lambda^2$	a_μ	m_c	m_d	Λ	$G_S\Lambda^2$	a_μ
200	G	9.7	318	651.9	18.82	357	9.8	1356	459.7	71.11	544
	L2	9.7	296	539.9	12.45	333	—	—	—	—	—
220	G	7.4	282	772	16.98	355	7.4	620	604	29.06	288
	L2	7.4	259	642.2	10.98	338	—	—	—	—	—
240	G	5.8	255	902.4	15.82	374	5.8	424	752.2	20.65	247
	L2	5.8	233	751.8	10.14	372	5.8	475	586.8	16.06	242
260	G	4.6	235	1042.2	15.08	412	4.6	339	903.4	17.53	261
	L2	4.6	216	868	9.61	412	4.6	330	736.1	11.77	234

In [34], two kinds of form factors are used. One is of the Gaussian type

$$g_G(p^2) = \left[r_G(p^2)\right]^2 = \exp\left(\frac{-p^2}{\Lambda^2}\right), \tag{95}$$

and another is of the n-Lorentzian type

$$g_{\rm Ln}(p^2) = \left[r_{\rm Ln}(p^2)\right]^2 = \frac{1}{1+(p^2/\Lambda^2)^n}, \quad n \geqslant 2. \tag{96}$$

Somewhat different nonlocal model is suggested in [35] with the effective action

$$S_E = \int d^4x \left\{ \overline{\psi}(x)(-i\hat{\partial} + m_c)\psi(x) - \frac{G_S}{2}\left[j_a(x)j_a(x) - j_P(x)j_P(x)\right]\right\} \tag{97}$$

and the nonlocal currents

$$\begin{aligned} j_a(x) &= \int d^4z\, g(z)\, \overline{\psi}\left(x+\frac{z}{2}\right)\Gamma_a\psi\left(x-\frac{z}{2}\right), \\ j_P(x) &= \int d^4z\, f(z)\, \overline{\psi}\left(x+\frac{z}{2}\right)\frac{i\overleftrightarrow{\hat{\partial}}}{2\varkappa_p}\psi\left(x-\frac{z}{2}\right). \end{aligned} \tag{98}$$

Due to the current with derivative $j_P(x)$, the renormalization of the wave function appears in the leading in $1/N_c$ order, and the quark propagator becomes

$$\begin{aligned} S(p) &= \frac{Z(p)}{-\hat{p}+M(p)}, \; Z(p) = (1-\bar{\sigma}_2 f(p))^{-1}, \\ M(p) &= Z(p)\left[m_c + \bar{\sigma}_1 g(p)\right]. \end{aligned} \tag{99}$$

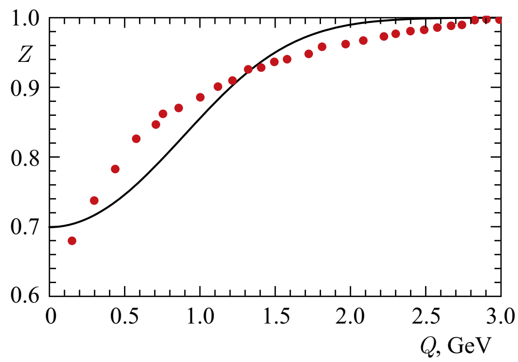


Fig. 12. A fit of the function $Z(p)$ for the form factors (100). The model parameters are $m_c = 5.7$ MeV, $\bar{\sigma}_1 = 529$ MeV, $\Lambda_0 = 814.42$ MeV, $\Lambda_1 = 1034.5$ MeV, $\bar{\sigma}_2 = -0.43$

By using the Gaussian form factors

$$g(p) = \exp\left(\frac{-p^2}{\Lambda_0^2}\right),\ f(p) = \exp\left(\frac{-p^2}{\Lambda_1^2}\right), \tag{100}$$

it is possible to calculate the quark vacuum polarization contribution to a_μ.

In [35], a choice of the model parameters $\bar{\sigma}_1$, $\bar{\sigma}_2$, Λ_0, Λ_1, m_c is obtained by fitting the function $Z(p)$ to the data of the lattice calculations [36] with condition that $Z(0) = 0.7$ (Fig. 12).

The result for a_μ for this model by using (69) is

$$a_\mu = 218 \cdot 10^{-10}. \tag{101}$$

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