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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

INVESTIGATION OF THE SEMILEPTONIC WEAK DECAYS OF $B_{(s)}$ TO S (SCALAR MESON) VIA LIGHT-CONE QCD SUM RULES

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In this paper, the semileptonic weak decay process of $B_{(s)} \to S$ scalar mesons is investigated by using the light-cone QCD sum rules (LCSR) in the nonperturbative part. The corresponding transition matrix elements leading to form factors and the branching ratio of this process are determined.

Представлено исследование полулептонных слабых распадов $B_{(s)}$ в S (скалярные мезоны) с помощью правил сумм КХД на световом конусе. Вычисляются элементы матрицы перехода, из которых можно получить формфакторы, и соотношение разветвления.

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INTRODUCTION

For several consecutive decades, the structure of scalar mesons has been investigated. It was assumed that the scalar mesons with the masses greater than 1 GeV, such as $f_0(1370)$, can form the nonet based upon their spectrum, and those with masses less than 1 GeV, such as $a_0(980)$, $K_0^*(800)$, $f_0(980)$, and $f_0(600)$, form another nonet.

Investigation of strong and electromagnetic decays of scalar mesons, apart from the production properties of scalar mesons in πN scattering, $p\bar{p}$ annihilation, $\gamma\gamma$ formation, and heavy meson decays, has been the subject of interest in literature. Belle and BaBar investigated the scalar meson decays in the mass range of 1.0 and 1.5 GeV experimentally [1,2].

We would like to investigate the semileptonic decay of $B_{(s)}$ to scalar meson (S) in order to calculate the form factors and the branching ratio of this decay process.

The QCD sum rules method, which is based on relativistic quantum field theory, has been successful in calculation of form factors in the perturbative part, but in the nonperturbative part, namely for large momentum transfers or in the decay of scalar mesons with heavy masses, confronts difficulties. Correlation of the standard QCD sum rules (QCDSR) and hard exclusive processes theory leads to the light cone QCD sum rules (LCSR) in which such difficulties are overcome by performing the operator product expansion in terms of twists rather than dimensions [3].

Instead of using the vaccum condensation, which is used in QCDSR, in LCSR, the hadronic distribution amplitudes are used. The light cone distribution amplitudes are nonperturbative functions used to describe the hadronic structure.

This paper is organized as follows. In Sec. 1, the effective Hamiltonian responsible for the transition of $b \to u, s$ in the Standard Model, matrix elements parameterizations, and relations between the form factors and the heavy quarks in the large recoil regions are discussed. In Sec. 2, the distribution amplitudes of twist-2 and twist-3 obtained via QCDSR are given for scalar mesons. Then LCSR method is used for the determination of the form factors. The numerical calculation of the form factors and branching ratios as well as their application in decays of $B_s \to K_0^*(1430)l\bar{\nu}_l$ and $B_s \to K_0^*(1450)l\bar{\nu}_l$, and also leptonic longitudinal asymmetry are discussed in Secs. 3 and 4. In the last section, discussion of results and a conclusion are presented.

1. EFFECTIVE HAMILTONIAN AND PARAMETERIZATIONS OF MATRIX ELEMENT

In order to find the effective Hamiltonian for transition of $b \to u$, we integrate out the particles, such as top quark, $W^{\pm}$ and Z bosons above the scale of $\mu = O(m_b)$, and we get

$$\mathcal{H}_{\text{eff}}(b \to ul\bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub}\bar{u}\gamma_\mu(1-\gamma_5)b\,\bar{l}\gamma^\mu(1-\gamma_5)\nu_l + \text{h.c.}, \tag{1}$$

where V_{ub} denotes CKM matrix elements, and $l = (e, \mu, \tau)$. Similarly, the effective Hamiltonian for flavor-changing neutral current (FCNC) for $b \to s$ transition is obtained as follows:

$$\mathcal{H}_{\text{eff}}(b \to sl\bar{l}) = \frac{G_F}{\sqrt{2}}\, V_{tb}V_{ts}^*\Big[C_9^{\text{eff}}(\mu)\,\bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{l}\gamma^\mu(1-\gamma_5)\,l + \\ + C_{10}\bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{l}\gamma^\mu\gamma_5 l - \frac{2m_bC_7(\mu)}{q^2}\sigma_{\mu\nu}(1-\gamma_5)q^\nu b\,\bar{l}\gamma^\mu l\Big] + \text{h.c.}, \tag{2}$$

where the term $V_{ub}V_{us}^*$ is neglected because $|V_{ub}V_{us}^*/V_{tb}V_{ts}^*| < 0.02$. C_i denotes the Wilson coefficients given in [4]. Wilson coefficient C_{10} does not depend on scale $\mu \simeq O(m_b)$, and $C_9^{\text{eff}}(\mu)$ is defined as follows ([5–11]):

$$C_9^{\text{eff}}(\mu) = C_9(\mu) + Y_{\text{SD}}(z, s') + Y_{\text{LD}}(z, s'), \tag{3}$$

where $z = m_c/m_b$, $s' = q^2/m_b^2$, and $Y_{\text{SD}}(z, s')$ describes the short-distance contributions from four-quark operators far away from the $c\bar{c}$ resonance and is precisely determined by perturbative QCD. The long-distance contributions $Y_{\text{LD}}(z, s')$ from four-quark operators near $c\bar{c}$ are parameterized in the form of a phenomenological Breit–Wigner formula, and due to the lack of sufficient data, is not considered. Therefore, we have [12]:

$$Y_{\text{SD}}(z, s') = h(z, s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + \\ + C_6(\mu)) - \frac{1}{2}h(1, s')(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) - \\ - \frac{1}{2}h(0, s')(C_3(\mu) + 3C_4(\mu)) + \frac{2}{9}(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)), \tag{4}$$

where

$$h(z,s') = -\frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x) \mid 1-x \mid^{1/2} \times \\ \times \begin{cases} \ln \mid \frac{\sqrt{1-x}+1}{\sqrt{1-x}-1} \mid - i\pi \text{ for } x \equiv \frac{4z^2}{s' < 1}, \\ 2 \arctan \frac{1}{\sqrt{x-1}} \text{ for } x \equiv \frac{4z^2}{s' > 1}, \end{cases} \tag{5}$$

$$h(0,s') = \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s' + \frac{4}{9}i\pi. \tag{6}$$

In addition to that, the charm quark presence causes more corrections in the radiative transition $b \longrightarrow s\gamma$, which can be absorbed into C_7^{eff} and is given by [13]

$$C_7^{\text{eff}}(\mu) = C_7(\mu) + C'_{b\to s\gamma}(\mu), \tag{7}$$

where

$$C'_{b\to s\gamma}(\mu) = i\alpha_s\left[\frac{2}{9}\eta^{\frac{14}{23}}(G_1(x_t) - 0.1687) - 0.03C_2(\mu)\right], \tag{8}$$

and

$$C'_{b\to\gamma}G_1(x_t) = \frac{x(x^2-5x-2)}{8(x-1)^3} + \frac{3x^2\ln{}^2x}{4(x-1)^4}, \tag{9}$$

where $x_t = m_t^2/m_W^2$ and $\eta = \alpha_s(m_W)/\alpha_s(\mu)$. As can be seen from Eqs. (1) and (2), it is obvious that in order to calculate decay amplitudes for the semileptonic decays of $B_{q'} \to S$, the following hadronic matrix elements should be determined:

$$\langle S(p)|\bar{s}\gamma_\mu\gamma_5|B_{q'}(p+q)\rangle \text{ and } \langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B_{q'}(p+q)\rangle. \tag{10}$$

These two matrix elements are parameterized in the following form:

$$\begin{aligned} \langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B_{q'}(p+q)\rangle &= -i[f_+(q^2)p_\mu + f_-(q^2)q_\mu], \\ \langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B_{q'}(p+q)\rangle &= -\frac{1}{m_B+m_S}\left[(2p+q)_\mu q^2 - \left(m_B^2 - m_S^2\right)q_\mu\right] f_T\left(q^2\right). \end{aligned} \tag{11}$$

The covariant trace formalism [14] causes the form factors at large recoils and should satisfy the following relations:

$$f_+(q^2) = \frac{2m_B}{m_B+m_S}f_T(q^2), \quad f_-(q^2) = 0, \quad \text{and} \quad f_T(q^2) = -\frac{m_b-m_q}{m_b-m_s}f_-(q^2), \tag{12}$$

in which the hard gluon exchanges are neglected.

2. SCALAR MESON DISTRIBUTION AMPLITUDES AND LIGHT-CONE SUM RULES FOR FORM FACTORS

The scalar meason light-cone amplitudes for $q_2\bar{q}_1$ are defined [2, 15] as

$$\begin{aligned}
\langle S(p)|\bar{q}_2(x)\gamma_\mu q_1(y)|0\rangle &= p_u \int_0^1 du\, \mathrm{e}^{i(up\cdot x+\bar{u}p\cdot y)}\,\Phi_S(u,\mu), \\
\langle S(p)|\bar{q}_2(x)\; q_1(y)|0\rangle &= m_S \int_0^1 du\, \mathrm{e}^{i(up\cdot x+\bar{u}p\cdot y)}\,\Phi_S^s(u,\mu), \text{ and} \qquad (13)\\
\langle S(p)|\bar{q}_2(x)\sigma_{\mu\nu} q_1(y)|0\rangle &= -m_S(p_\mu z_\nu - p_\nu z_\mu)\int_0^1 du\, \mathrm{e}^{i(up\cdot x+\bar{u}p\cdot y)}\,\Phi_S^\sigma(u,\mu),
\end{aligned}$$

where $z = x - y$, m_S is the meson mass and u denotes q, which is momentum fraction carried by q_2. $\Phi_S(u,\mu)$ is of twist-2, $\Phi_S^s(u,\mu)$ and $\Phi_S^\sigma(u,\mu)$ are of twist-3. These are scalar meson distribution amplitudes with the following normalizations:

$$\int_0^1 du\,\Phi_S(u,\mu) = f_S, \quad \int_0^1 du\,\Phi_S^s(u,\mu) = \int_0^1 du\,\Phi_S^\sigma(u,\mu) = \bar{f}_S, \tag{14}$$

the vector current decay constant f_S is defined as

$$\langle S(p)|\bar{q}_2\gamma^\mu q_1|0\rangle = f_S p^\mu, \tag{15}$$

and $\bar{f}_S$, the scalar density decay constant, is given by

$$\langle S(p)|\bar{q}_2 q_1|0\rangle = m_S\bar{f}_S, \tag{16}$$

with

$$\bar{f}_S = \mu_S f_S, \text{ and } \mu_S = \frac{m_S}{m_2(\mu) - m_1(\mu)}, \tag{17}$$

where m_1 and m_2 denote masses of quarks q_1 and q_2, respectively. The distribution amplitudes are expanded in terms of Jacobi polynomials in Hilbert space as

$$\begin{aligned}
\Phi_S(u,\mu) &= \bar{f}_S(\mu)\,6u\bar{u}\left[B_0(\mu) + \sum_{m=1}^{\infty} B_m(\mu)\; C_m^{3/2}(2u-1)\right], \\
\Phi_S^s(u,\mu) &= \bar{f}_S(\mu)\left[1 + \sum_{m=1}^{\infty} a_m(\mu)\, C_m^{1/2}(2u-1)\right], \text{ and} \qquad (18)\\
\Phi_S^\sigma(u,\mu) &= \bar{f}_S(\mu)\,6u\bar{u}\left[1 + \sum_{m=1}^{\infty} b_m(\mu)\; C_m^{3/2}(2u-1)\right],
\end{aligned}$$

where polynomials $C_m^{3/2}(x)$ are special cases of Jacobi polynomials. The fundamental quantity in QCD light-cone sum rules is the correlation function, which expresses a hadron as the corresponding interpolating current with proper quantum number, such as spin, isospin, (charge) parity and so on. The transition form factors are determined by calculating both the phenomenological and the theoretical parts of correlation function and using dispersion relation and quark–hadron duality approximation.

The correlation function corresponding to form factors $f_+(q^2)$ and $f_-(q^2)$ is definded as

$$\Pi_\mu\left(p,q\right) = -\int d^4x\,\mathrm{e}^{iqx}\left\langle S\left(p\right)\left|T\left\{j_{2\mu}(x), j_1(0)\right\}\right|0\right\rangle, \tag{19}$$

where $j_{2\mu}(x) = \bar{q}_2\left(x\right)\gamma_\mu\gamma_5 b\left(x\right)$ correspond, to weak transition of b to q_2, and $j_1(0)$ correspond, to B_{q_1} channel:

$$j_{2\mu}(x) = \overline{q}_2(x)\gamma_\mu\gamma_5 b(x), \quad j_1(0) = \overline{b}(0)i\gamma_5 q_1(0). \tag{20}$$

The physical (phenomenological) part of correlation function for a complete set of states with the same quantum numbers as B_{q_1} is obtained as

$$\Pi_\mu(p,q) = i\frac{\langle S(p)|\overline{q}_2(0)\gamma_\mu\gamma_5 b(0)|B_{q1}(p+q)\rangle\langle B_{q1}(p+q)|\overline{b}(0)i\gamma_5 q_1(0)|0\rangle}{m_{Bq1}^2-(p+q)^2} + \\ + \sum_h i\frac{\langle S(p)|\overline{q}_2(0)\gamma_\mu\gamma_5 b(0)|h(p+q)\rangle\langle h(p+q)|\overline{b}(0)i\gamma_5 q_1(0)|0\rangle}{m_h^2-(p+q)^2}, \tag{21}$$

where the ground-state contribution is separated from the higher states, corresponding to the B_{q_1}-meson channel. The vacuum-to-meson matrix element for B_{q_1} meson is

$$\langle B_{q1}(p+q)|\overline{b}i\gamma_5 q_1|0\rangle = \frac{m^2 B_{q_1}}{m_b+m_{q1}} f_{B_{q_1}}. \tag{22}$$

Substituting Eqs. (11) and (22) into Eq. (21), the physical part of correlation function appears as follows:

$$\Pi_\mu(p,q) = \frac{m_{B_{q1}}^2 f_{B_{q1}}}{(m_b+m_{q1})[m_{B_{q1}}^2-(p+q)^2]}[f_+(q^2)p_\mu + f_-(q^2)q_\mu] + \\ + \int\limits_{S_0^{B_{q1}}}^{\infty} ds\frac{\rho_+^h(s,q^2)p_\mu + \rho_-^h(s,q^2)q_\mu}{s-(p+q)^2}. \tag{23}$$

Now let us find the theoretical part of the correlation function. By using the perturbative QCD and Operator Product Expansion (OPE) in the deep Euclidean region p^2, $q^2 = -Q^2 \ll 0$, we arrive at the theoretical part of correlation function:

$$\Pi_\mu(p,q) = \Pi_+^{\mathrm{QCD}}(q^2,(p+q)^2)p_\mu + \Pi_-^{\mathrm{QCD}}(q^2,(p+q)^2)q_\mu = \\ = \int\limits_{(m_{b+m_{q1}})^2}^{\infty} ds\frac{1}{\pi}\frac{\mathrm{Im}\,\Pi_+^{\mathrm{QCD}}(s,q^2)}{s-(p+q)^2}p_\mu + \int\limits_{(m_{b+m_{q1}})^2}^{\infty} ds\frac{1}{\pi}\frac{\mathrm{Im}\,\Pi_-^{\mathrm{QCD}}(s,q^2)}{s-(p+q)^2}q_\mu.$$

Using the quark–hadron duality assumption,

$$\rho_i^h(s,q^2) = \frac{1}{\pi}\mathrm{Im}\,\Pi_i^{\mathrm{QCD}}(s,q^2)\Theta\,(s-s_0^h), \tag{24}$$

where $i =$ «+», «−». Let us use the Borel transformation as

$$\hat{\mathcal{B}}_{M^2} = \lim_{\substack{-(p+q)^2,n\to\infty \\ -(p+q)^2/n=M^2}} \frac{(-(p+q)^2)^{(n+1)}}{n!}\left(\frac{d}{d(p+q)^2}\right)^n, \tag{25}$$

and the form factors are finally obtained as follows:

$$f_i(q^2) = \frac{m_b+m_{q_1}}{\pi f_{B_{q_1}} m^2_{B_{q_1}}} \int\limits_{(m_b+m_{q_1})^2}^{s_0^{B_{q_1}}} \mathrm{Im}\,\Pi_i^{\mathrm{QCD}}(s,q^2)\exp\left(\frac{m^2_{B_{q_1}}-s}{M^2}\right)ds. \tag{26}$$

Let us substitute Eq. (20) into Eq. (19) to get

$$\Pi_\mu(p,q) = -\int d^4x\, \mathrm{e}^{iqx}\langle S(p)|\overline{q}_2(x)\gamma_\mu\gamma_5 \underbrace{b(x)\overline{b}(0)}\, i\gamma_5 q_1(0)|0\rangle. \tag{27}$$

Howerer, the full quark propagator is given as [16, 17]:

$$\langle 0|T\{b_i(x)\overline{b_j}(0)\}|0\rangle = \delta_{ij}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{e}^{-ikx}\frac{i}{\not k - m_b} - ig\int\frac{d^4k}{(2\pi)^4}\,\mathrm{e}^{-ikx}\times$$
$$\times\int\limits_0^1 dv\left[\frac{1}{2}\frac{\not k + m_b}{(m_b^2-k^2)^2}G_{ij}^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{1}{m_b^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu\right],$$

where the first term corresponds to free quark propagator, $G_{ij}^{\mu\nu} = G^a_{\mu\nu}T^a_{ij}$ and $\mathrm{Tr}\,[T^aT^b] = (1/2)\delta^{ab}$. Neglecting the LCDAs of higher excited states [31], and by substituting the first term of Eq. (28) into Eq. (27), we get:

$$\Pi_\mu(p,q) = -\int d^4x\int\frac{d^4k}{(2\pi)^4}\,\mathrm{e}^{i(q-k)x}\frac{i}{\not k - m_b}\langle S(p)|\overline{q}_2(x)\gamma_\mu\gamma_5 i\gamma_5 q_1(0)|0\rangle. \tag{28}$$

Now by substituting Eq. (13) into Eq. (28) and calculating the integrals, the correlation function is found as

$$\Pi_\mu(p,q) = p_\mu\int\limits_0^1 du\frac{1}{m_b^2-(q+up)^2}\Bigg\{-m_b\Phi_S(u)+$$
$$+um_S\Phi_S^s(u) + 4m_S\Phi_S^\sigma(u)\frac{q^2+up\cdot q}{m_b^2-(q+up)^2}\Bigg\}+$$
$$+q_\mu\int\limits_0^1 du\frac{1}{m_b^2-(q+up)^2}\left\{m_S\Phi_S^s(u) - 4m_S\Phi_S^\sigma(u)\frac{q\cdot p+up^2}{m_b^2-(q+up)^2}\right\}\equiv$$
$$\equiv \Pi_+^{\mathrm{QCD}}(q^2,(p+q)^2)p_\mu + \Pi_-^{\mathrm{QCD}}(q^2,(p+q)^2)q_\mu \tag{29}$$

from which the form factors are found as follows:

$$f_+(q^2) = \frac{(m_b + m_{q_1})}{m_{B_{q_1}}^2 f_{B_{q_1}}} \exp\left(\frac{m_B^2}{M^2}\right)\Bigg\{\int_{u_0}^{1} \frac{du}{u} \exp\left(-\frac{S(u)}{M^2}\right)\times$$
$$\times\left[\left(-m_b\Phi_S(u) + m_S\left(u\Phi_S^s(u) - 2\Phi_S^\sigma(u)\right)\right) + \frac{2m_S}{uM^2}\Phi_S^\sigma(u)\left(m_b^2 - u^2p^2 + q^2\right)\right]+$$
$$+\frac{m_S}{6}\Phi_S^\sigma(u_0)\exp\left(-\frac{s_0}{M^2}\right)\frac{m_b^2 - u_0p^2 + q^2}{m_b^2 + u_0^2p^2 - q^2}\Bigg\}, \quad (30)$$

$$f_-(q^2) = \frac{(m_b + m_{q_1})}{m_{B_{q_1}}^2 f_{B_{q_1}}} \exp\left(\frac{m_B^2}{M^2}\right)\Bigg\{\int_{u_0}^{1} \frac{du}{u} \exp\left(-\frac{S(u)}{M^2}\right)\times$$
$$\times\left[\left(m_S\left(\Phi_S^s(u) + \frac{2}{u}\Phi_S^\sigma(u)\right)\right) - \frac{2m_S}{u^2M^2}\Phi_S^\sigma(u)(m_b^2 + u^2p^2 - q^2)\right]-$$
$$-\frac{m_S}{6u_0}\Phi_S^\sigma(u_0)\exp\left(-\frac{s_0}{M^2}\right)\Bigg\}, \quad (31)$$

where

$$u_0 = \frac{-(S_0 + Q^2 - m_{\text{meson}}^2) + \sqrt{(S_0 + Q^2 - m_{\text{meson}}^2)^2 + 4m_{\text{meson}}^2(m_b^2 + Q^2)}}{2m_{\text{meson}}^2}. \quad (32)$$

Now in order to find the form factor $f_T(q^2)$ for the transition $b \to s$, we start with the following correlation function:

$$\tilde{\Pi}_\mu(p,q) = -\int d^4x\, e^{iqx}\left\langle S(p)\left|T\left\{\tilde{j}_{2\mu}(x), j_1(0)\right\}\right|0\right\rangle, \quad (33)$$

where

$$\tilde{j}_{2\mu}(x) = \bar{q}_2(x)\sigma_{\mu\nu}q^\nu\gamma_5 b(x). \quad (34)$$

The phenomenological part of the correlation function is

$$\tilde{\Pi}_\mu(p,q) = \frac{\langle S(p)|\overline{q}_2(x)\sigma_{\mu\nu}q^\nu\gamma_5 b(x)|B_{q'}(p+q)\rangle\langle B_{q'}(p+q)|\overline{b}(0)i\gamma_5 q_1(0)|0\rangle}{m_{Bq1}^2 - (p+q)^2}+$$
$$+\sum_h i\frac{\langle S(p)|\overline{q}_2(x)\sigma_{\mu\nu}q^\nu\gamma_5 b(x)|0\rangle|h(p+q)\rangle\langle h(p+q)|\overline{b}(0)i\gamma_5 q_1(0)|0\rangle}{m_h^2 - (p+q)^2}, \quad (35)$$

and the theoretical part of the correlation function in the perturbative context is given by

$$\tilde{\Pi}_\mu(p,q) = [-p_\mu q^2 + q_\mu(q\cdot p)]\int_0^1 du\frac{1}{m_b^2 - (q+up)^2}\times$$
$$\times\left\{\Phi_S(u) - 4m_b m_S\frac{\Phi_S^\sigma(u)}{(m_b^2 - (q+up)^2)}\right\}. \quad (36)$$

By using Borel transformation in terms of variable $(p+q)^2$ and dispersion relation, the form factors are found:

$$f_T(q^2) = \frac{(m_b + m_{q_1})\,(m_B + m_S)}{m_{B_{q_1}}^2 f_{B_{q_1}}} \exp\left(-\frac{m_B^2}{M^2}\right) \times$$
$$\times \left\{ \int_{u_0}^{1} \frac{-du}{2u} \left[\Phi_S(u) - 4\frac{m_b m_S \Phi_S^\sigma(u)}{uM^2}\right] \exp\left(-\frac{S(u)}{M^2}\right) + \right.$$
$$\left. + \frac{m_b m_S}{6} \Phi_S^\sigma(u_0) \exp\left(-\frac{S_0}{M^2}\right) \frac{1}{m_b^2 + u_0^2 p^2 - q^2} \right\}. \quad (37)$$

3. NUMERICAL ANALYSIS

The following values have been used for parameters in our numerical calculations [18–25]:

$$\begin{array}{ll} G_F = 1.166 \cdot 10^{-2}\ \text{GeV}^{-2}, & |V_{ub}| = 3.96^{+0.09}_{-0.09} \cdot 10^{-3}, \\ |V_{tb}| = 0.9991, & |V_{ts}| = 41.61^{+0.10}_{-0.80} \cdot 10^{-3}, \\ m_b = (4.68 \pm 0.03)\ \text{GeV}, & m_s(1\ \text{GeV}) = 142\ \text{MeV}, \\ m_u(1\ \text{GeV}) = 2.8\ \text{MeV}, & m_d(1\ \text{GeV}) = 6.8\ \text{MeV}, \\ m_{B_0} = 5.279\ \text{GeV}, & m_{B_s} = 5.368\ \text{GeV}, \\ f_{B_0} = (0.19 \pm 0.02)\ \text{GeV}, & f_{B_s} = (0.23 \pm 0.02)\ \text{GeV}. \end{array}$$

From Eqs. (30), (31) and (37), it is seen that the sum rules for form factors also depend upon the continuity threshold s and Borel parameter M^2. Since M^2 is not physical, physical quantities, such as form factors, should not depend upon M^2. In order to find the acceptable

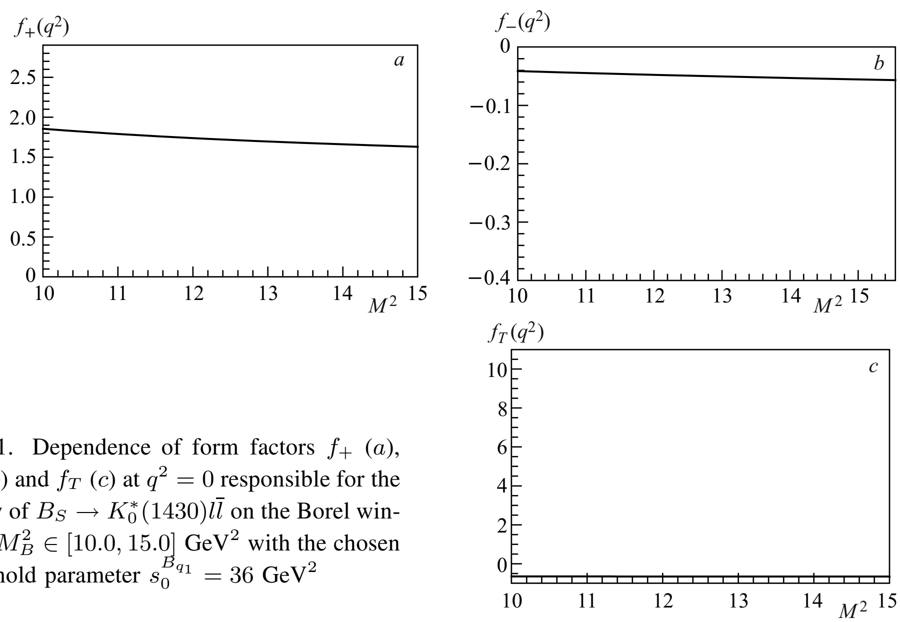


Fig. 1. Dependence of form factors f_+ (*a*), f_- (*b*) and f_T (*c*) at $q^2 = 0$ responsible for the decay of $B_S \to K_0^*(1430)l\bar{l}$ on the Borel window $M_B^2 \in [10.0, 15.0]$ GeV^2 with the chosen threshold parameter $s_0^{B_{q_1}} = 36$ GeV^2

region for M^2, the form factors at zero momentum are plotted versus M^2, and the region, in which the slope is zero, is selected as acceptable range for M^2. In addition, we should not forget that M^2 must be large enough to suppress higher order twists and small enough to suppress the higher excited states. The standard value of the threshold in the X channel is

$$s_{0X} = (m_X + \Delta_X)^2, \tag{38}$$

where Δ_X is taken from [26, 27]. The continuity thresholds are $s_0^{B_0} = (35 \pm 2)$ GeV^2 and $s_0^{B_s} = (36 \pm 2)$ GeV^2 for B_0 and B_s channels. Once the region for M^2 is found, the form factors are plotted versus Q^2 for each value of acceptable M^2. The range of

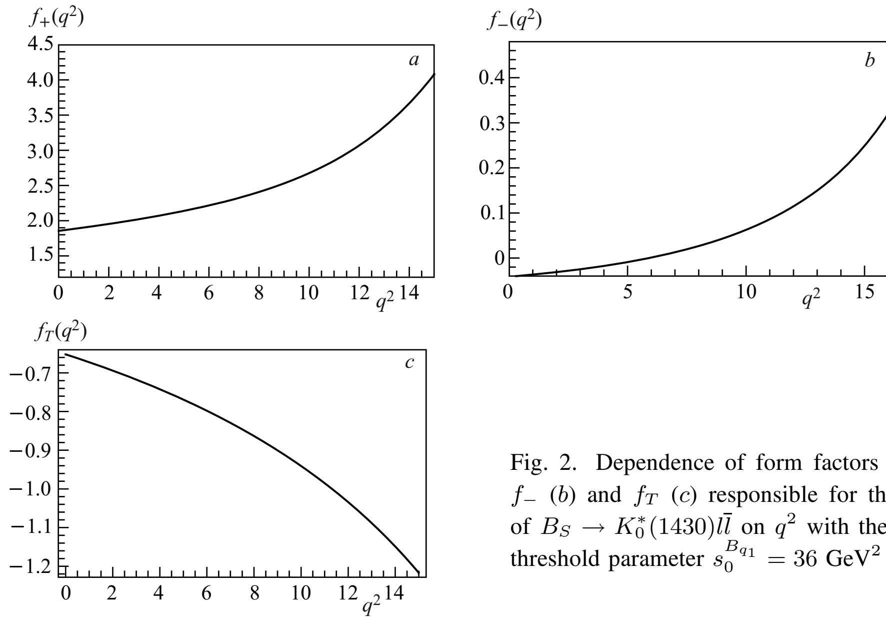


Fig. 2. Dependence of form factors f_+ (a), f_- (b) and f_T (c) responsible for the decay of $B_S \to K_0^*(1430)l\bar{l}$ on q^2 with the chosen threshold parameter $s_0^{B_{q_1}} = 36$ GeV^2

Table 1. **Decay constants and Gegenbauer moments for the twist-2 distribution amplitude Φ_S of scalar mesons at the scale $\mu = 1$ GeV [2]**

State	$\bar{f}$, MeV	B_1	B_3
$a_0(1450)$	460 ± 50	-0.58 ± 0.12	-0.49 ± 0.15
$K_0^*(1430)$	445 ± 50	-0.57 ± 0.13	-0.42 ± 0.22
$f_0(1500)$	490 ± 50	-0.48 ± 0.11	-0.37 ± 0.20

Table 2. **Gegenbauer moments for the twist-3 distribution amplitudes Φ_S^s and Φ_S^σ of scalar mesons at the scale $\mu = 1$ GeV [15]**

State	$a_1 \cdot 10^{-2}$	a_2	a_4	$b_1 \cdot 10^{-2}$	b_2	b_4
a_0	0	$-0.33 \sim -0.18$	$-0.11 \sim 0.39$	0	$0 \sim 0.058$	$0.070 \sim 0.20$
K_0^*	$1.8 \sim 4.2$	$-0.33 \sim -0.025$	—	$3.7 \sim 5.5$	$0 \sim 0.15$	—
f_0	0	$-0.33 \sim -0.18$	$0.28 \sim 0.79$	0	$-0.15 \sim -0.088$	$0.044 \sim 0.16$

Q^2 is chosen between 0 and 15. These procedures are given in Figs. 1 and 2. In our calculations based upon the mentioned criteria, the value of M^2 is chosen to be 10 GeV^2. The values of required constants and coefficients used in our numerical analysis are given in Tables 1 and 2.

4. BRANCHING RATIO AND POLARIZATION ASYMMETRY CALCULATION

Once the transition form factors are found, it is possible to determine quantities, such as branching ratio and polarization asymmetry in the phenomenological region. Due to lack of scalar-type coupling between the lepton pairs, the forward–backward asymmetry for decay modes $\bar{B}_0 \to K_0^*(1430)l\bar{l}$ and $B_s \to f_0(1500)l\bar{l}$ is exactly equal to zero [29, 30].

The semileptonic decay $\bar{B}_0 \to K_0^*(1430)l\bar{l}$ involves the flavour changing neutral interpolating current. In the rest frame of $\bar{B}_0$ meson, the decay width is given in [18] as

$$\frac{d\Gamma(\bar{B}_0 \to K_0^*(1430)l\bar{l})}{dq^2} = \frac{1}{(2\pi)^3}\frac{1}{32m_{\bar{B}_0}}\int\limits_{u_{\min}}^{u_{\max}} |\widetilde{M}_{\bar{B}_0\to K_0^*(1430)l\bar{l}}|^2\, du, \tag{39}$$

where $u = (p_{K_0^*(1430)} + p_l)^2$, $q^2 = (p_l + p_{\bar{l}})^2$ and $p_{K_0^*(1430)}$, p_l and $p_{\bar{l}}$ are four momenta of $K_0^*(1430)$, l and $\bar{l}$, respectively. $|\widetilde{M}_{\bar{B}_0\to K_0^*(1430)l\bar{l}}|^2$ denote the decay amplitude after integraton with respect to the angles between lepton l and the meson $K_0^*(1430)$. The upper and lower limits of integration are as follows:

$$\begin{aligned} u_{\max} &= (E^*_{K_0^*(1430)} + E_l^*)^2 - \left(\sqrt{E^{*2}_{K_0^*(1430)} - m^2_{K_0^*(1430)}} - \sqrt{E_l^{*2} - m_l^2}\right)^2, \\ u_{\min} &= (E^*_{K_0^*(1430)} + E_l^*)^2 - \left(\sqrt{E^{*2}_{K_0^*(1430)} - m^2_{K_0^*(1430)}} + \sqrt{E_l^{*2} - m_l^2}\right)^2, \end{aligned} \tag{40}$$

where $E^*_{K_0^*(1430)}$ and E_l^* are the rest frame energies of $K_0^*(1430)$ and l, which are obtained from the following relations:

$$E^*_{K_0^*(1430)} = \frac{m^2_{\bar{B}_0} - m^2_{K_0^*(1430)} - q^2}{2\sqrt{q^2}}, \quad E_l^* = \frac{q^2}{2\sqrt{q^2}}. \tag{41}$$

Considering all the parameters involved, the differential decay rate for $B_{q'} \to Sl\bar{l}$ is given as [28]

$$\frac{d\Gamma\left(B_{q'} \to Sl\bar{l}\right)}{ds'} = \frac{G_F^2\left|V_{tb}V_{ts}\right|^2 m_B^5 \alpha_{\text{em}}^2}{1536\pi^5}\left(1 - \frac{4r_l}{s'}\right)^{1/2} \times \\ \times \varphi_S^{1/2}\left[\left(1 + \frac{2r_l}{s'}\right)\alpha_S + r_l\delta_S\right], \tag{42}$$

where

$$\alpha_{\rm em} = \frac{1}{129}, \quad s' = \frac{q^2}{m_B^2}, \quad r_l = \frac{m_l^2}{m_B^2}, \quad r_S = \frac{m_S^2}{m_B^2},$$

$$\varphi_S = (1 - r_S)^2 - 2s(1 + r_S) + s^2,$$

$$\alpha_S = \varphi_S \left(\left| C_9^{\rm eff} \frac{f_+\left(q^2\right)}{2} - 2\frac{C_7 f_T(q^2)}{1+\sqrt{r_S}} \right|^2 + \left| C_{10} \frac{f_+\left(q^2\right)}{2} \right|^2 \right),$$

$$\delta_S = 6\left|C_{10}\right|^2 \left\{ \left[2\left(1 + r_S\right) - s\right] \left| \frac{f_+\left(q^2\right)}{2} \right|^2 + \right.$$

$$\left. + \left(1 - r_S\right) \mathrm{Re}\left[f_+\left(q^2\right) \left(f_-(q^2) - \frac{f_+\left(q^2\right)}{2} \right)^* \right] + s \left| f_-(q^2) - \frac{f_+\left(q^2\right)}{2} \right|^2 \right\}.$$

It is obvious that the decay rates for electron- and muon-pair final states are considerably higher than the decay rate for tauon-pair due to the heavily suppressed phase space. Finally, let us consider the polarization asymmetry for a decay $B_{q'} \to Sl\bar{l}$. The four-spin vector s^μ in the rest frame is defined as

$$(s^\mu)_{\rm r.s} = (0,\ \hat{\xi}). \tag{43}$$

The unit vector along the longitudinal direction of leptons' polarization is given by

$$\hat{e}_L = \frac{\mathbf{p_L}}{\mid \mathbf{p_L} \mid}, \tag{44}$$

where $P_L(s')$ is given as

$$P_L(s') = \frac{\dfrac{d\Gamma\left(\hat{e}_L\hat{\xi} = 1\right)}{ds'} - \dfrac{d\Gamma\left(\hat{e}_L\hat{\xi} = -1\right)}{ds'}}{\dfrac{d\Gamma\left(\hat{e}_L\hat{\xi} = 1\right)}{ds'} + \dfrac{d\Gamma\left(\hat{e}_L\hat{\xi} = -1\right)}{ds'}}, \tag{45}$$

which is similar to forward–backward asymmetry $P_L(s')$ that has odd parity, but even CP.

An explicit expression for $P_L(s')$ for the decay $B_{q'} \to Sl\bar{l}$ in the rest frame of lepton pair is given as [28]

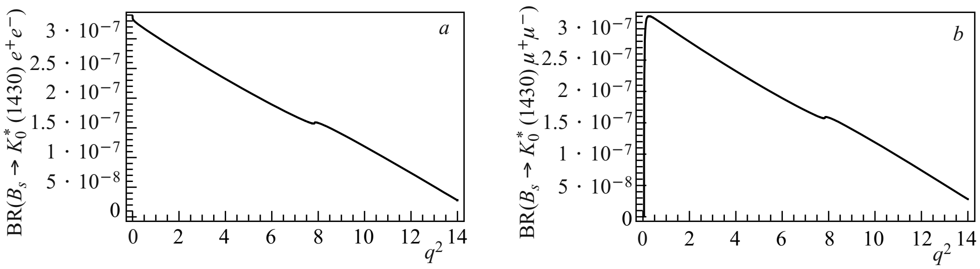
$$P_L(s') = \frac{2\left(1 - \dfrac{4r_l}{s'}\right)^{1/2}}{\left(1 + \dfrac{2r_l}{s'}\right)\alpha_S + r_l\delta_S} \times$$

$$\times\ \mathrm{Re}\left[\varphi_S \left(C_9^{\rm eff} \frac{f_+\left(q^2\right)}{2} - 2\frac{C_7 f_T\left(q^2\right)}{1+\sqrt{r_S}} \right) \left(C_{10} \frac{f_+\left(q^2\right)}{2} \right)^* \right]. \tag{46}$$

Equation (46) is an indication of the fact that in the limit of zero lepton mass, the asymmetry is independent of form factors, and due to small value of Wilson coefficient C_7, compared with C_9^{eff} and C_{10}, it is approximately given as

$$P_L(s') = \frac{2\text{Re}[C_9^{\text{eff}} C_{10}^*]}{|C_9^{\text{eff}}|^2 + |C_{10}|^2} + O(C_7) \simeq -1. \tag{47}$$

The results of our calculations are given in Figs. 3 and 4 and summarized in Table 3 in which our results are compared with the values obtained with other approaches.



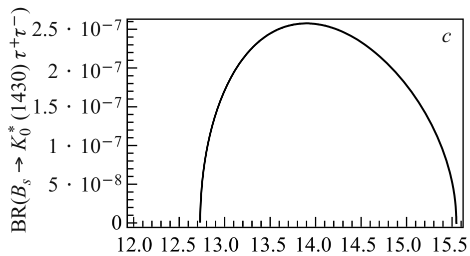
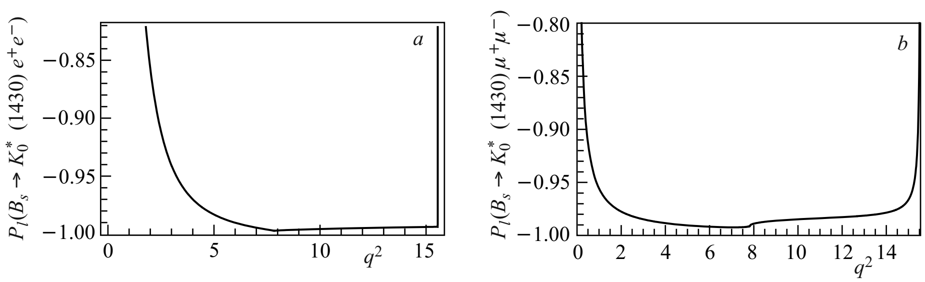


Fig. 3. The branching ratio of $B_S \to K_0^*(1430)e^+e^-$ (*a*), $B_S \to K_0^*(1430)\mu^+\mu^-$ (*b*), and $B_S \to K_0^*(1430)\tau^+\tau^-$ (*c*) as functions of squared momentum transfer q^2 based on light-cone QCD sum rules



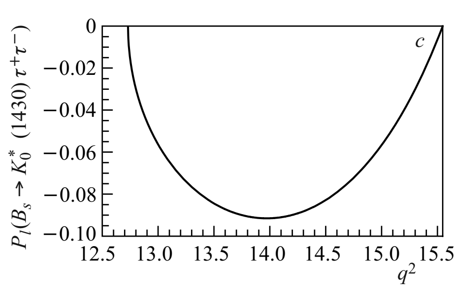


Fig. 4. Lepton polarization asymmetries for $B_S \to K_0^*(1430)e^+e^-$ (*a*), $B_S \to K_0^*(1430)\mu^+\mu^-$ (*b*), and $B_S \to K_0^*(1430)\tau^+\tau^-$ (*c*) as functions of squared momentum transfer q^2 based on light-cone QCD sum rules

Table 3. **Numerical results for the total decay width of $B_0 \to a_0(1450)l\bar{l}$, $\bar{B}_0 \to K_0^*(1430)l\bar{l}$, and $B_s \to K_0^*(1430)l\bar{l}$ with $l = e, \mu, \tau$ in the light-cone sum rules approach, together with the numbers estimated in QCD sum rules [32, 33] and LFQM [28]**

State	$\bar{B}_0 \to a_0(1450)$	$\bar{B}_0 \to K_0^*(1430)$	$B_s \to K_0^*(1430)$
		e^+e^-	
OUR	$7.15 \cdot 10^{-4}$	$6.28 \cdot 10^{-7}$	$2.47 \cdot 10^{-6}$
LCSR	$1.8^{+0.9}_{-0.6} \cdot 10^{-4}$	$5.7^{+3.4}_{-2.4} \cdot 10^{-7}$	
LFQM		$1.63 \cdot 10^{-7}$ [28]	
QCDSR		$(2.09-2.68) \cdot 10^{-7}$ [32]	
		$\mu^+\mu^-$	
OUR	$7.09 \cdot 10^{-4}$	$6.28 \cdot 10^{-7}$	$2.45 \cdot 10^{-6}$
LCSR	$1.8^{+0.9}_{-0.7} \cdot 10^{-4}$	$5.6^{+3.1}_{-2.3} \cdot 10^{-7}$	
LFQM		$1.62 \cdot 10^{-7}$ [28]	
QCDSR		$(2.07-2.66) \cdot 10^{-7}$ [32]	
		$\tau^+\tau^-$	
OUR	$9.17 \cdot 10^{-5}$	$1.16 \cdot 10^{-9}$	$5.93 \cdot 10^{-8}$
LCSR	$6.3^{+3.4}_{-2.5} \cdot 10^{-5}$	$9.8^{+12.4}_{-5.5} \cdot 10^{-9}$	
LFQM		$2.86 \cdot 10^{-9}$ [28]	
QCDSR		$(1.70-2.20) \cdot 10^{-9}$ [32]	

CONCLUSIONS

The values of form factors for different q^2 values obtained here are compared with the results obtained with other approaches. There are some differences, small but noticeable, though experimental data will clarify the correct one in the future.

Here by applying the light-cone QCD sum rules, we investigated $B_s \to K_0^*(1430)l\bar{l}$, $B_0 \to K_0^*(1430)l\bar{l}$ and $B_0 \to a_0(1450)l\bar{l}$ decays with distribution amplitudes up to the twist-3. In general, it is concluded that form factors for decay of $B \to S$ are almost twice the value of $B \to P$.

The form factors $f_+(q^2)$, $f_-(q^2)$ and $f_T(q^2)$ found here are for large recoil in heavy quark regions. The branching ratios were determined for the three decays mentioned above. Our findings indicate that the values of branching ratios for $\bar{B}_S \to K_0^*(1430)e^+e^-$ and $\bar{B}_S \to K_0^*(1430)\mu^+\mu^-$ are of order 10^{-6}, for $\bar{B}_0 \to a_0(1450)e^+e^-$ and $\bar{B}_0 \to a_0(1450)\mu^+\mu^-$ are of order 10^{-4}, while for $\bar{B}_0 \to K_0^*(1430)e^+e^-$ and $\bar{B}_0 \to K_0^*(1430)\mu^+\mu^-$ are of order 10^{-7}, for $\bar{B}_S \to K_0^*(1430)\tau^+\tau^-$ is of order 10^{-8}, for $\bar{B}_0 \to K_0^*(1430)\tau^+\tau^-$ is of order 10^{-9}, and, finally for $\bar{B}_0 \to a_0(1450)\tau^+\tau^-$ is of order 10^{-5}.

Finally, the longitudinal lepton polarization asymmetry of the three decays mentioned above was calculated and is in good agreement with the results obtained with light-front quark model (LFQM) approaches. The lepton polarization asymmetry for tauon is much smaller than that of muon and electron and cannot be measured due to the efficiency for the detectability of the tauon.

REFERENCES

1. *Close F. E., Törnqvist N. A.* // J. Phys. G. 2002. V. 28. P. R249.
2. *Cheng H. Y., Chua C. K., Yang K. C.* Charmless Hadronic B Decays Involving Scalar Mesons: Implications to the Nature of Light Scalar Mesons // Phys. Rev. D. 2006. V. 73. P. 014017.
3. *Braun V. M.* Light-Cone Sum Rules // Plenary talk given at the IV Intern. Workshop on Progress in Heavy Quark Physics, Rostock, Germany, 1997.
4. The Exclusive Rare Decays in a Relativistic Quark Model // Eur. Phys. J. C. 2002. V. 4. P. 18.
5. *Kim C. S., Morozumi T., Sanda A. I.* // Phys. Lett. B. 1989. V. 218. P. 343.
6. *He X. G., Nguyen T. D., Volkas R. R.* B Meson Rare Decays in Two Higgs Doublets Models // Phys. Rev. D. 1988. V. 38. P. 814.
7. *Grinstein B., Savage M. J., Wise M. B.* // Nucl. Phys. B. 1989. V. 319. P. 271.
8. *Deshpande N. G., Trampetic J., Panose K.* Resonance Background to the Decays $b \to s$ Lepton$^+$ Lepton$^-$, $B \to K^*$ Lepton$^+$ Lepton$^-$ and $B \to K$ Lepton$^+$ Lepton$^-$ // Phys. Rev. D. 1989. V. 39. P. 1461.
9. *O'Donnell P. J., Tung H. K. K.* Resonance Contributions to the Decay $b \to s(l^+l^-)$ // Phys. Rev. D. 1991. V. 43. P. 2067.
10. *Paver N., Riazuddin.* On the Interference between Short Distance and Long Distance Contributions to $b \to sl^+l^-$ // Phys. Rev. D. 1992. V. 45. P. 978.
11. *Ali A., Mannel T., Morozumi T.* // Phys. Lett. B. 1991. V. 273. P. 505.
12. *Buchalla G., Buras A. J., Lautenbacher M. E.* // Rev. Mod. Phys. 1996. V. 68. P. 1230.
13. *Chen C. H., Geng C. Q.* Baryonic Rare Decays of Lambda/$b \ \to$ Lambda l^+l^- // Phys. Rev. D. 2001. V. 64. P. 074001.
14. *Falk A. F. et al.* Heavy Meson Form Factors from QCD // Nucl. Phys. B. 1990. V. 343. P. 1.
15. *Lü C. D., Wang Y. M., Zou H.* // Phys. Rev. D. 2007. V. 75. P. 056001.
16. *Balitsky I. I., Braun V. M.* // Nucl. Phys. B. 1989. V. 311. P. 541.
17. *Khodjamirian A., Ruckl R.* QCD Sum Rules for Exclusive Decays of Heavy Mesons // Adv. Ser. Direct. High Energy Phys. 1998. V. 15. P. 345.
18. *Yao W. M. et al.* // J. Phys. G. 2006. V. 33. P. 1.
19. *Kiselev V. V.* // Cent. Eur. J. Phys. 2004. V. 2. P. 523.
20. *Ivanov M. A., Körner J. G., Santorelli P.* // Phys. Rev. D. 2006. V. 73. P. 054024.
21. *Bauer C. W. et al.* // Phys. Rev. D. 2004. V. 70. P. 094017.
22. *Ioffe B. L.* // Prog. Part. Nucl. Phys. 2006. V. 56. P. 232.
23. *Gray A. et al.* (*HPQCD Collab.*). The B-Meson Decay Constant from Unquenched Lattice QCD // Phys. Rev. Lett. 2005. V. 95. P. 212001.
24. *Penin A. A., Steinhauser M.* Heavy-Light Meson Decay Constant from QCD Sum Rules in Three-Loop Approximation // Phys. Rev. D. 2002. V. 65. P. 054006.
25. *Jamin M., Lange B. O.* $f(B)$ and $f(B/s)$ from QCD Sum Rules // Ibid. P. 056005.
26. *Dosch H. G. et al.* // Phys. Rev. D. 2002. V. 65. P. 114002.
27. *Navarra F. S. et al.* // Phys. Lett. B. 2000. V. 489. P. 319.
28. *Chen C. H. et al.* Study of $B \to K_0^*(1430)\ell\bar{\ell}$ Decays // Phys. Rev. D. 2007. V. 75. P. 074010.
29. *Belanger G., Geng C. Q., Turcotte P.* // Nucl. Phys. B. 1993. V. 390. P. 253.

30. *Geng C. Q., Kao C. P.* // Phys. Rev. D. 1996. V. 54. P. 5636.
31. *Diehl M. et al.* Linking Parton Distributions to Form Factors and Compton Scattering // Eur. Phys. J. C. 1999. V. 8. P. 409.
32. *Aliev T. M., Azizi K., Savci M.* Analysis of Rare $(B \to K_0^*(1430)l^+l^-)$ Decay within QCD Sum Rules // Phys. Rev. D. 2007. V. 76. P. 074017.
33. *Yang M. Z.* Semileptonic Decay of B and $D \to K_0^*(1430)\ \bar{l}\nu$ from QCD Sum Rule // Phys. Rev. D. 2006. V. 73. P. 034027; Erratum // Ibid. P. 079901.

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