# PROTON–ELECTRON ELASTIC SCATTERING AND THE PROTON CHARGE RADIUS

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It is suggested that proton elastic scattering on atomic electrons allows a precise measurement of the proton charge radius. The main advantage is that inverse kinematics allows one to access with a huge cross section very small values of transferred momenta, up to four orders of magnitude smaller than the ones presently achieved.

Рассматривается возможность измерения радиуса протона на основе данных по рассеянию протона на покоящихся (атомных) электронах, что позволяет достичь малых значений переданного импульса, вплоть до четырех порядков меньших достижимых в настоящее время. Данная постановка эксперимента потенциально может увеличить точность экспериментальной оценки радиуса протона.

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## INTRODUCTION

The problem of the proton size has recently been object of large interest, due to the recent experiment on muonic hydrogen by laser spectroscopy measurement of the $\nu p(2S-2P)$ transition frequency [1]. The result on the proton charge radius $r_c = 0.84184(67)$ fm obtained in this experiment is one order of magnitude more precise but smaller by five standard deviations compared to the best value previously assumed $r_c = 0.8768(69)$ fm [2] (CODATA). Previous best measurements include techniques based on hydrogen spectroscopy, which are more precise than, but compatible with, electron–proton elastic scattering at small values of the four-momentum transfer squared $Q^2$. The most recent result from electron–proton elastic scattering, $r_c = 0.879(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}}(4)_{\text{group}}$ fm, can be found in [3].

While corrections to the laser spectroscopy experiments seem well under control in the framework of QED and may be estimated with a precision better than 0.1%, in case of $ep$ elastic scattering the best precision that has been achieved is of the order of few percent.

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Different sources of possible systematic errors to the muonic experiment have been discussed; however, no definite explanation of this difference has been given yet (see [5] and references therein).

Recent works have been devoted to the scattering of a proton projectile on an electron target (see [6] and references therein). The possibility to build beam polarimeters for high-energy polarized (anti)proton beams has been discussed [7]. Experiments have been done [8, 9] and are ongoing with the aim to understand the experimental fact that a proton beam circulating through a polarized hydrogen target gets polarized [10]. The possibility to polarize antiproton beams would open a wide domain of polarization studies at the GSI Facility for Antiproton and Ion Research (FAIR) [11, 12]. Assuming $C$-invariance in electromagnetic interactions, the (elastic and inelastic) reactions $p + e^-$ and $\bar{p} + e^+$ are strictly equivalent.

In [6], the cross section and the polarization observables for proton electron–elastic scattering, in a relativistic approach assuming the Born approximation, were derived. The relations connecting kinematical variables in direct and inverse kinematics were given. In particular, it was shown that large polarization effects appear at beam energies around 15 GeV. Moreover, the transferred momenta are very small even when the proton energy is in the GeV range. In this work, we focus on the second issue and apply to the problem of a precise and consistent determination of the proton radius. The kinematics of proton–electron scattering is extremely peculiar and interesting in this respect.

In the elastic interaction between a proton and an electron, assuming that the interaction occurs through the exchange of a virtual photon of four-momentum $k = (\omega, \mathbf{k})$, the observables can be expressed as functions of two form factors, electric $G_E$ and magnetic $G_M$, which are functions of $Q^2 = -k^2$ only.

The electric form factor $G_E(Q^2)$ in the nonrelativistic limit is related to the charge distribution through a Fourier transform. For small values of $Q^2$, one can develop $G_E(Q^2)$ in a Taylor series expansion:

$$G_E(Q^2) = 1 - \frac{1}{6}Q^2\langle r_c^2\rangle + O(Q^2), \tag{1}$$

where one takes into account the fact that the density (being the square of the wave function) is an even function of the spatial distance $r$, whereas the scalar product $\mathbf{k}\cdot\mathbf{r}$ is an odd function. The root mean square radius is the derivative of the form factor at $Q^2 = 0$:

$$\langle r_c^2\rangle = -6\left.\frac{dG_E(Q^2)}{dQ^2}\right|_{Q^2=0}. \tag{2}$$

The value itself of $G_E(Q^2 = 0) = Ze = 1$ is given by the normalization to the charge (assuming $e = 1$); $Z$ is the atomic number.

Form factors are derived from unpolarized $ep$ scattering through the Rosenbluth separation method: measurements of the unpolarized cross section at fixed $Q^2$ for different angles allow one to extract the electric and magnetic form factors. The polarization method [13] has recently been applied [14], providing very precise measurements of the ratio $G_E/G_M$ up to large values of $Q^2 \simeq 9$ GeV$^2$. The larger precision comes from the fact that in this case one measures a polarization ratio in which radiative corrections (at first order) cancel and the systematics effects related to the beam polarization and to polarimetry are essentially reduced.

Radiative corrections and Coulomb corrections have to be applied to $ep$ scattering experiments, in particular for unpolarized measurements. Besides the problems related to the fact that there is no model-independent way to calculate those radiative corrections which depend on the hadron structure and that correlations exist in extracting form factors from the Rosenbluth fit [15], one has to face the extrapolation of the data to $Q^2 = 0$ as discussed in [3]. The smallest value of $Q^2$ reached in that experiment was 0.004 GeV$^2$. The precision on the measurement of the proton radius is strongly related to the extrapolation for $Q^2 \to 0$. Recent analysis [4] shows that, although very precise data have been obtained for the electric form factor of the proton at low $Q^2$, there is still place for model dependence. The functional form of the fit function is essential to get the slope at $Q^2 \to 0$. According to the model used, the same data may be consistent with the estimation of the radius from muonic atom or from CODATA, quoted above, which are themselves inconsistent at $5\sigma$ level. Keeping the systematic error of the measurement as in [3], using electron at rest or moving in a low-energy collider, one could extend the range of $Q^2$ from $10^{-3}$ to $10^{-7}$ GeV$^2$, giving severe constraints to the fitting procedure. The possibility to access much smaller values of $Q^2$ is offered by the elastic reaction induced by a proton beam on an electron target.

## DIFFERENTIAL CROSS SECTION AND INVERSE KINEMATICS

Let us consider the reaction

$$p(p_1) + e(k_1) \to p(p_2) + e(k_2), \tag{3}$$

where particle momenta are indicated in parentheses and $k = k_1 - k_2 = p_2 - p_1$. The expression of the differential cross section for unpolarized proton–electron scattering, in the coordinate system where the electron is at rest, can be written, in the Born approximation, as

$$\frac{d\sigma}{dQ^2} = \frac{\pi\alpha^2}{2m^2|\mathbf{p}|^2} \frac{\mathcal{D}}{Q^4}, \tag{4}$$

$$\mathcal{D} = -Q^2(-Q^2 + 2m^2)G_M^2 + 2\left[G_E^2 + \tau G_M^2\right]\left[-Q^2M^2 + \frac{1}{1+\tau}\left(2mE - \frac{Q^2}{2}\right)^2\right], \tag{5}$$

where $\alpha = 1/137$ is the electromagnetic fine constant; $\tau = Q^2/4M^2$ and $G_{E,M}$ are the Sachs electric and magnetic form factors; $m$ $(M)$ is the electron (proton) mass; $\mathbf{p}$ is the three-momentum of the proton beam; and $E$ is the total energy of the proton beam.

Similarly to $ep$ scattering, the differential cross section diverges as $(Q^2)^2$ when $Q^2 \to 0$. This is a well-known result, which is a consequence of the one-photon exchange mechanism and allows one to reach very large cross sections. The expression (5) differs from the Rosenbluth formula [16], as additional terms depending on the electron mass cannot be neglected. The electric contribution to the cross section dominates, being in all the allowed $Q^2$ range, $\sim 10^7$ times larger than the magnetic one.
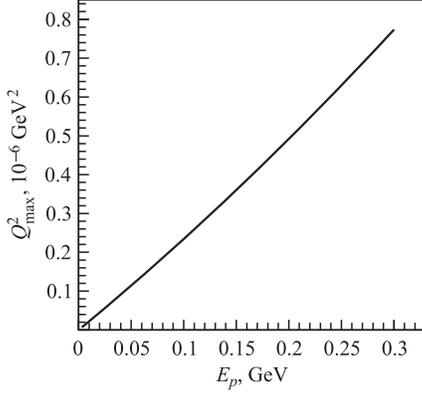
Fig. 1. Maximum value of the four-momentum transfer squared as a function of the proton beam kinetic energy, $E_p$
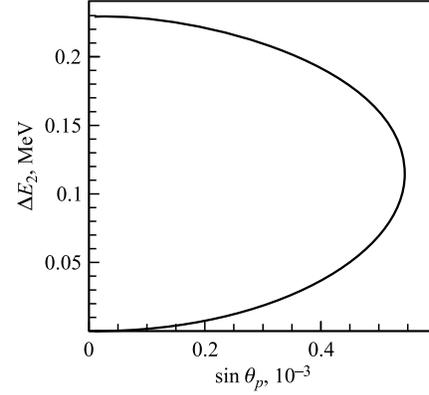


Fig. 2. Difference of the kinetic energy of the scattered proton from the beam kinetic energy, $\Delta E_2$, for $E_p = 100$ MeV, as a function of the sine of the proton scattering angle

Let us consider the case when the proton-beam kinetic energy, $E_p$, is under the pion threshold for $pp$ reactions, $E_p = 100$ MeV. This helps in reducing the hadronic background.

The properties of inverse kinematics has been discussed in [6]. It has been shown that for a given value of $E$, the maximum four-momentum transfer squared is

$$(Q^2)_{\mathrm{max}} = \frac{4m^2(E^2 - M^2)}{M^2 + 2mE + m^2}. \tag{6}$$

Being proportional to the electron mass squared, $Q^2$ is restricted to very small values. In Fig. 1, we report $Q^2_{\mathrm{max}}$ as a function of the proton kinetic energy, in the MeV range. One can see that the values of transferred momenta are very small: for a proton beam with kinetic energy $E_p = 100$ MeV, $(Q^2)_{\mathrm{max}} = 0.2 \cdot 10^{-6}$ GeV$^2$.

From energy and momentum conservation, one finds the following relation between the angle $\theta_e$ and the energy $\epsilon_2$ of the scattered electron:

$$\cos\theta_e = \frac{(E + m)(\epsilon_2 - m)}{|\mathbf{p}|\sqrt{(\epsilon_2^2 - m^2)}}, \tag{7}$$

which shows that $\cos\theta_e \geqslant 0$ (the electron can never be scattered backward). In the inverse kinematics, the available kinematical region is reduced to small values of $\epsilon_2$:

$$\epsilon_{2,\mathrm{max}} = m\frac{2E(E + m) + m^2 - M^2}{M^2 + 2mE + m^2}, \tag{8}$$

which is proportional to the electron mass. From momentum conservation, on can find the following relation between the kinetic energy $E_2$ and the angle $\theta_p$ of the scattered proton (Fig. 2):

$$E_2^{\pm} + M = \frac{(E + m)(M^2 + mE) \pm M(E^2 - M^2)\cos\theta_p\sqrt{\dfrac{m^2}{M^2} - \sin^2\theta_p}}{(E + m)^2 - (E^2 - M^2)\cos^2\theta_p}, \tag{9}$$

which shows that for one proton angle there may be two values of the proton energy (and two corresponding values for the recoil-electron energy and angle, and for the transferred momentum $Q^2$). The two solutions coincide when the angle between the initial and final proton takes its maximum value, which is determined by the mass ratio of the electron and the scattered proton, $\sin\theta_{p,\max} = m/M = 0.544 \cdot 10^{-3}$. Hadrons are scattered from atomic electrons at very small angles, and the larger is the hadron mass, the smaller is the available angular range for the scattered hadron. The difference between the scattered proton kinetic energy and the beam kinetic energy is shown as a function of the proton scattering angle in Fig. 2. The proton kinematics is very close to the beam, which makes the detection very challenging. However, a magnetic system with momentum resolution of the order of $10^{-4}$ can provide at least the measurement of the energy of the scattered proton. This would allow a coincidence measurement which helps in reducing the possible background.

While the proton is emitted in a narrow cone, the electron is scattered up to $90°$. The energy dependence as a function of the cosine of the angle for the recoil electron is shown in Fig. 3. The detection of electrons in the MeV energy range is currently available.

The differential cross section as a function of $\cos\theta_e$ is shown in Fig. 4 in the angular range $10 \leqslant \theta_e \leqslant 80°$. It is large when the electron angle is close to $90°$ and monotonically decreasing. The cross section, integrated in this angular range, is $25 \cdot 10^4$ mb. Assuming a luminosity $\mathcal{L} = 10^{32}$ cm$^{-2} \cdot$ s$^{-1}$ with an ideal detector with an efficiency of 100%, a number of $\simeq 25 \cdot 10^9$ events can be collected in one second. Therefore, the reaction (3) allows one to reach very small momenta with huge cross section. The very specific kinematics requires a dedicated experiment. One possibility is to detect the correlation between angle and energy of the recoil electron. The detection of the energy of the scattered proton in coincidence is feasible, in principle, with a magnetic system.

Let us stress the importance of taking into account the lepton mass. The approximation of zero electron mass, commonly used in lepton–hadron scattering, should be carefully considered. Another example is given by low-energy $\mu + p$ elastic scattering. In the hundred MeV
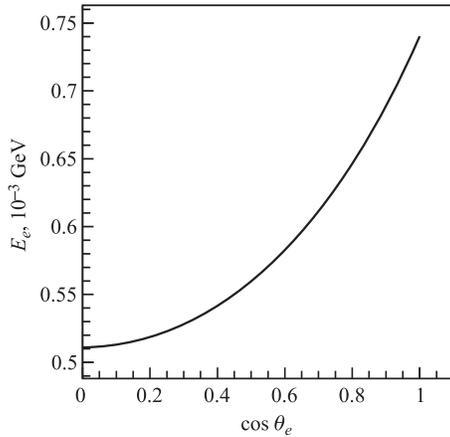


Fig. 3. Kinetic energy of the recoil electron as a function of the cosine of the electron scattering angle for beam kinetic energy $E_p = 100$ MeV
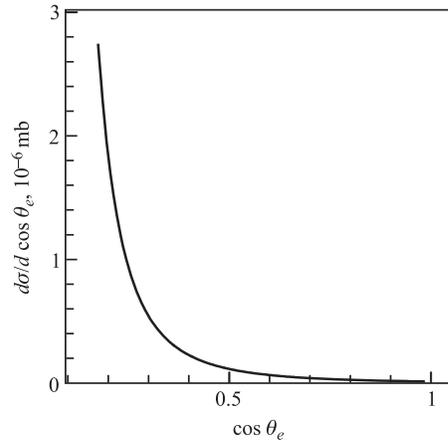
Fig. 4. Differential cross section as a function of the cosine of the electron scattering angle for beam kinetic energy $E_p = 100$ MeV
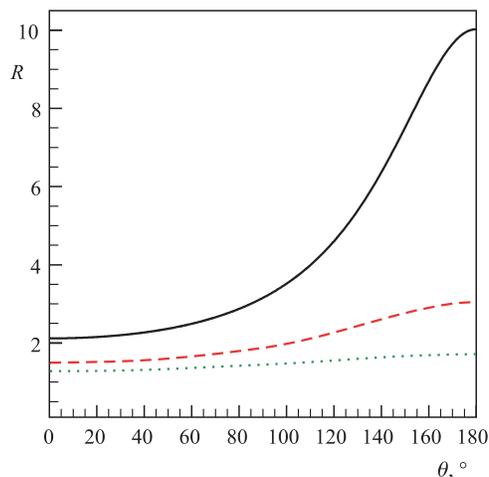
Fig. 5. (Color online) Ratio $R$ of the differential cross section for muon to electron elastic scattering, as a function of the lepton scattering angle for three values of the beam momentum $|\mathbf{p}| = 100$ MeV (black, solid line), $|\mathbf{p}| = 150$ MeV (red, dashed line), $|\mathbf{p}| = 200$ MeV (green, dotted line)

region, the $\mu p$ cross section largely exceeds the $e + p$ cross section. The corrections due to the mass may reach one order of magnitude, in particular at backward angles. The ratio of the differential cross section, $R = \sigma(\mu p)/\sigma(ep)$, is shown in Fig. 5 for three values of the beam momentum. One can see that this ratio varies monotonically with the angle, reaching its maximum (up to a factor of ten) for backward scattering.

## CONCLUSIONS

We suggested a possibility to measure the proton radius, based on the small value of the transfer momentum squared (even for relatively large energies of colliding hadrons) achievable in $pe$ elastic scattering. This is a general characteristic of all reactions of elastic and inelastic hadron scattering by atomic electrons (which can be considered at rest). We illustrated the accessible kinematical $Q^2$ range and showed that one could improve by four orders of magnitudes the lower limit at which elastic $ep$ scattering experiments have been done. In such kinematical conditions, the electric contribution to the cross section dominates and the magnetic contribution can be safely neglected. Therefore, there is no need for Rosenbluth separation and/or polarization method to determine $G_E$. This allows a precise measurement of the proton radius, decreasing the errors due to the extrapolation for $Q^2 \to 0$. For completeness, let us mention polarization effects in proton–electron scattering, which have indeed been measured [10]. In [6], it was shown that polarization observables are very small at low energy, but sizable in the GeV range. As for $ep$ elastic scattering, the ratio $G_E/G_M$ can be derived from the ratio of double spin observables [13], in our case two correlation coefficients, for example, $C_{tl}/C_{tt}$. Having a proton beam and an electron target both polarized in the direction normal to the scattering plane gives access to the product of $G_E$ and $G_M$,

once the unpolarized cross section is known:

$$\mathcal{D}C_{nn} = -4mMQ^2 G_E G_M. \tag{10}$$

For the problem discussed in this paper, any heavy target, Au, Pb, W, can be considered as a good target. Electron jet targets, or low-energy electron beams in a collider have already been used. Low-energy electron–ion colliders are under study [17]. Note that in the case of colliding ions, tuning the energy of the proton and electron beams, one could reach larger $Q^2$ values and fill the gap between the very low value discussed here $10^{-6}$ and the minimum value experimentally reached up to now $10^{-3}$. The measurement requires the selection of elastic events with the energy and angle correlation for the electron. The protons are emitted in a narrow cone around the beam direction, with energy close to the beam one.

The theoretical limit of our approach as discussed in [6] is related to the validity of the Born approximation (scattering through one-photon exchange): at very small energies, multiphoton exchanges may lead to a quasi-bound state $p + e$, at higher energies, over the pion threshold, inelastic reactions start to contribute. Therefore, we suggested a kinematical range below pion emission threshold.

In summary, the main advantages of the proposed reaction are the possibility of accessing low $Q^2$ values with high statistics, and negligible physical background. A momentum resolution of the order of $10^{-4}$ for an emitted proton has been achieved in high-resolution spectrometers, for example, the dispersive spectrometer SPES1 (Saturne) [18]. More recently, high-resolution detection for protons at zero degrees is reported for the facility RIBF, RIKEN [19].

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