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SELF-SIMILARITY OF LOW-p_T CUMULATIVE PION PRODUCTION IN PROTON–NUCLEUS COLLISIONS AT HIGH ENERGIES

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The experimental data on inclusive spectra of the π^+ mesons produced in $p + A$ collisions at an incident proton momentum of 400 GeV/c obtained by G. Leksin group at FNAL are analyzed in the framework of z-scaling. Self-similarity of the hadron production in the low-p_T cumualtive region is verified. Scaling function $\psi(z)$ for the Li, Be, C, Al, Cu, Ta nuclei is constructed. It is expressed via the invariant cross section and average multiplicity density of charged particles. Results of the analysis of the low-p_T data are compared with the high-p_T data sets obtained by J. Cronin, R. Sulyaev and D. Jaffe groups. A microscopic scenario of $p + A$ interactions in terms of momentum fractions x_1, x_2 is discussed. Indication of self-similarity of the cumulative pion production in $p + A$ collisions over a wide kinematical range has been found. Based on the universality of the shape of the scaling function, the inclusive cross sections of the π^+ mesons produced in $p + A$ collisions on the Li, Be, C, Al, Cu, Ta targets in deep-cumulative region ($x_2 \gg 1/A$) are predicted.

Анализируются экспериментальные данные по инклюзивным сечениям рождения π^+-мезонов в столкновениях $p + A$ при импульсе протона 400 ГэВ/c, полученные группой Г. Лексина во FNAL в рамках теории z-скейлинга. Проверяется самоподобие рождения кумулятивных пионов с малыми поперечными импульсами. Построена скейлинговая функция $\psi(z)$ для ядер Li, Be, C, Al, Cu, Ta. Она выражается через инвариантное сечение и среднюю плотность множественности заряженных адронов. Результаты анализа данных при малых p_T сравниваются с данными, полученными группами Дж. Кронина, Р. Суляева и Д. Джаффе, при больших поперечных импульсах. Обсуждается микроскопический сценарий взаимодействий $p + A$ в терминах долей импульсов x_1, x_2. Получено указание на самоподобие рождения кумулятивных пионов во взаимодействиях $p + A$ в широкой кинематической области. На основе универсальности формы скейлинговой функции $\psi(z)$ сделаны предсказания поведения инклюзивных спектров рождения π^+-мезонов в столкновениях $p + A$ на ядрах Li, Be, C, Al, Cu, Ta в глубоко кумулятивной области ($x_2 \gg 1/A$).

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INTRODUCTION

Search for clear signatures of the phase transtion of the nuclear matter in collisions of hadrons and nuclei is the main goal of the heavy-ion experimental programmes at the Relativistic Heavy Ion Collider at BNL [1, 2], Super Proton Synchrotron [3], and Large

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Hadron Collider at CERN [4–8]. The hypothesis of self-similarity of the hadron production is an important concept for data analysis in searching for new physics. The hypothesis is related with the established scaling laws such as the Bjorken scaling — in the deep inelatic scattering, Feynman scaling — in the inclusive hadron production, P-KNO scaling — in the multiparticle production, and others. Among the others there are quark counting rules describing the power asymptotics of the electromagnetic form factors of hadrons and cross sections of exclusive processes [9–18]. The phase transitions in the nuclear matter produced in the heavy-ion collisions at the high energy density and temperature are new phenomena related to collective interactions of quarks and gluons near the phase boundaries and the critical point.

It is well known that the general concepts in the critical phenomena are related with the notions of «scaling» and «universality» [19]. Scaling means that the system near the critical point exhibiting self-similar properties is invariant under transformation of the scale. According to universality, quite different systems behave in a remarkably similar way near the respective critical point. It is assumed that transition of the nuclear matter from hadron to quark and gluon degrees of freedom near the critical point should reveal large fluctuations, correlations and discontinuty of some experimental quantities characterizing the system.

The high-density nuclear matter can be produced in cumulative proceeses. Production of any inclusive particle with a momentum far beyond the nucleon–nucleon kinematic region is accompanied by cumulation of a nucleus. The effect does not contradict the momentum conservation law. The cumulative processes have been extensively studied mainly at JINR, ITEP, IHEP (see [13–15, 20] and references therein). After commissioning of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven and the Large Hadron Collider (LHC) at CERN, the cumulative processes can be investigated in a new kinematic region [21]. High sensitivity of elementary constituent interactions to properties of the compressed nuclear matter is expected to be in this region [22, 23].

The concept of z-scaling [24, 25] is based on principles of self-similarity, locality, scale relativity and fractality reflecting the general features of constituent interactions in small scales. A-dependence of z-scaling in inclusive hadron production in $p+A$ collisions at a high transverse momentum and $\theta_{\rm cms} \simeq 90^\circ$ was studied in [26]. The independence of function $\psi(z)$ from center-of-mass energy $\sqrt{s}$ and the angle of the produced particle for different nuclei from D up to Pb was shown. The symmetry transformation, $z \to \alpha z$, $\psi \to \alpha^{-1}\psi$ was used to determine A-dependence of transformation parameter α.

In the paper we analyze spectra of the cumulative pions produced in $p + A$ collisions at FNAL at energy $\sqrt{s} \simeq 27.4$ GeV and angle $\theta_{\rm lab} = 70{-}160^\circ$. A microscopic scenario of constituent interactions in the framework of z-scaling has been developed for cumulative processes. We have verified the hypothesis of self-similarity of the hadron production in $p + A$ collisions in the cumulative region and properties of z-scaling.

The present paper is organized as follows. Main ideas of z-scaling and the method of construction of the scaling function for hadron–nucleus collisions are briefly described in Sec. 1. The data on inclusive spectra of the cumulative pions produced in $p + A$ collisions obtained at FNAL are discussed in Sec. 2. Results of analysis of pion spectra in z presentation are given in Sec. 3. They are compared with the data sets measured by J. Cronin, R. Sulyaev and D. Jaffe groups. Universality of the scaling function are used to calculate high-momentum spectra of pion production in $p + A$ collisions. The obtained results are presented in Sec. 4. A microscopic scenario of the elementary subprocess is discussed in Sec. 5. Discussion is presented in Sec. 6. Conclusions are summarized in the final section.

1. z-SCALING

In this section, we would like to remind the basic ideas of z-scaling dealing with the investigation of the inclusive process. We follow the approach developed in [26]. The main idea of z-scaling is based on the assumptions that the gross feature of the inclusive particle distribution of process $P_1 + P_2 \rightarrow p + X$ at high energies can be described in terms of the corresponding kinematic characteristics of the constituent subprocess written in the symbolic form:

$$(x_1 M_1) + (x_2 M_2) \rightarrow m_1 + (x_1 M_1 + x_2 M_2 + m_2), \tag{1}$$

satisfying the following condition:

$$(x_1 P_1 + x_2 P_2 - p)^2 = (x_1 M_1 + x_2 M_2 + m_2)^2. \tag{2}$$

The equation is the expression of locality of the hadron interaction at a constituent level. The x_1 and x_2 are the fractions of incoming momenta P_1 and P_2 of the colliding objects with masses M_1 and M_2. They determine the minimum energy necessary to produce the secondary particle with mass m_1 and four-momentum p. The parameter m_2 is introduced to satisfy the internal conservation laws (for charge, baryon, isospin, strangeness numbers, and so on).

Equation (2) reflects the minimum recoil mass hypothesis in the elementary subprocess. To connect kinematic and structural characteristics of the interaction, quantity Ω is introduced. It is chosen in the following form:

$$\Omega(x_1, x_2) = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}. \tag{3}$$

Here δ_1 and δ_2 are the fractal dimensions of the colliding objects. The fractions x_1 and x_2 are determined to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling condition (2):

$$\left. \frac{d\Omega(x_1, x_2)}{dx_1} \right|_{x_2 = x_2(x_1)} = 0. \tag{4}$$

The fractions x_1 and x_2 are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

Self-similarity is a scale-invariant property related to dropping of certain dimensional quantities out of the physical picture of the interactions. It means that dimensionless quantities to describe the physical processes are used. Scaling function $\psi(z)$ depends in a self-similar manner on single dimensionless variable z. The function is expressed via the measurable quantities and written in the following form:

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{\text{in}}} J^{-1} E \frac{d^3\sigma}{dp^3}. \tag{5}$$

Here, $Ed^3\sigma/dp^3$ is the invariant cross section, $dN/d\eta$ is the multiplicity density as a function of the center-of-mass collision energy squared s and pseudorapidity η, σ_{in} is the inelastic cross section, J is the corresponding Jacobian. Factor J is the known function of the kinematic variables, the momenta and masses of the colliding and produced particles.

Function $\psi(z)$ is normalized as follows:

$$\int_0^\infty \psi(z)\, dz = 1. \tag{6}$$

The relation allows us to interpret the function as a probability density to produce a particle with corresponding value of variable z. We note that the existence of function $\psi(z)$ depending on the single dimensionless variable z and revealing scaling properties (independence of $\psi(z)$ from collision energy $\sqrt{s}$, an angle of the produced particle) is not evident in advance. The validity of the scaling is confirmed a posteriori.

Self-similarity of an object revealing itself over a wide scale range is the general property of fractality. It means that the measure corresponding to the object diverges in terms of the resolution. In our case this measure is variable z which has the following form:

$$z = z_0 \Omega^{-1}. \tag{7}$$

Here, $z_0 = \sqrt{\hat{s}_\perp}/[m(dN_{\rm ch}/d\eta)]$ is the finite part of z. It is expressed via the ratio of the transverse energy $\sqrt{\hat{s}_\perp}$ released in the binary collision of constituents and the average multiplicity density $dN_{\rm ch}/d\eta$ at $\eta = 0$ and the nucleon mass m. The divergent part Ω^{-1} describes the resolution at which the collision of the constituents can be singled out of this process. The $\Omega(x_1, x_2)$ represents a relative number of all initial configurations containing the constituents which carry fractions x_1 and x_2 of the incoming momenta. The δ_1 and δ_2 are fractal dimensions of the colliding objects. The momentum fractions x_1 and x_2 are determined to minimize the resolution $\Omega^{-1}(x_1, x_2)$ of measure z with respect to all possible subprocesses (1) under condition (2).

Note that ψ and z are the scale-dependent quantities. The both ones depend on the dimensional variables $\sqrt{s}$ and p_T. We assume that the hadron and nucleus interactions at high energies and transverse momenta are interactions of fractals. In this region the internal structure of hadrons, interactions of their constituents and mechanism of hadronization reveal self-similarity.

2. CUMULATIVE HADRON PRODUCTION IN $p + A$ COLLISIONS AT FNAL

Cumulative particles are the particles produced in the kinematical region forbidden for free nucleon–nucleon interactions [13–15] (see also [20]). Such particles can be produced only in the processes with participation of nuclei. Production of such particles does not contradict momentum conservation laws. The interest in the study of cumulative processes is motivated by searching for signatures of the phase transition in the high-compressed nuclear matter.

The data on inclusive invariant cross sections for $\pi^\pm, K^\pm, p^\pm$ hadrons produced in a backward hemisphere in $p + A$ collisions at $p_L = 400$ GeV/c and at angle $\theta_{\rm lab}$ of 70, 90, 118 and 160° were presented in [27]. The measurements were performed over the momentum range of $0.2 < p < 1.25$ GeV/c using the Li, Be, C, Al, Cu, Ta nuclear targets. These data cover, in particular, the low-p_T kinematical rigion forbidden for particle production in free nucleon–nucleon collisions, known as a cumulative region.

Figure 1 shows the inclusive cross sections for the π^+ mesons produced in the backward hemisphere in $p + A$ collisions at momentum $p_{\rm lab} = 400$ GeV/c and angle $\theta_{\rm lab}$ of 70–160°. As seen from Fig. 1, the strong dependence of the cross section on angle $\theta_{\rm lab}$ is observed for all Li, Be, C, Al, Cu, Ta nuclear targets. The difference of cross sections at $\theta_{\rm lab} = 90$ and 160° at momentum $p = 1{-}1.2$ GeV/c reaches 2–3 orders of the magnitude.

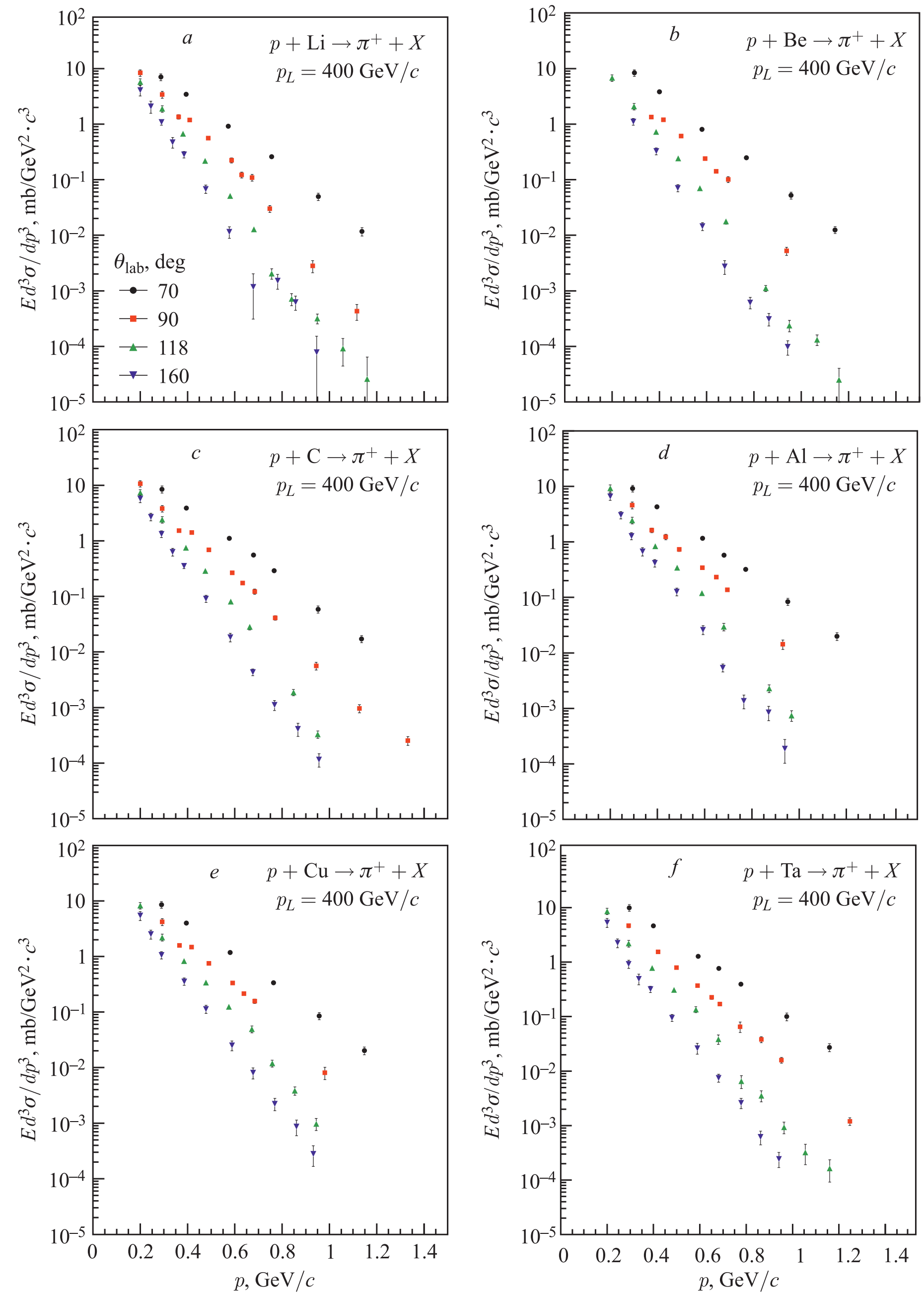

Fig. 1. Transverse momentum spectra of the π^+ mesons produced in $p + A$ collisions at incident proton momentum $p_L = 400$ GeV/c and angle $\theta_{\rm lab} = 70$, 90, 118 and 160° in p presentation. The experimental data are taken from [27]

3. INCLUSIVE $p + A$ SPECTRA IN z PRESENTATION

Below we follow the procedure of the data analysis used in [26]. Function ψ is calculated for every nucleus using Eq. (5) with normalization factor $\sigma_{\rm in}^{pA}/\sigma_{\rm in}^{pp}$ instead of $\sigma_{\rm in}$. Factor $\sigma_{\rm in}^{pA}$ is the total inelastic cross section for $p + A$ interactions. The multiplicity density of charged particles for different nuclei is parametrized by the following formula:

$$\rho_A(s) \simeq 0.67 A^{0.18} \cdot s^{0.105}, \quad A \geqslant 2. \tag{8}$$

The scaling functions for different nuclei obtained in this way have revealed the energy and angular independence [26].

Figure 2 shows z presentation of the data from [27]. The data from [28, 29] and [30] for the deuteron target are given for comparison. One can see that the curves found for the Li, Be, C, Al, Cu, Ta nuclei are in agreement with the data z presentation for D nucleus at p_L = 70, 400 GeV/c and $\theta_{\rm cms} = 90°$.

We assume that the shape of the scaling curve should be the same as for the data points corresponding to the high-p_T region and $\theta_{\rm cms} \simeq 90°$. This hypothesis corresponds to validity of self-similarity of the hadron production over a wider kinematical region.

The symmetry transformation

$$z \to \alpha(A) \cdot z, \quad \psi \to \alpha^{-1}(A) \cdot \psi \tag{9}$$

of function $\psi(z)$ and argument z was used to compare functions ψ for different nuclei. A-dependence of the parameter α was found in [26] and described by $\alpha(A) = 0.9A^{0.15}$.

There are no experimental data on the angular dependence of $\rho(s, \eta, A)$ for particles produced in a backward hemisphere to obtain a normalization factor for the scaling function. Therefore, we have verified a possibility to restore the shape of the $\psi(z)$ found from the analysis of high-p_T data [28–30] using low-p_T data [27].

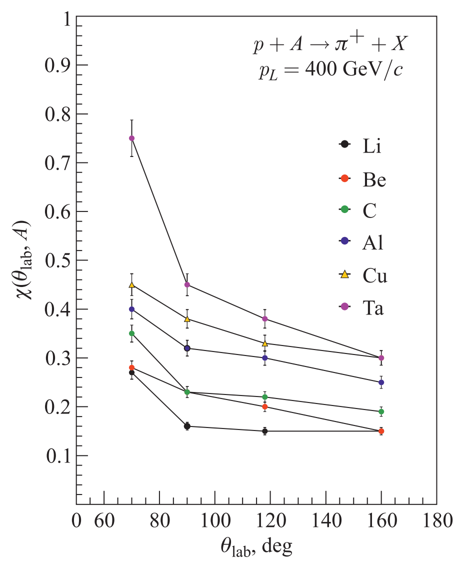

Fig. 2. The dependence of $\chi(\theta_{\rm lab}, A)$ on angle $\theta_{\rm lab}$ of a produced particle and atomic weight A

Function $\rho(s, \eta, A)$ has been parameterized in the form of $\rho(s, \eta, A) = \rho(s, A) \cdot \chi(\theta_{\rm lab}, A)$, where the angular dependence is described by $\chi(\theta_{\rm lab}, A)$. It was found that $\chi(\theta_{\rm lab}, {\rm Ta}) = 0.75$ and 0.3 at $\theta_{\rm lab}$ = 70 and 160° in the laboratory system frame (the fixed target frame), respectively. Ratio $\chi(\theta_{\rm lab}, {\rm Ta})/\chi(\theta_{\rm lab}, {\rm Li})$ decreases from 3.5 to 1.5 as the angle increases from 70 to 160°. A-dependence of the ratio has demonstrated saturation for the backward particle production.

The angular dependence $\chi(\theta_{\rm lab}, A)$ which restores the scaling function is shown in Fig. 2. We see that saturation of χ increases with $\theta_{\rm lab}$. It should be noted that function $\rho(s, \eta, A)$ is a normalization factor for the inclusive cross section. The fact that the single factor can restore the shape of $\psi(z)$ over a wide range of z is an unexpected result. Experiments are necessary to verify this observation.

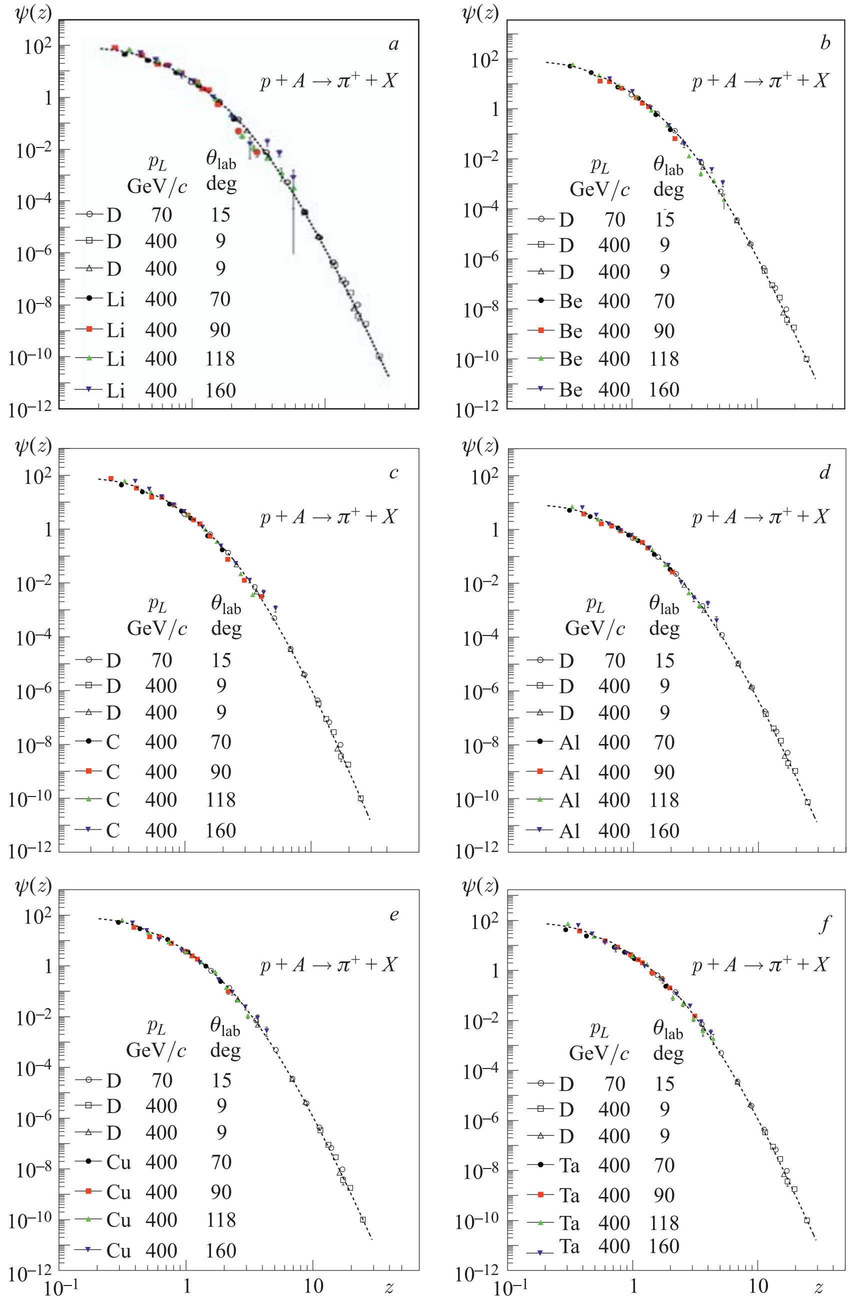

Fig. 3. Scaling function $\psi(z)$ versus variable z for the π^+ mesons produced in $p + A$ collisions at momentum $p_L = 400$ GeV/c and angle $\theta_{\rm lab} = 70$, 90, 118 and 160°. The dotted line is the result of fitting. It is the same on all graphs. The points are z presentation of the experimental data taken from [27–30]

Figure 3 shows the data from [27] on the inclusive spectra of the π^+ mesons produced in $p + A$ collisions at $p_L = 400$ GeV/c and $\theta_{\rm lab} = 70,\ 90,\ 118,\ 160°$ for the Li, Be, C, Al, Cu, Ta nuclei in z presentation. Taking into account experimental errors, we can conclude that the shape of $\psi(z)$ can be restored using the angular dependence of the multiplicity density. We would like to note that all the data points [27] are out of the asymptotic region of $\psi(z)$ described by the power law. Nevertheless, the low- and high-p_T data overlap each other. They have demonstrated a deviation from the law $\psi(z) \sim z^{-\beta}$ for $z < 4$. Verification of the power law in cumulative processes for $z > 4$ is of interest to search for the signatures of the phase transition (see [22, 23] and references therein).

It was noted in [24, 25] that the scaling function reveals the power behavior $\psi(z) \simeq z^{-\beta}$ in both the low- and high-z range for particle production in the central rapidity range. The value of slope parameter β was found to be about zero for low z and a nonzero constant value for $z > 4$. The first regime corresponds to exponential dependence of the inclusive cross section on p_T. The second one is described by the power law.

To approximate the experimentally established shape of $\psi(z)$, we have used the parametrization of the scaling function in the Tsallis form [31]:

$$\psi(z) = C[1 + (q-1)z/T]^{1/(1-q)}. \tag{10}$$

This form is flexible enough to describe inclusive cross sections [32]. All the fitting parameters, C, T and q, are dimensionless. We have used only high-p_T data for D nucleus and low-p_T data for Al nucleus for the fitting procedure. The values of these parameters were found to be as follows: $q = 1.083$, $T = 0.2189$, $C = 177.4$. The dotted lines in Fig. 3 correspond to our calculations of $\psi(z)$ with the found parameters.

4. HIGH MOMENTUM SPECTRA IN $p + A$ COLLISIONS IN THE CUMULATIVE REGION

The universal shape of the scaling function allows us to predict inclusive spectra in the region which has not been available up to now. It is expected that mechanisms of hadron production in the cumulative and noncumulative regions differ from each other. As a result, the additive law, $\delta_A = A{\cdot}\delta$, for fractal dimensions of nuclei could be violated. The cumulative region corresponds to the regime of particle production in the strongly compressed nuclear matter. We assume that a nucleus size could be of the order of a nucleon size in the deep cumulative region (the region near the kinematic boundary of the reaction). For the process the momentum of the inclusive particle should be fully balanced by the momentum of the recoil system consisting of very slow constituents. The system in the state should demonstrate the property of collectivity. Therefore, a transition regime from single constituent interactions to collective phenomena is expected.

Our predictions are based on self-similarity of constituent interactions at high p_T. Therefore, extrapolation of a cross section far from the nucleon–nucleon kinematical boundary could allow us to verify simultaneously the power law, $\psi(z) \simeq z^{-\beta}$, and search for its violation. Verification of self-similarity of hadron production in a cumulative region at a constituent level could give us a new insight into collective phenomena of the nuclear matter.

Figure 4 demonstrates the dependence of the inclusive cross section $Ed^3\sigma/dp^3$ on the momentum p of the particle produced in $p + A$ collisions at incident proton momentum

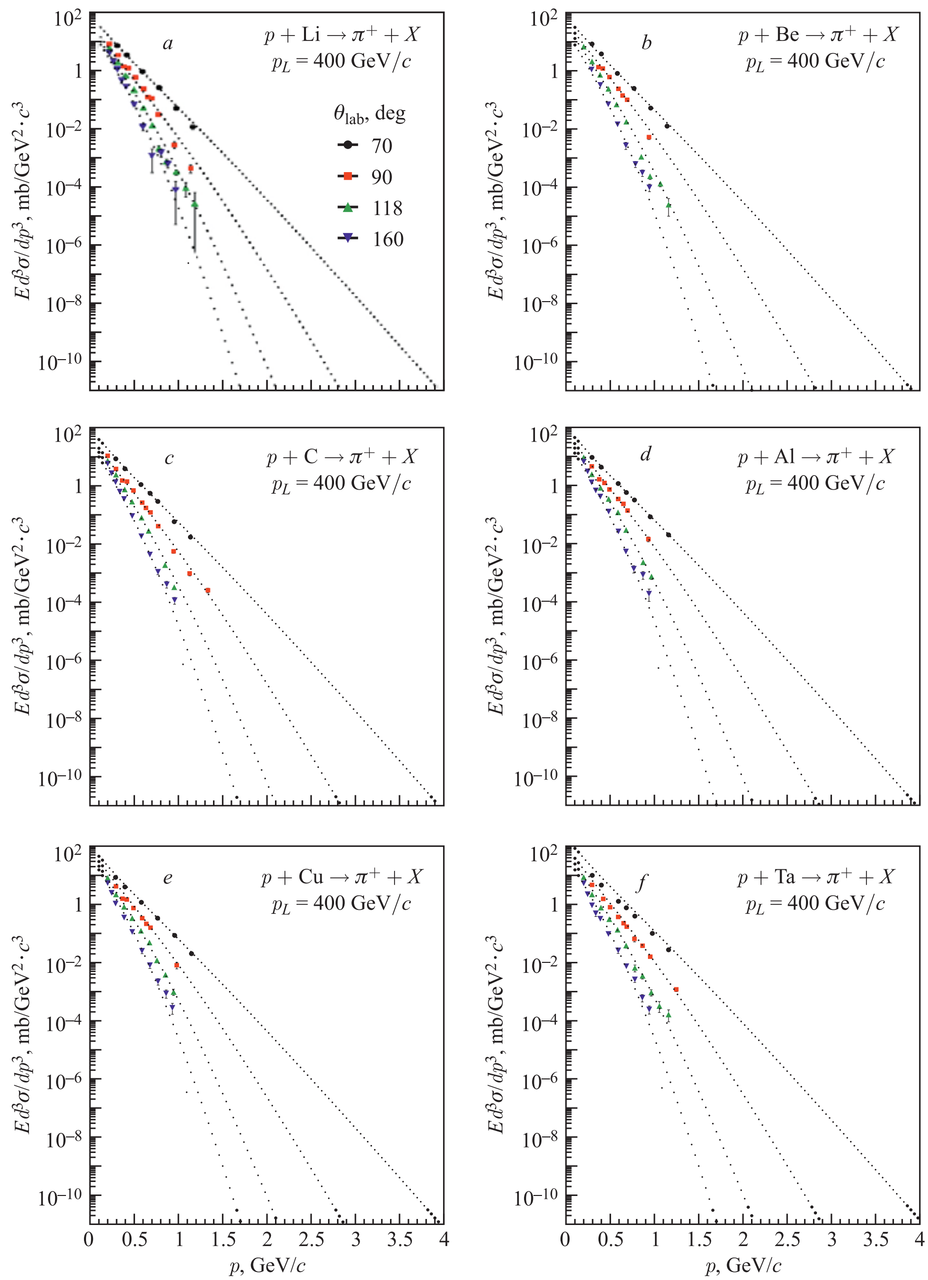

Fig. 4. Transverse momentum spectra of the π^+ mesons produced in $p + A$ collisions at momentum $p_L = 400$ GeV/c and angle $\theta_{\text{lab}} = 70$, 90, 118 and 160° in p presentation. The dotted lines are predictions based on z-scaling. The points are the experimental data taken from [27]

Kinematical boundary for the π mesons produced in $p + A$ collisions at $p_L = 400$ GeV/c for $\theta_{\rm lab} = 70,\ 90,\ 118$ and $160°$

A	Proton	Li	Be	C	Al	Cu	Ta
$\theta_{\rm lab}$, deg	$p_{\max}$, GeV/c						
70	1.41	9.82	12.6	16.6	36.2	79.9	190.0
90	0.93	6.51	8.35	11.1	24.6	56.1	144.0
118	0.63	4.45	5.72	7.62	17.0	39.8	109.0
160	0.47	3.38	4.35	5.80	13.0	30.9	87.1

$p_L = 400$ GeV/c and angle $\theta_{\rm lab} = 70,\ 90,\ 118, 160°$. The experimental data [27] are shown by symbols. Our calculations are drawn by the dotted lines. From Fig. 4 one can see that the cross section rapidly decreases with the momentum. For the Be target and $\theta_{\rm lab} = 160°$ it drops by more than six orders of magnitude for $p = 1$ and 1.5 GeV/c, respectively. Kinematical boundaries for cumulative production in these processes are shown in table. We expect that experimental measurements of spectra up to $p = 1.7$ and 4 GeV/c at $\theta_{\rm lab} = 160$ and $70°$ allow us to test the power law up to $z \simeq 30$.

5. MOMENTUM FRACTIONS x_1, x_2 AND $z{-}p$ PLOT

Firure 5 shows the dependence of fractions x_1 and x_2 on momentum p for $p+C$ collisions at $p_L = 400$ GeV/c and $\theta_{\rm lab} = 70,\ 90,\ 118$ and $160°$. Fraction x_1 corresponds to fragmentaion of the incident proton and x_2 to fragmentaion of the nucleus. The both ones are restricted:

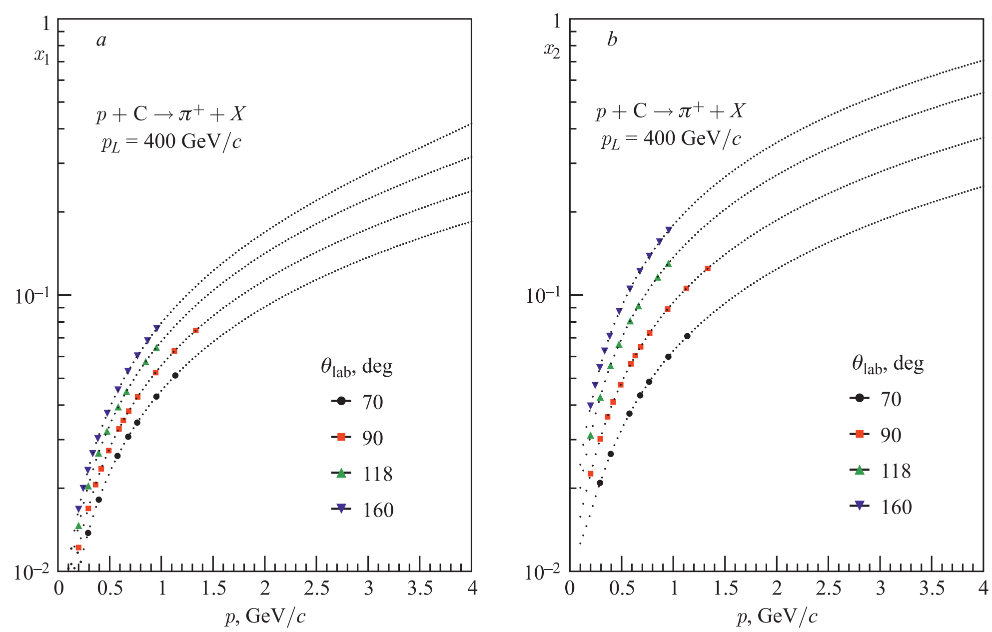

Fig. 5. Dependence of fractions x_1 (*a*) and x_2 (*b*) on momentum p for $p + C$ collisions at $p_L = 400$ GeV/c and $\theta_{\rm lab} = 70,\ 90,\ 118$ and $160°$

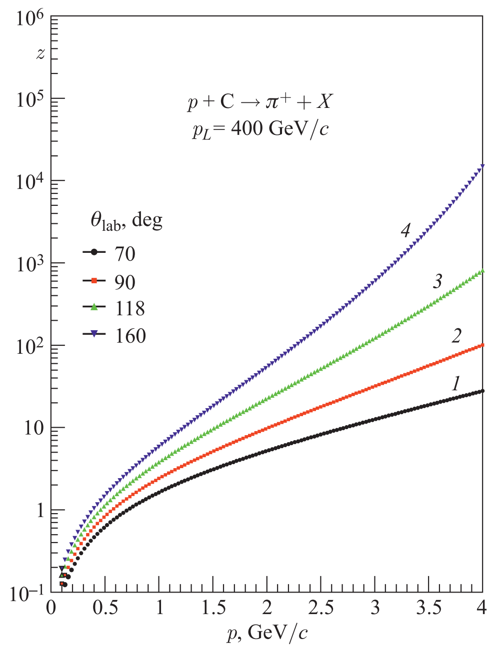

Fig. 6. The $z{-}p$ plot for $p + C$ collisions at $p_L = 400$ GeV/c. Dotted lines *1, 2, 3* and *4* correspond to $\theta_{\rm lab} = 70$, 90, 118 and 160°, respectively

$0 < x_1, x_2 < 1$. The normalization conditions — the momentum fraction x_1 per nucleon and x_2 per nucleus, were used in the paper. We see that the fractions increase with the transverse momentum and registered angle. The cumulative region covers the range $x_2 > 1/A$ and noncumulative — $x_2 < 1/A$ for fragmentation of a nucleus.

Figure 6 demonstrates the dependence of scaling variable z on momentum p for different values of angle $\theta_{\rm lab}$. The plot allows us to choose the kinematic region in order to verify the power law for $\psi(z)$. For $\theta_{\rm lab} = 160°$ and $p > 0.47$ GeV/c the function is determined in the cumulative region.

6. DISCUSSION

Self-similarity of constituent interactions at high energies was studied in z-scaling approach for different inclusive processes in pp and $\bar{p}p$ collisions. The flavor independence of the scaling function over a wide range of z has been found [25]. This result cogently indicates fractal properties of the internal hadron structure and constituent interactions. A more sophisticated approach developed in [24] allowed us to analyze the hadron spectra in AA collisions as well [33–35]. The main goal of the study is to search for signatures of the critical point and phase transitions of the nuclear matter. A constituent energy loss as a function of energy and centrality of the collision and transverse momentum of the inclusive particle is assumed to be a good feature of the produced medium. The energy loss was found to increase with energy $\sqrt{s}$ and multiplicity density $dN_{\rm ch}/d\eta$, and decrease while momentum p_T increasing.

Complementary information on constituent interactions and properties of multiparticle system can be obtained in $p + A$ collisions, too. We expect that such information allows us to clarify properties of transition from the hadron to nuclear medium at different scales. Modification of an elementary subprocess is assumed to be stronger in the region forbidden for particle production on free nucleons. This region is known as a cumulative one. It can be experimentally reached at a collider in the central rapidity range and in backward semisphere production in the fixed target experiments. Thus, both the high-p_T and low-p_T regions can be studied in the z-scaling approach. The experiment performed by Leksin group [15] is the experiment of the fixed target type. Note that the value of transverse momentum p_T does not exceed 0.35 and 1.3 GeV/c at 160 and 90°, respectively. As seen from Fig. 2, the data points lie in the range $0.2 < z < 3$ beyond the power law found for $z < 0.1$ and $z > 4$. We see that the cross sections are sensitive to p in the high momentum range. The decrease of the section is more than seven orders of magnitude for $p = 1 - 1.7$ GeV/c at $\theta_{\rm cms} = 90°$. The fact that the shape of $\psi(z)$ found at $\theta_{\rm lab} = 70{-}160°$ coincides with the shape at $\theta_{\rm lab} = 9°$ is an unexpected result. These kinematic regions are quite different. Therefore, mechanisms of constituent interactions could also be different. Nevertheless, the both ones have demonstrated the self-similarity property. It is of interest to verify the power behavior of $\psi(z)$ for cumulative production in $p + A$ collisions.

CONCLUSIONS

The experimental data on inclusive spectra of the pions produced in $p+A$ collisions at the incident proton momentum $p_L = 400$ GeV/c obtained by G. Leksin group were analyzed in the framework of z-scaling. Self-similarity of cumulative hadron production in low-p_T region was verified. Scaling function $\psi(z)$ is expressed via invariant cross section $Ed^3\sigma/dp^3$ and average multiplicity density $dN_{\rm ch}/d\eta$ of charged particles. It is interpreted as a probability density to produce a particle with a given value of z. Quantity z has a property of the fractal measure, and δ_1, δ_2 are the fractal dimensions of colliding particles.

We have verified the hypothesis that z-scaling reflects the fundamental symmetries such as locality, self-similarity, and fractality of hadron interactions at a constituent level in cumulative processes. The results of our analysis have been compared with the high-p_T data sets obtained by J. Cronin, R. Sulyaev and D. Jaffe groups. We have found that the shape of $\psi(z)$ is the same in the overlapping region. We conclude that the obtained results are in agreement with a microscopic scenario of $p + A$ interactions at a constituent level.

The parameterization of the scaling function in the Tsalis form was used to predict inclusive cross sections of the π^+-meson production in $p + A$ collisions on the Li, Be, C, Al, Cu, Ta targets in the deep-cumulative region ($x_2 \gg 1/A$).

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