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ROBUST FITTING FOR THE ESTIMATION OF HIDDEN PARAMETERS IN EXPERIMENTAL DISTRIBUTIONS ON THE PLANE

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The registration of two event characteristics, for example, e (energy) and t (time), gives us a two-dimensional experimental distribution $A(e,t)$, where the extraction of the information of interest stumbles upon the absense of the appropriate parametrical model. A significant advantage of using the autocorrelation spectrometer of delayed coincidences is the possibility to determine the scintillator decay time τ from an energy dependence $E = g(t,\tau)$, which, unlike the classical time decay model $N(t) = f(t,\tau)$, allows us to establish its parametrical correspondence to some elements of the two-dimensional data and so to estimate τ by a procedure of the robust fitting, which automatically suppresses the impact of parts of data not adequate to the model.

Регистрация двух характеристик событий, например, e (энергия) и t (время), дает двумерное экспериментальное распределение $A(e,t)$, где извлечение интересующей нас информации наталкивается на отсутствие соответствующей параметрической модели. Существенным преимуществом использования автокорреляционного спектрометра задержанных совпадений является возможность определять время высвечивания сцинтиллятора τ из энергетической зависимости $E = g(t,\tau)$, которая, в отличие от классической модели $N(t) = f(t,\tau)$, позволяет установить его параметрическое соответствие некоторым элементам двумерных данных и оценить τ с помощью процедуры робастной подгонки, которая автоматически подавляет действие тех частей данных, которые не адекватны модели.

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INTRODUCTION

The registration of two event characteristics, for example, e (energy) and t (time), gives us a two-dimensional experimental distribution $A(e,t)$, where the extraction of the information of interest stumbles upon the absense of the appropriate parametrical model. Let us consider an experimental distribution $A(e,t)$, where e (energy) and t (time) — coordinates from the axes $[0,E)$ and $[0,T)$ — are connected by a parametric dependence

$$f(e,t,P) = 0. \tag{1}$$

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Typical objectives of the analysis of this distribution are as follows:

1) to estimate the vector of the parameters P from the experimental values of $A(i,j)$ and to determine the precision of such estimates; here i, j are channels, corresponding to the physical values of e, t.

2) to test the hypothesis of $P = P_0$, where P_0 is an a priori estimate of the vector P.

However, the solution of these tasks meets serious difficulties. The dependence (1) in the experimental data $A(e,t)$ is not always visible directly. The parameters P can be defined, if one could select (or build) a component $f_{\exp}(e,t)$ of $A(e,t)$, which would approximate the function $f(e,t,P)$ with an acceptable accuracy, and on the basis of this approximation one would build an estimate of the parameters P. But this is not a very simple case — in such a function $f_{\exp}(e,t)$ there will be a lot of elements, which are not adequate to the model $f(e,t,P)$.

Let us assume that we can consider (1) as the relation between the variables e and t, that will allow us to transform $f(e,t,P)$ to the function $f(e(t),t,P)$, i.e., in the function of one variable $g(t,P)$. Similarly, instead of $f_{\exp}(e,t)$ we will obtain the function $u_{\exp}(t)$.

1. THE SOLUTION

Methods of semimanual recognition of functional dependences that would allow us to identify and delete (or suppress) the disturbing elements in $u_{\exp}(t)$, are very time- and effort-consuming and are not suitable for a mass analysis of spectra.

We use the following approach:

• A formal definition of the background object of a one-dimensional distribution (taking into account that $e = e(t)$) is formulated; this function $\mathrm{err}\,(t)$ will be later either suppressed or significantly diminished. In the general case $\mathrm{err}\,(t)$ contains both deterministic and stochastic components.

• A function $u_{\exp}(t)$ is found, which within the accuracy defined by the disturbing elements $\mathrm{orr}\,(t)$, is close to the desired $u(t,P)$. We can denote the sum of both errors as $d(t) = \mathrm{err}\,(t) + \mathrm{orr}\,(t)$ and consider it as a total error of the approximating function $g(t,P)$.

• The robust approximation (stable to various inadequacies between the data and its model) of the experimental function $u_{\exp}(t) = u(t) + d(t)$ by the model $g(t,P)$ is used.

2. THE FITTING

The robust approximation (fitting) in the quadratic metric [2] is the most natural when the deformed data are fitted. The idea of the method used can be illustrated by an example of a variational task: to find a function, which is the closest to a given one in terms of the quadratic metric.

So, we have a function $u_{\exp}(t)$ and the class of functions $G = (g(t))$, where we look for an element $g_0(t)$ such that it is the most close to $u_{\exp}(t)$ in the sense of some weighted quadratic metric on a set of points $T = t_1, t_2, t_3, \ldots, t_m$:

$$g_0(t) = \mathrm{ARG}\min \sum_{i=1}^{m} w(t_i)(u_{\exp}(t_i) - g(t_i))^2, \tag{2}$$

where the minimum is sought in the class G, and $w(t)$ is a weight function, taking into account $d(t)$, i.e., the fact that in the class G there is no function exactly coinciding with $u_{\exp}(t)$ at all points of the T set.

It is obvious that the function $g_0(t)$ satisfying (2) should also satisfy the condition

$$\sum_{i=1}^{m} w(t_i)(u_{\exp}(t_i) - g_0(t_i)) = 0. \tag{3}$$

This shows that our task is to build such a weight function $w(t_i)$, which would have the following property:

$$w(t_i) = \begin{cases} 1 & \text{if } u_{\exp}(t_i) = u(t_i), \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The equality to zero or the unit in the case of perturbed data will naturally hold only approximately and, therefore, the solution of the problem will also be approximate. To find such a weight function, we can use different algorithms of pattern recognition, but if the class G is parameterized, i.e., $G = g(t, P)$ and all its functions differ only by the values of the parameters P, then the idea of generalized weight function (4) can be the basis of the method of robust fitting [1].

Let us consider this question in detail. In the fitting procedure the solution $g(t, P_0)$ will fulfill the requirement: in the space of the parameter values (P) minimize the expression

$$S(P) = \sum_{i=1}^{m} w(t_i)(u_{\exp}(t_i) - g(t_i, P))^2. \tag{5}$$

A necessary condition of this minimum is

$$\frac{\partial S}{\partial p_j} = \sum_{i=1}^{m} w(t_i)(u_{\exp}(t_i) - g(t_i, P))\frac{\partial g(t_i, P_0)}{\partial p_i}) = 0, i = 1, 2, \ldots, n, \tag{6}$$

where p_i are components of the vector P, and the function

$$w(t, P_a) = \begin{cases} \dfrac{1}{\|d(t)\|^2} & \text{if } \mid h(t) \mid < c, \\ \dfrac{(1+\beta)}{\|e(t)\|^2 (h(t)/c)^2 + \beta} & \text{otherwise,} \end{cases} \tag{7}$$

is taken as such a weight function. Here $h(t) = u_{\exp}(t) - g(t, P)$, P_a is an a priori estimate of P_0 (in particular, the value at the previous iteration in a nonlinear process), c and β are constants determining the strategy of the robust fitting.

The weights (7) suppress a substantial part of the noise automatically, i.e., at those points of t, where the function $u_{\exp}(t)$ obviously cannot be fitted by the function $g(t, P)$; its impact on the process of minimization is diminished so that there is a chance to approximate the function $u(t, P)$ correctly, at least, on a part of the T set.

The filtering mechanism for the correct decision in this approach, unlike the method of penalty functions, where large deviations from the true functions are punished, here is encouraged but at the same time neutralized.

Additionally, the formalism of the quadratic fitting allows us to estimate at least a part of the statistical accuracy of the obtained parameters.

The described method was implemented as part of the DELPHI program suite VMRIA [3].

3. EXAMPLES

In the autocorrelation method using the delayed coincidences for the determination of the decay constants by different scintillators [1] we register the distribution $A(e,t)$ as a function of the two variables e and t, where the exponentials from which these constants may be derived are not presented explicitly; they may be obtained from $A(e,t)$ by different smoothing and averaging procedures, but the obtained functions are not pure exponentials — they contain a lot of «interfering» elements. And here the use of robust fitting was the only effective and at the same time an automatic means to obtain the estimates of parameters with an acceptable accuracy. (See also Figs. 1 and 2.)

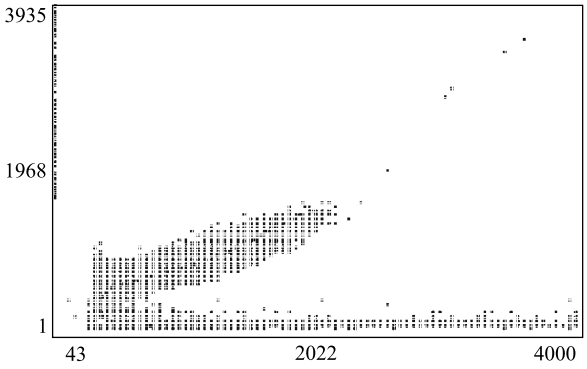

Fig. 1. A two-dimensional distribution $A(e,t)$

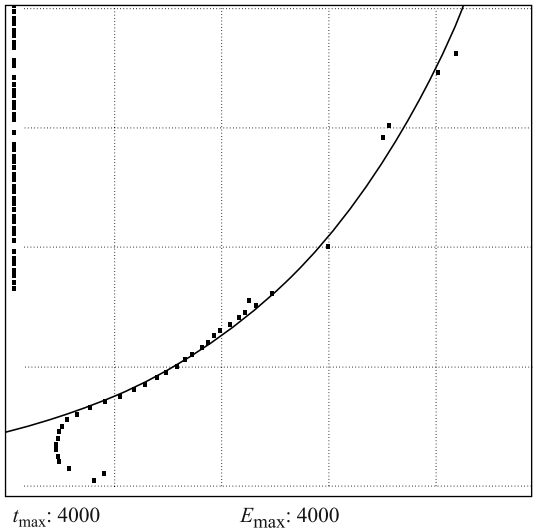

Fig. 2. The fitting function goes correctly through the points where it is adequate to the model, and ignores those where it is not (on the left side and at the bottom). The obtained result is in agreement with the known data ($\tau = 230$ ns) [4]

CONCLUSION

This is an approach which allows us to perform a mass analysis of two-dimensional spectra from the scintillator detectors with a satisfactory accuracy in an automatic mode. Its methodical advantage consists in the possibility to determine the scintillator decay time τ from an energy dependence $E = g(t, \tau)$, which, unlike the classical time model $N(t) = f(t, \tau)$, allows us to establish its parametrical correspondence to informative elements of the two-dimensional data and so to estimate τ by a procedure of the robust fitting.

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