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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

DYNAMICAL REALIZATIONS OF NONRELATIVISTIC CONFORMAL GROUPS

A. Galajinsky [1], *I. Masterov*

Laboratory of Mathematical Physics, Tomsk Polytechnic University, Tomsk, Russia

Dynamical realizations of the l-conformal Galilei algebra and its Newton–Hooke counterpart in terms of the second-order differential equations are discussed.

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INTRODUCTION

Conformal many-body mechanics in one dimension is being extensively investigated for more than four decades. In the early days, the issues of integrability and exact solvability were dominant [1, 2]. More recently, various supersymmetric extensions [3–5] attracted considerable interest (for a review and references to the original literature, see [6]). As $d > 1$ is physically more realistic, it is natural to wonder what happens beyond one dimension. This invokes nonrelativistic conformal algebras. Such algebras also play an important role within the context of the nonrelativistic AdS/CFT correspondence [7] which motivated extensive recent studies in [8–32].

In general, conformal extensions of the Galilei algebra [33] or its Newton–Hooke counterpart [2] [21] are parameterized by a positive half-integer l such that $(2l + 1)$ vector generators enter the algebra. Along with spatial translations and the Galilei boosts, they involve accelerations. In constructing dynamical realizations, generators in the algebra of symmetries are linked to constants of the motion. As the number of functionally independent constants of the motion needed to integrate a differential equation correlates with its order, dynamical realizations of the l-conformal Galilei algebra or its Newton–Hooke counterpart in general involve higher derivative terms (see, e.g., [8, 9, 20, 23, 24, 28, 29, 31]). Dynamical realizations without higher derivatives have been constructed quite recently in [19, 26, 27] within the method of nonlinear realizations.

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[1] E-mail: galajin@tpu.ru

[2] The Newton–Hooke algebra is an analogue of the Galilei algebra in the presence of a universal cosmological repulsion or attraction [34]. It is derived from the (anti-) de Sitter algebra by the nonrelativistic contraction in the same way as the Galilei algebra is obtained from the Poincaré algebra. In the limit in which a cosmological constant tends to zero the Newton–Hooke algebra reproduces the Galilei algebra.

The purpose of this work is to review the peculiar features of the formulations proposed in [26,27] with a particular emphasis on the qualitative difference between the results obtained for the l-conformal Galilei algebra and its Newton–Hooke counterpart. The work is organized as follows. In Sec. 1, the method of nonlinear realizations is applied to construct a dynamical realization of the l-conformal Galilei algebra in terms of the second-order differential equations. Similar analysis for the l-conformal Newton–Hooke algebra is accomplished in Sec. 2. We confront the results obtained for the two algebras in the concluding Sec. 3.

1. DYNAMICAL REALIZATION OF l-CONFORMAL GALILEI ALGEBRA

The l-conformal Galilei algebra includes the generators of time translations, dilatations, special conformal transformations, spatial rotations, spatial translations, Galilei boosts and accelerations. Denoting the generators by $(H, D, K, M_{ij}, C_i^{(n)})$, respectively, where $i = 1, \ldots, d$ is a spatial index and $n = 0, 1, \ldots, 2l$, one has the structure relations [33]:

$$
\begin{gathered}
[H, D] = iH, \quad [H, C_i^{(n)}] = inC_i^{(n-1)}, \\
[H, K] = 2iD, \quad [D, K] = iK, \\
[D, C_i^{(n)}] = i(n-l)C_i^{(n)}, \quad [K, C_i^{(n)}] = i(n-2l)C_i^{(n+1)}, \\
[M_{ij}, C_k^{(n)}] = -i(\delta_{ik}C_j^{(n)} - \delta_{jk}C_i^{(n)}), \\
[M_{ij}, M_{kl}] = -i(\delta_{ik}M_{jl} + \delta_{jl}M_{ik} - \delta_{il}M_{jk} - \delta_{jk}M_{il}).
\end{gathered} \tag{1}
$$

Note, that (H, D, K) form $so(2,1)$ subalgebra, which is the conformal algebra in one dimension. The instances of $n = 0$ and $n = 1$ in $C_i^{(n)}$ correspond to the spatial translations and Galilei boosts. Higher values of n are linked to accelerations.

In order to construct a dynamical realization of this algebra, let us apply the method of nonlinear realizations along the lines proposed in [19]. As the first step, one considers the coset space [1]

$$
\tilde{G} = \mathrm{e}^{itH}\mathrm{e}^{izK}\mathrm{e}^{iuD}\mathrm{e}^{ix_i^{(n)}C_i^{(n)}} \tag{2}
$$

parameterized by the coordinates $(t, z, u, x_i^{(n)})$. Left multiplication by a group element $g = \mathrm{e}^{iaH}\mathrm{e}^{ibK}\mathrm{e}^{icD}\mathrm{e}^{i\lambda_i^{(n)}C_i^{(n)}}\mathrm{e}^{\frac{i}{2}\omega_{ij}M_{ij}}$ determines the action of the group on the coset space. Then, one considers the subgroup $G = \mathrm{e}^{itH}\mathrm{e}^{izK}\mathrm{e}^{iuD}\mathrm{e}^{ix_i^{(n)}C_i^{(n)}}$ and constructs the left-invariant Maurer–Cartan one-forms

$$
G^{-1}dG = i(\omega_H H + \omega_K K + \omega_D D + \omega_i^{(n)}C_i^{(n)}), \tag{3}
$$

where

$$
\begin{gathered}
\omega_H = \mathrm{e}^{-u}dt, \quad \omega_K = \mathrm{e}^{u}(z^2dt + dz), \quad \omega_D = du - 2z\,dt, \\
\omega_i^{(n)} = dx_i^{(n)} - (n-l)x_i^{(n)}\omega_D - (n+1)x_i^{(n+1)}\omega_H - (n-2l-1)x_i^{(n-1)}\omega_K.
\end{gathered} \tag{4}
$$

[1] As usual, summation over repeated indices is understood.

In the last line it is assumed that $x_i^{(-1)} = x_i^{(2l+1)} = 0$. Finally, let us impose the constraints on the Maurer–Cartan one-forms

$$\omega_D = 0, \quad \gamma^{-1}\omega_K - \gamma\omega_H = 0, \quad \omega_i^{(n)} = 0, \tag{5}$$

where γ is an arbitrary (coupling) constant. Taking t to be the temporal coordinate and introducing the new variable $\rho = \mathrm{e}^{\frac{u}{2}}$, one can get rid of the variable z via $z = \dot{\rho}/\rho$, while the equations which govern the dynamics read

$$\ddot{\rho} = \frac{\gamma^2}{\rho^3}, \qquad \rho^2 \dot{x}_i^{(n)} = (n+1)x_i^{(n+1)} - (2l-n+1)\gamma^2 x_i^{(n-1)}. \tag{6}$$

Note, that ρ describes the conformal particle in one dimension [35] and the first two constraints in (5) coincide with those in [36].

Let us rewrite the rightmost equation in (6) in the matrix form

$$\rho^2 \frac{d}{dt} x^{(n)} = x^{(m)} A^{mn}, \tag{7}$$

where $x^{(n)} = (x^{(0)}, \ldots, x^{(2l)})$. For the discussion to follow, the spatial index i carried by $x^{(n)}$ is inessential and will be omitted. For integer l the matrix A^{mn} is degenerate and has the following eigenvalues:

$$(0, \pm 2i\gamma, \pm 4i\gamma, \pm 6i\gamma, \ldots, \pm 2li\gamma). \tag{8}$$

As A^{mn} is real, all the eigenvectors occur in complex conjugate pairs, but for the eigenvector corresponding to the zero eigenvalue, which is real. Let us denote the eigenvectors by $v^0_{(n)}$, $v^1_{(n)}$, $\bar{v}^1_{(n)}$, $\ldots$, $v^l_{(n)}$, $\bar{v}^l_{(n)}$, where the superscript refers to the number of the corresponding eigenvalue, the bar stands for complex conjugate, and $n = 0, \ldots, 2l$. In particular, in this notation, $v^1_{(n)}$ is related to the eigenvalue $2i\gamma$, while $\bar{v}^1_{(n)}$ is linked to $-2i\gamma$. As usual, the eigenvectors are defined up to a factor.

Contracting the master equations (7) with the eigenvectors of A^{mn}, one gets

$$\begin{aligned} &\rho^2 \frac{d}{dt}\left[x^{(n)} v^0_{(n)}\right] = 0, \\ &\rho^2 \frac{d}{dt}\left[x^{(n)}(v^p_{(n)} + \bar{v}^p_{(n)})\right] = 2p\gamma\left[ix^{(n)}(v^p_{(n)} - \bar{v}^p_{(n)})\right], \\ &\rho^2 \frac{d}{dt}\left[ix^{(n)}(v^p_{(n)} - \bar{v}^p_{(n)})\right] = -2p\gamma\left[x^{(n)}(v^p_{(n)} + \bar{v}^p_{(n)})\right], \end{aligned} \tag{9}$$

where $p = 1, \ldots, l$. Thus, it is natural to introduce the new fields

$$x^{(n)} v^0_{(n)}, \quad x^{(n)}(v^p_{(n)} + \bar{v}^p_{(n)}), \quad ix^{(n)}(v^p_{(n)} - \bar{v}^p_{(n)}). \tag{10}$$

As $x^{(n)} v^0_{(n)}$ obeys the first-order equation, on physical grounds it seems reasonable to discard it. The second line in (9) allows one to express $ix^{(n)}(v^p_{(n)} - \bar{v}^p_{(n)})$ via $x^{(n)}(v^p_{(n)} + \bar{v}^p_{(n)})$. The latter define a set of dynamical fields

$$\chi^p_i = x^{(n)}_i(v^p_{(n)} + \bar{v}^p_{(n)}), \tag{11}$$

where $p = 1, \ldots, l$ and $i = 1, \ldots, d$, which obey the equations of motion

$$\rho^2 \frac{d}{dt}\left(\rho^2 \frac{d}{dt}\chi_i^p\right) + (2\gamma p)^2 \chi_i^p = 0. \tag{12}$$

It is to be remembered that (12) should be solved jointly with $\ddot{\rho} = \gamma^2/\rho^3$.

Note that, given l, (12) contains a chain of oscillator-like equations with growing frequency. In particular, the value $(2\gamma)^2$ appears for any l, the equation involving $(4\gamma)^2$ is common for all $l > 1$, the frequency $(6\gamma)^2$ is shared by all $l > 2$, etc. The reason why one can realize different l-conformal Galilei groups in one and the same equation is that all the vector generators $C_i^{(n)}$ with $n > 1$ prove to be functionally dependent on $C_i^{(0)}$ and $C_i^{(1)}$. To put it in other words, although $C_i^{(n)}$ with $n > 1$ are involved in the formal algebraic structure behind the equations of motion (12), they prove to be irrelevant for an actual solving thereof (see also [37]).

Now let us turn to a half-integer l. In this case, the matrix A^{mn} is nondegenerate and its eigenvalues read

$$(\pm i\gamma, \pm 3i\gamma, \pm 5i\gamma, \ldots, \pm 2li\gamma). \tag{13}$$

As before, the eigenvectors of A^{mn} occur in complex conjugate pairs $v_{(n)}^p$, $\bar{v}_{(n)}^p$, where $p = 1, 3, 5, \ldots, 2l$, which prompt one to introduce the new dynamical fields $\chi_i^p = x_i^{(n)}(v_{(n)}^p + \bar{v}_{(n)}^p)$ and bring (7) to a set of decoupled generalized oscillators

$$\rho^2 \frac{d}{dt}\left(\rho^2 \frac{d}{dt}\chi_i^p\right) + (\gamma p)^2 \chi_i^p = 0, \tag{14}$$

which are accompanied by $\ddot{\rho} = \gamma^2/\rho^3$. Making use of the l-conformal Galilei symmetry, one can readily solve the equations of motion by purely algebraic means

$$\chi_i^p = \alpha_i^p \cos(p\gamma s(t)) + \beta_i^p \sin(p\gamma s(t)), \quad s(t) = \frac{1}{\gamma}\arctan\left(\frac{\mathcal{D} + t\mathcal{H}}{\gamma}\right),$$
$$\rho(t) = \sqrt{\frac{(\mathcal{D} + t\mathcal{H})^2 + \gamma^2}{\mathcal{H}}}, \tag{15}$$

where $\mathcal{D}$, $\mathcal{H}$, α_i^p and β_i^p are constants of integration.

2. DYNAMICAL REALIZATION OF l-CONFORMAL NEWTON–HOOKE ALGEBRA

The l-conformal Newton–Hooke algebra involves the same set of generators as its Galilei counterpart. As compared to (1), only the first line is modified [1]

$$[H, D] = i\left(H \mp \frac{2}{R^2}K\right), \quad [H, C_i^{(n)}] = i\left(nC_i^{(n-1)} \pm \frac{(n-2l)}{R^2}C_i^{(n+1)}\right). \tag{16}$$

[1] As is known, the l-conformal Newton–Hooke algebra and its Galilei counterpart are isomorphic (see, e.g., [21]). It is to be remembered, however, that, as far as dynamical realizations are concerned, a linear change of basis, which links up the algebras, implies a change of the Hamiltonian and alters the dynamics.

Here, R is the dimensionful constant called the characteristic time. In (16) the upper/lower sign corresponds to a negative/positive cosmological constant. Below we construct a dynamical realization of the l-conformal Newton–Hooke algebra in terms of the second-order differential equations for the case of a negative cosmological constant.

Repeating the steps outlined in the preceding section, one constructs the Maurer–Cartan one-forms

$$\begin{gathered} w_H = \mathrm{e}^{-u} dt, \quad w_D = du - 2z\, dt, \quad w_K = \mathrm{e}^{u} \left(dz + z^2 dt\right) + \frac{2}{R^2} \sinh u\, dt, \\ w_i^{(n)} = dx_i^{(n)} - (n+1)\, x_i^{(n+1)} w_H - (n-l)\, x_i^{(n)} w_D - (n-2l-1)\, x_i^{(n-1)} \left(w_K + \frac{1}{R^2} w_H\right). \end{gathered} \tag{17}$$

Imposing the constraints

$$w_D = 0, \quad \tilde{\gamma}^{-1} w_K - \tilde{\gamma} w_H = 0, \quad w_i^{(n)} = 0, \tag{18}$$

where $\tilde{\gamma}$ is an arbitrary (coupling) constant, from the first two restrictions in (18) one removes z and derives the equation of motion for the one-dimensional conformal mechanics in the harmonic trap

$$\ddot{\rho} = \frac{\gamma^2}{\rho^3} - \frac{\rho}{R^2}, \tag{19}$$

where $\gamma^2 = \tilde{\gamma}^2 + 1/R^2$. The last constraint in (18) yields the equation which coincides with the rightmost equation in (6), but for $\gamma^2 = \tilde{\gamma}^2 + 1/R^2$. Thus, the analysis in the preceding section can be repeated for this case as well, which yields the general solution of the equations of motion

$$\begin{gathered} \chi_i^p = \alpha_i^p \cos\left(p\gamma s(t)\right) + \beta_i^p \sin\left(p\gamma s(t)\right), \quad s(t) = \frac{1}{\gamma} \arctan \frac{\mathcal{D}\mathcal{K} + (\mathcal{D}^2 + \gamma^2) R \tan \dfrac{t}{R}}{\gamma \mathcal{K}}, \\ \rho(t) = \sqrt{\frac{\left(\mathcal{D} R \sin \dfrac{t}{R} + \mathcal{K} \cos \dfrac{t}{R}\right)^2 + \left(\gamma R \sin \dfrac{t}{R}\right)^2}{\mathcal{K}}}, \end{gathered} \tag{20}$$

where $\mathcal{D}$, $\mathcal{K}$, α_i^p and β_i^p are constants of integration.

3. DISCUSSION

Let us confront qualitative behaviors of particles described by (20) and (15). For definiteness, in what follows we assume γ to be positive, choose $p = 2$ ($l = 1$), and stick to the case of three spatial dimensions. Making use of the rotation invariance, one can choose a coordinate system, in which the motion occurs in the xy-plane, with α_i [1] in (15) being parallel to the

[1] Here and in what follows, we omit the superscript $p = 2$ attached to α_i and χ_i.

x-axis. The orbit is an ellipse with one point corresponding to the polar angle $\phi = 2\gamma s = \pi$ removed. One can verify that, being initially at rest close to $\chi_i = -\alpha_i$ (as $t \to -\infty$), the particle starts moving towards $\chi_i = \alpha_i$ with growing angular velocity $\frac{d\phi}{dt} = 2\gamma\frac{ds}{dt} = \frac{2\gamma}{\rho^2(t)}$. Given the initial data $\mathcal{D}$, $\mathcal{H}$, it arrives there at $t = -\mathcal{D}/\mathcal{H}$, which corresponds to the polar angle $\phi = 2\gamma s = 0$. Then, it continues to move towards $\chi_i = -\alpha_i$ with decreasing angular velocity and freezes up as $t \to \infty$.

For the case of the l-conformal Newton–Hooke group the shape of the orbit is the same, but a qualitative behavior is different. The range of the temporal coordinate is $-\frac{\pi R}{2} < t < \frac{\pi R}{2}$. As $t \to \pm\frac{\pi R}{2}$, the angular velocity $\frac{d\phi}{dt} = 2\gamma\frac{ds}{dt} = \frac{2\gamma}{\rho^2}$ tends to the constant value $\left(\frac{2\gamma\mathcal{K}}{\mathcal{D}^2+\gamma^2}\right)\frac{1}{R^2}$, which is proportional to the cosmological constant. If $\mathcal{D} > 0$, one reveals three regimes, in which the angular velocity first increases, then decreases and then increases again. Likewise, for $\mathcal{D} < 0$ two phases of decelerated motion are separated by the acceleration phase in the middle. In both cases, the three regimes are separated by two roots of $\tan\frac{2t}{R} = \frac{2\mathcal{D}\mathcal{K}R}{\mathcal{K}^2-(\mathcal{D}^2+\gamma^2)R^2}$. For $\mathcal{D} = 0$ there are two regimes, in which acceleration is followed by deceleration for $\gamma^2 - \mathcal{K}^2/R^2 > 0$, and vice versa for $\gamma^2 - \mathcal{K}^2/R^2 < 0$. If $\mathcal{D} = 0$ and $\mathcal{K} = \gamma R$, the motion is uniform. Note that, as the angular velocity is fully determined by the conformal mode, the qualitative difference in the motion of a particle along the orbit in the case of the l-conformal Galilei symmetry and its Newton–Hooke counterpart correlates with the dynamics of $\rho(t)$. For the former case, the conformal mode is scattered off the repulsive potential γ^2/ρ^2 and its motion is unbounded, while for the latter case, it is confined to move in the potential well $\gamma^2/\rho^2 + \rho^2/R^2$. One can verify that for $p > 2$ the qualitative picture is similar, but a particle makes more than one revolution in the ellipse.

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