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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

NEW APPROACH TO $\mathcal{N}$-EXTENDED CONFORMAL SUPERGRAVITY IN THREE DIMENSIONS

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We briefly review the novel off-shell formulation for $\mathcal{N}$-extended conformal supergravity in three space-time dimensions developed in [1]. Our approach is based on gauging the $\mathcal{N}$-extended superconformal algebra $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$ in superspace. A special feature of the formulation is that the constraints imposed imply that the covariant derivative algebra is given in terms of a single curvature superfield, the super-Cotton tensor. We also elaborate on the component structure of the Weyl multiplet.

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INTRODUCTION

Pure $\mathcal{N}$-extended conformal supergravity in three dimensions (3D) is a supersymmetric Chern–Simons theory. It was originally engineered in the 1980s [2–4] (see also [5]) by gauging the $\mathcal{N}$-extended superconformal algebra $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$ in ordinary space-time. The resulting theory was off-shell only for $\mathcal{N}=1$ [2] and $\mathcal{N}=2$ [3], and on-shell for $\mathcal{N}>2$ [4,5]. We discuss this important point in more detail below.

According to [4], $\mathcal{N}$-extended conformal supergravity is described by the set of gauge fields, which are in one-to-one correspondence with the generators of $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$ and which may naturally be split into three subsets. The first subset consists of the dynamical fields: the vielbein $e_m{}^a$, the $\mathcal{N}$ gravitino $\psi_m{}^\alpha_I$ and the $SO(\mathcal{N})$ gauge field $V_m{}^{IJ}=-V_m{}^{JI}$. The second subset is given by the dilatation field b_m, which is a pure gauge degree of freedom (one may completely gauge away b_m by using the local conformal boosts). The third subset consists of the following composite fields: the spin connection $\omega_m{}^{ab}$, the special conformal connection $\mathfrak{f}_m{}^a$ and the S-supersymmetry connection $\phi_m{}^\alpha_I$. There are two ways to make the latter fields composite: either by imposing covariant constraints within the second-order formalism or by enforcing certain equations of motion using the first-order formalism.

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The action for $\mathcal{N}$-extended conformal supergravity given in [4] is

$$S = \frac{1}{4}\int d^3x\, e\, \Big\{ \varepsilon^{abc} \Big(\omega_a{}^{fg}\mathcal{R}_{bcfg} - \frac{2}{3}\omega_{af}{}^{g}\omega_{bg}{}^{h}\omega_{ch}{}^{f} - \\ -\frac{i}{2}\Psi_{bc}{}^{\alpha}_{I}(\gamma_d)_{\alpha}{}^{\beta}(\gamma_a)_{\beta}{}^{\gamma}\varepsilon^{def}\Psi_{ef}{}^{I}_{\gamma} - 2\Big(\mathcal{R}_{ab}{}^{IJ}V_{cIJ} + \frac{2}{3}V_a{}^{IJ}V_{bI}{}^{K}V_{cKJ}\Big)\Big)\Big\}. \quad (1)$$

Here $\mathcal{R}_{ab}{}^{cd}$ and $\mathcal{R}_{ab}{}^{IJ}$ are the Lorentz and $SO(\mathcal{N})$ curvature tensors, and $\Psi_{ab}{}^{\gamma}_{K}$ — the gravitino field strength.

It is a simple exercise to count the number of off-shell degrees of freedom which are contained in the dynamical fields, the bosonic $e_m{}^a$ and $V_m{}^{IJ}$ and the fermionic $\psi_m{}^{\alpha}_{I}$ ones. The result is $\mathcal{N}(\mathcal{N}-1)+2$ bosonic and $2\mathcal{N}$ fermionic off-shell degrees of freedom. Thus, the number of bosonic degrees of freedom matches that of the fermionic ones only in the cases $\mathcal{N}=1$ and $\mathcal{N}=2$. Since the formulation of [4,5] is on-shell for $\mathcal{N}>2$, it is not suitable for many interesting applications such as the construction of matter couplings. Auxiliary fields are required for $\mathcal{N}>2$.

1. THE WEYL MULTIPLET IN $SO(\mathcal{N})$ SUPERSPACE

In 1995, Howe et al. [6] proposed a curved superspace geometry with structure group $SL(2,\mathbb{R})\times SO(\mathcal{N})$, which is suitable to describe off-shell 3D $\mathcal{N}$-extended conformal supergravity. Specifically, the authors of [6] postulated the superspace constraints and determined all components of the superspace torsion of dimension-1. They also identified the $\mathcal{N}$-extended Weyl multiplet, that is the off-shell superconformal multiplet that contains all the independent gauge fields of $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$. At the same time, crucial elements of the formalism (including the explicit structure of super-Weyl transformations and the solution of the dimension-3/2 and dimension-2 Bianchi identities) did not appear in [6]. The geometry of $\mathcal{N}$-extended conformal supergravity has been fully developed in [7] and then applied to construct general supergravity-matter couplings in the cases $\mathcal{N}\leqslant 4$ (the simplest extended case $\mathcal{N}=2$ was studied in more detail in [8]). Below we review the salient points of the formalism.

Since the structure group is $SL(2,\mathbb{R})\times SO(\mathcal{N})$, the superspace geometry is described by covariant derivatives of the form

$$\mathcal{D}_A = E_A{}^M\partial_M - \frac{1}{2}\Omega_A{}^{bc}M_{bc} - \frac{1}{2}\Phi_A{}^{IJ}N_{IJ}, \quad \partial_M = \frac{\partial}{\partial z^M}, \quad (2)$$

with local coordinates $z^M=(x^m,\theta^{\mu}_{I})$ chosen to parameterize the curved superspace $\mathcal{M}^{3|2\mathcal{N}}$. Here $E_A = E_A{}^M\partial_M$ is the supervielbein; M_{ab} and $\Omega_A{}^{bc}$ are the Lorentz generators and connection, respectively; and N_{IJ} and $\Phi_A{}^{IJ}$ are respectively the $SO(\mathcal{N})$ generators and connection. The covariant derivatives obey (anti)commutation relations of the form

$$[\mathcal{D}_A,\mathcal{D}_B\} = -T_{AB}{}^C\mathcal{D}_C - \frac{1}{2}R_{AB}{}^{cd}M_{cd} - \frac{1}{2}R_{AB}{}^{IJ}N_{IJ}, \quad (3)$$

where $T_{AB}{}^C$ is the torsion, $R_{AB}{}^{cd}$ is the Lorentz curvature and $R_{AB}{}^{IJ}$ is the $SO(\mathcal{N})$ curvature.

The torsion is subject to the *conventional* constraints [6]:

$$T_{\alpha\beta}^{IJc} = -2i\delta^{IJ}(\gamma^c)_{\alpha\beta}, \quad T_{\alpha\beta K}^{IJ\gamma} = T_{\alpha b}^{I}{}^c = T_{ab}{}^c = \varepsilon^{\beta\gamma}T_{a\beta}{}^{[JK]}_{\gamma} = 0. \tag{4}$$

For $\mathcal{N} > 1$, the complete solution to the constraints (4), derived in [7], is given in terms of three dimension-1 tensor superfields $W^{IJKL} = W^{[IJKL]}$, $S^{IJ} = S^{(IJ)}$ and $C_a{}^{IJ} = C_a{}^{[IJ]}$, which appear in the anticommutator

$$\begin{aligned}\{\mathcal{D}_\alpha^I, \mathcal{D}_\beta^J\} &= 2i\delta^{IJ}(\gamma^c)_{\alpha\beta}\mathcal{D}_c - 2i\varepsilon_{\alpha\beta}C^{\gamma\delta IJ}M_{\gamma\delta} - 4iS^{IJ}M_{\alpha\beta} + \\ &+ \Big(i\varepsilon_{\alpha\beta}W^{IJKL} - 4i\varepsilon_{\alpha\beta}S^{K[I}\delta^{J]L} + iC_{\alpha\beta}{}^{KL}\delta^{IJ} - 4iC_{\alpha\beta}{}^{K(I}\delta^{J)L}\Big)N_{KL}. \end{aligned}\tag{5}$$

The tensor W^{IJKL} is absent for $\mathcal{N} < 4$. The Bianchi identities imply constraints on the curvature superfields W^{IJKL}, S^{IJ} and $C_a{}^{IJ}$ that are given in [7]. We refer to the superspace geometry described above as $SO(\mathcal{N})$ superspace.

Although the $\mathcal{N} = 1$ case is not described by (5), it can be obtained from the $\mathcal{N} > 1$ algebra by performing a certain limit [7]. The algebra of $\mathcal{N} = 1$ covariant derivatives [9] is

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 2i\mathcal{D}_{\alpha\beta} - 4iS\mathcal{M}_{\alpha\beta}, \tag{6a}$$

$$[\mathcal{D}_a, \mathcal{D}_\beta] = S(\gamma_a)_\beta{}^\gamma\mathcal{D}_\gamma - (\gamma_a)_\beta{}^\gamma C_{\gamma\delta\rho}\mathcal{M}^{\delta\rho} - \frac{2}{3}\left(\eta_{ab}\mathcal{D}_\beta S + 2\varepsilon_{abc}(\gamma^c)_{\beta\gamma}\mathcal{D}^\gamma S\right)\mathcal{M}^b. \tag{6b}$$

Unlike the space-time approaches that gauge the entire superconformal algebra [2–5], the structure group of $SO(\mathcal{N})$ superspace is a subgroup of $\mathfrak{osp}(\mathcal{N}|4, \mathbb{R})$. In particular, the dilatation symmetry and S-supersymmetry are not gauged in this approach. The reason why $SO(\mathcal{N})$ superspace is suitable to describe conformal supergravity is that the constraints (4) are invariant under arbitrary super-Weyl transformations of the form [7]:

$$\delta_\sigma\mathcal{D}_\alpha^I = \frac{1}{2}\sigma\mathcal{D}_\alpha^I + (\mathcal{D}^{\beta I}\sigma)M_{\alpha\beta} + (\mathcal{D}_{\alpha J}\sigma)N^{IJ}, \tag{7a}$$

$$\delta_\sigma\mathcal{D}_a = \sigma\mathcal{D}_a + \frac{i}{2}(\gamma_a)^{\gamma\delta}(\mathcal{D}_\gamma^K\sigma)\mathcal{D}_{\delta K} + \varepsilon_{abc}(\mathcal{D}^b\sigma)M^c + \frac{i}{16}(\gamma_a)^{\gamma\delta}([\mathcal{D}_\gamma^K, \mathcal{D}_\delta^L]\sigma)N_{KL}, \tag{7b}$$

where the parameter σ is a real unconstrained superfield. Under (7), W^{IJKL} transforms homogeneously, while the transformations of S^{IJ} and $C_a{}^{IJ}$ are inhomogeneous [7],

$$\delta_\sigma W^{IJKL} = \sigma W^{IJKL}, \tag{7c}$$

$$\delta_\sigma S^{IJ} = \sigma S^{IJ} - \frac{i}{8}[\mathcal{D}^{\gamma(I}, \mathcal{D}_\gamma^{J)}]\sigma, \tag{7d}$$

$$\delta_\sigma C_a{}^{IJ} = \sigma C_a{}^{IJ} - \frac{i}{8}(\gamma_a)^{\gamma\delta}[\mathcal{D}_\gamma^{[I}, \mathcal{D}_\delta^{J]}]\sigma. \tag{7e}$$

The superfield W^{IJKL} is called the super-Cotton tensor, since it transforms as a primary field under the super-Weyl group and contains the ordinary Cotton tensor among its component fields. In the cases $\mathcal{N} < 4$, the superfield W^{IJKL} vanishes and instead the super-Cotton tensor is constructed from the curvature superfields as follows [1, 10, 11]:

$$\mathcal{N}=1: \quad W_{\alpha\beta\gamma} = -i\mathcal{D}^{\delta}\mathcal{D}_{\delta}C_{\alpha\beta\gamma} - 2\mathcal{D}_{(\alpha\beta}\mathcal{D}_{\gamma)}S - 8SC_{\alpha\beta\gamma}, \tag{8a}$$

$$\mathcal{N}=2: \quad W_{\alpha\beta} = \frac{i}{8}[\mathcal{D}^{\gamma}_{I}, \mathcal{D}^{I}_{\gamma}]C_{\alpha\beta} - \frac{i}{4}\varepsilon_{IJ}[\mathcal{D}^{I}_{(\alpha}, \mathcal{D}^{J}_{\beta)}]\mathcal{S} + 2\mathcal{S}C_{\alpha\beta}, \tag{8b}$$

$$\mathcal{N}=3: \quad W_{\alpha} = \frac{i}{12}\varepsilon_{IJK}\mathcal{D}^{\beta I}C_{\alpha\beta}{}^{JK}, \tag{8c}$$

where for $\mathcal{N}=2$ we have defined $C_{\alpha\beta} := (1/2)\,\varepsilon_{IJ}C_{\alpha\beta}{}^{IJ}$ and $\mathcal{S} := (1/2)\,\delta^{IJ}S_{IJ}$. The symmetric spinors (8a)–(8c) transform homogeneously under the super-Weyl transformations (7).

The ordinary Weyl and local S-supersymmetry transformations are generated by the lowest components of σ:

$$\sigma|_{\theta=0}, \quad \mathcal{D}^{I}_{\alpha}\sigma|_{\theta=0}. \tag{9}$$

The appearance of super-Weyl transformations is a common feature of conventional approaches to conformal supergravity in diverse dimensions.

The $SO(\mathcal{N})$ superspace has proven powerful for the construction of general supergravity-matter couplings in the cases $\mathcal{N} \leqslant 4$ [7, 8]. However, the problem of constructing off-shell conformal supergravity actions was not considered in these papers. As follows from the analyses in [1, 10], $SO(\mathcal{N})$ superspace is not an optimum setting to address this problem.

2. THE WEYL MULTIPLET IN CONFORMAL SUPERSPACE

In this section, we present the new off-shell formulation for $\mathcal{N}$-extended conformal supergravity developed in [1] and elaborate on the component structure (see also [14]). It is a generalization of the off-shell formulations for $\mathcal{N}=1$ and $\mathcal{N}=2$ conformal supergravity theories in four dimensions [12, 13].

2.1. The Geometry of Conformal Superspace. The 3D $\mathcal{N}$-extended superconformal algebra, $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$, contains the super-Poincaré translation $P_A = (P_a, Q^I_\alpha)$, special (super)conformal generators[1] $K_A = (K_a, S^I_\alpha)$, Lorentz (M_{ab}), dilatation ($\mathbb{D}$) and $SO(\mathcal{N})$ or R-symmetry (N_{KL}) generators. Their (anti)commutation relations are given explicitly in [1]. The covariant derivatives are chosen to have the form

$$\nabla_A = E_A - \omega_A{}^{\underline{b}}X_{\underline{b}} = E_A - \frac{1}{2}\Omega_A{}^{bc}M_{bc} - \frac{1}{2}\Phi_A{}^{JK}N_{JK} - B_A\mathbb{D} - \mathfrak{F}_A{}^{B}K_B. \tag{10}$$

The action of the generators $X_{\underline{a}} = (M_{ab}, N_{IJ}, \mathbb{D}, K_A)$ on the covariant derivatives,

$$[X_{\underline{a}}, \nabla_B\} = -f_{\underline{a}B}{}^{C}\nabla_C - f_{\underline{a}B}{}^{\underline{c}}X_{\underline{c}}, \tag{11}$$

resembles that with P_A in the superconformal algebra

$$[X_{\underline{a}}, P_B\} = -f_{\underline{a}B}{}^{C}P_C - f_{\underline{a}B}{}^{\underline{c}}X_{\underline{c}}. \tag{12}$$

[1] In line with usual nomenclature we may refer to S^I_α as the S-supersymmetry generator.

The supergravity gauge group $\mathcal{G}$ is generated by local transformations of the form

$$\delta_{\mathcal{G}}\nabla_A = [\mathcal{K}, \nabla_A], \quad \mathcal{K} = \xi^B\nabla_B + \frac{1}{2}\Lambda^{bc}M_{bc} + \frac{1}{2}\Lambda^{JK}N_{JK} + \sigma\mathbb{D} + \Lambda^B K_B. \tag{13}$$

Such a gauge transformation is a combination of: (i) a *covariant general coordinate transformation* associated with ξ^B; and (ii) a *standard superconformal transformation* associated with $\Lambda^{\underline{b}} = (\Lambda^{bc}, \Lambda^{JK}, \sigma, \Lambda^B)$. The covariant derivatives satisfy the (anti)commutation relations

$$[\nabla_A, \nabla_B\} = -T_{AB}{}^C\nabla_C - \frac{1}{2}R(M)_{AB}{}^{cd}M_{cd} - \frac{1}{2}R(N)_{AB}{}^{PQ}N_{PQ} - \\ - R(\mathbb{D})_{AB}\mathbb{D} - R(S)_{AB}{}^{\gamma}_{K}S^K_\gamma - R(K)_{AB}{}^c K_c, \tag{14}$$

where $T_{AB}{}^C$ is the torsion and $R(X)_{AB}{}^{\underline{c}}$ is the curvature associated with $X_{\underline{c}}$.

The above geometry is too general and one needs to impose constraints. The constraints chosen are based on two principles: (i) the entire covariant derivative algebra should be expressed in terms of a single primary superfield, the $\mathcal{N}$-extended super-Cotton tensor; and (ii) the superspace geometry should resemble the one describing the Yang–Mills supermultiplet.

As discussed above, the super-Cotton tensor possesses a different index structure for different values of $\mathcal{N}$. For $\mathcal{N} > 3$ it corresponds to the $SO(\mathcal{N})$ superspace curvature W^{IJKL}. It is in this case that we take

$$\{\nabla^I_\alpha, \nabla^J_\beta\} = 2i\delta^{IJ}\nabla_{\alpha\beta} + 2i\varepsilon_{\alpha\beta}W^{IJ} \tag{15}$$

and require the operator W^{IJ} to be of dimension-1 and conformally primary,

$$[\mathbb{D}, W^{IJ}] = W^{IJ}, \quad [S^I_\alpha, W^{JK}] = 0. \tag{16}$$

The most general ansatz for W^{IJ} is

$$W^{IJ} = \frac{1}{2}W^{IJKL}N_{KL} + A(\nabla^\alpha_K W^{IJKL})S_{\alpha L} + Bi(\gamma^c)^{\alpha\beta}(\nabla_{\alpha K}\nabla_{\beta L}W^{IJKL})K_c, \tag{17}$$

with A and B some constants that turn out to be uniquely determined by (16). The resulting algebra of covariant derivatives for $\mathcal{N} > 3$ is

$$\{\nabla^I_\alpha, \nabla^J_\beta\} = 2i\delta^{IJ}\nabla_{\alpha\beta} + i\varepsilon_{\alpha\beta}W^{IJKL}N_{KL} - \frac{i}{\mathcal{N}-3}\varepsilon_{\alpha\beta}(\nabla^\gamma_K W^{IJKL})S_{\gamma L} + \\ + \frac{1}{2(\mathcal{N}-2)(\mathcal{N}-3)}\varepsilon_{\alpha\beta}(\gamma^c)^{\gamma\delta}(\nabla_{\gamma K}\nabla_{\delta L}W^{IJKL})K_c, \tag{18a}$$

$$[\nabla_a, \nabla^J_\beta] = \frac{1}{2(\mathcal{N}-3)}(\gamma_a)_{\beta\gamma}\Big((\nabla^\gamma_K W^{JPQK})N_{PQ} - \frac{1}{(\mathcal{N}-3)}(\nabla^\gamma_L\nabla^\delta_P W^{JKLP})S_{\delta K} - \\ - \frac{i}{2(\mathcal{N}-1)(\mathcal{N}-2)}(\gamma_a)_{\beta\gamma}(\gamma^c)_{\delta\rho}(\nabla^\gamma_K\nabla^\delta_L\nabla^\rho_P W^{JKLP})K_c\Big), \tag{18b}$$

$$
\begin{aligned}
[\nabla_a, \nabla_b] = {} & \frac{1}{8\mathcal{N}(\mathcal{N}-1)(\mathcal{N}-2)(\mathcal{N}-3)} \times \\
& \times \varepsilon_{abc}(\gamma^c)_{\alpha\beta}\Big(2i\mathcal{N}(\mathcal{N}-1)(\nabla_I^\alpha \nabla_J^\beta W^{PQIJ})N_{PQ} + 2i\mathcal{N}(\nabla_I^\alpha \nabla_J^\beta \nabla_K^\gamma W^{LIJK})S_{\gamma L} + \\
& + (\gamma^d)_{\gamma\delta}(\nabla_I^\alpha \nabla_J^\beta \nabla_K^\gamma \nabla_L^\delta W^{IJKL})K_d\Big), \quad (18c)
\end{aligned}
$$

where W^{IJKL} satisfies the Bianchi identity

$$
\nabla_\alpha^I W^{JKLP} = \nabla_\alpha^{[I} W^{JKLP]} - \frac{4}{\mathcal{N}-3}\nabla_{\alpha Q} W^{Q[JKL}\delta^{P]I}. \quad (19)
$$

In the $\mathcal{N} = 4$ case, $W^{IJKL} = \varepsilon^{IJKL} W$ and Eq. (19) is identically satisfied. For $\mathcal{N} = 4$ we instead have the Bianchi identity

$$
\nabla^{\alpha I}\nabla_\alpha^J W = \frac{1}{4}\delta^{IJ}\nabla_P^\alpha \nabla_\alpha^P W. \quad (20)
$$

Although we considered only the $\mathcal{N} > 3$ case, its algebra of covariant derivatives contains information about the lower $\mathcal{N}$ cases. This is discussed in detail in [1]. The important point is that in each case the algebra is expressed completely in terms of the super-Cotton tensor.

2.2. Degauging to $SO(\mathcal{N})$ Superspace. Under a K_A transformation, the dilatation gauge field $B = E^A B_A = E^a B_a + E_I^\alpha B_\alpha^I$ transforms as

$$
\delta_K(\Lambda)B = -2E^a\Lambda_a + 2E_I^\alpha \Lambda_\alpha^I, \quad (21)
$$

which permits the gauge choice $B_A = 0$. This removes the dilatation connection from all the covariant derivatives. Once the K_A symmetry has been fixed, it is natural to introduce the degauged covariant derivatives

$$
\tilde{\mathcal{D}}_A := \nabla_A + \mathfrak{F}_A{}^B K_B, \quad (22)
$$

whose structure group corresponds to $SL(2,\mathbb{R}) \times SO(\mathcal{N})$. The vanishing of all components of the dilatation curvature imposes constraints on the components of $\mathfrak{F}_A{}^B$. The solution of these constraints is

$$
\mathfrak{F}_{\alpha\beta}^{I\,J} = -\mathfrak{F}_{\beta\alpha}^{JI} = iC_{\alpha\beta}{}^{IJ} - i\varepsilon_{\alpha\beta}S^{IJ}, \quad (23a)
$$

$$
\mathfrak{F}_{\alpha\beta,}{}_\gamma^K = -\mathfrak{F}_\gamma^K{}_{,\alpha\beta} = C_{\alpha\beta\gamma}{}^K + \frac{2}{3}\varepsilon_{\gamma(\alpha}\Big(\frac{\mathcal{N}-1}{\mathcal{N}}\mathcal{S}_{\beta)}{}^J + \frac{\mathcal{N}}{\mathcal{N}+2}\tilde{\mathcal{D}}_{\beta)}^J\mathcal{S}\Big), \quad (23b)
$$

$$
\begin{aligned}
\mathfrak{F}_{ab} = {} & -\frac{i}{4\mathcal{N}}(\gamma_{(a})^{\alpha\beta}(\gamma_{b)})^{\gamma\delta}\tilde{\mathcal{D}}_{\alpha I}C_{\beta\gamma\delta}{}^I - \\
& - \frac{i(\mathcal{N}-1)}{6\mathcal{N}^2}\eta_{ab}\tilde{\mathcal{D}}_I^\alpha \mathcal{S}_\alpha{}^I - \frac{i}{6(\mathcal{N}+2)}\eta_{ab}\tilde{\mathcal{D}}_I^\alpha \tilde{\mathcal{D}}_\alpha^I\mathcal{S} - \\
& - \frac{1}{2\mathcal{N}}(\gamma_a)^{\alpha\beta}(\gamma_b)^{\gamma\delta}C_{\alpha\gamma}{}^{IJ}C_{\beta\delta IJ} + \frac{1}{\mathcal{N}}\eta_{ab}\mathcal{S}^{IJ}\mathcal{S}_{IJ} + \eta_{ab}\mathcal{S}^2, \quad (23c)
\end{aligned}
$$

where

$$
S^{IJ} = \mathcal{S}^{IJ} + \delta^{IJ}\mathcal{S}\ , \quad \mathcal{S} = \frac{1}{\mathcal{N}}\delta_{IJ}S^{IJ}. \quad (24)
$$

The superfields $C_{\alpha\beta}{}^{IJ}$, S^{IJ}, $C_{\alpha\beta\gamma}{}^{K}$, $\mathcal{S}_\alpha{}^I$ and $\mathcal{S}$ appear in the torsion and curvature tensors corresponding to the degauged covariant derivatives. To see this, it suffices to evaluate the action of $[\tilde{\mathcal{D}}_A, \tilde{\mathcal{D}}_B\}$ on an arbitrary conformal primary superfield. In particular, one finds

$$\{\tilde{\mathcal{D}}^I_\alpha, \tilde{\mathcal{D}}^J_\beta\} = 2i\delta^{IJ}(\gamma^c)_{\alpha\beta}\tilde{\mathcal{D}}_c - 2i\varepsilon_{\alpha\beta}C^{\gamma\delta IJ}M_{\gamma\delta} - 4iS^{IJ}M_{\alpha\beta} + \\ + \Big(i\varepsilon_{\alpha\beta}W^{IJKL} - 4i\varepsilon_{\alpha\beta}S^{K[I}\delta^{J]L} - 4iC_{\alpha\beta}{}^{K(I}\delta^{J)L}\Big)N_{KL}. \quad (25)$$

In fact, if we introduce a new vector covariant derivative defined by

$$\mathcal{D}_a = \tilde{\mathcal{D}}_a - \frac{1}{2}C_a{}^{IJ}N_{IJ}, \quad (26)$$

the algebra of the covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \tilde{\mathcal{D}}^I_\alpha)$ exactly coincides with that of $SO(\mathcal{N})$ superspace. The reason for having to introduce the new covariant derivatives $\mathcal{D}_A$ can be attributed to the appearance of the nonzero torsion component $\varepsilon^{\beta\gamma}\tilde{T}_{a\beta}{}^{[JK]}_{\gamma} = -2C_a{}^{JK}$ in the algebra corresponding to $\tilde{\mathcal{D}}_A$. Although this torsion component appears more complex than that of $SO(\mathcal{N})$ superspace, Eq. (4), it leads to a simpler covariant derivative algebra.

We conclude that the $\mathcal{N}$-extended conformal superspace describes the Weyl multiplet.

2.3. The Weyl Multiplet. The 3D $\mathcal{N}$-extended Weyl multiplet can be extracted from conformal superspace via component projections. It involves a set of gauge one-forms: the vielbein $e_m{}^a$, the gravitino $\psi_m{}^\alpha_I$, the $SO(\mathcal{N})$ gauge field $V_m{}^{IJ}$ and the dilatation gauge field b_m. They appear in the superspace formulation as the lowest components of their corresponding super one-forms,

$$e_m{}^a := E_m{}^a|, \quad \psi_m{}^\alpha_I := 2E_m{}^\alpha_I|, \quad V_m{}^{IJ} := \Phi_m{}^{IJ}|, \quad b_m := B_m|, \quad (27)$$

where the bar-projection [9] of a superfield $V(z) = V(x, \theta)$ is defined by the standard rule $V| := V(x,\theta)|_{\theta=0}$. The remaining connection fields are *composite* and their expressions in terms of the other fields are given in [14]. By adopting a Wess–Zumino gauge it is possible to see that the remaining physical fields are contained in the super-Cotton tensor.

Since one can deduce the lower $\mathcal{N}$ cases from the $\mathcal{N} > 3$ case, we focus on the $\mathcal{N} > 3$ case. For $\mathcal{N} > 3$ the additional fields are encoded in the super-Cotton tensor W^{IJKL} [6] (see also [15, 16]). The component fields are defined as

$$w_{IJKL} := W_{IJKL}|, \quad (28a)$$

$$w_\alpha{}^{IJK} := -\frac{i}{2(\mathcal{N}-3)}\nabla_{\alpha L}W^{IJKL}|, \quad (28b)$$

$$w_{\alpha\beta}{}^{IJ} := \frac{i}{2(\mathcal{N}-2)(\mathcal{N}-3)}\nabla_{(\alpha K}\nabla_{\beta)L}W^{IJKL}|, \quad (28c)$$

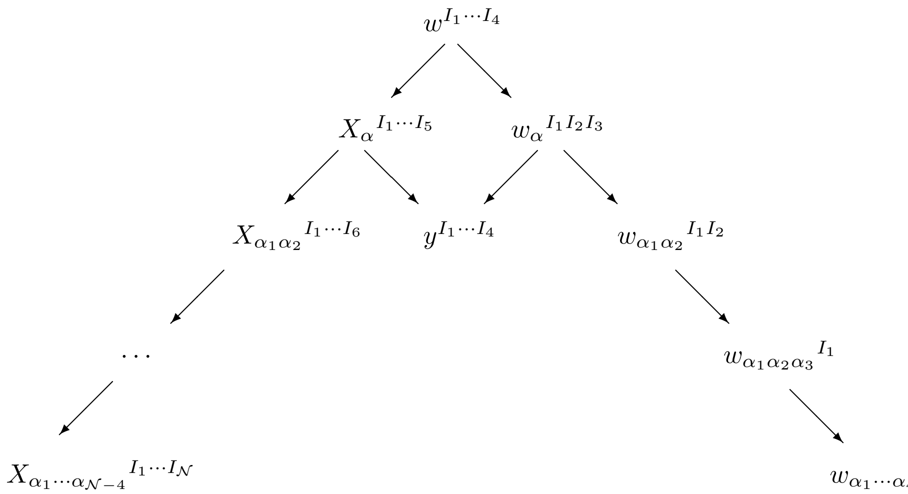
$$w_{\alpha\beta\gamma}{}^{I} := \frac{i}{(\mathcal{N}-1)(\mathcal{N}-2)(\mathcal{N}-3)}\nabla_{(\alpha J}\nabla_{\beta K}\nabla_{\gamma)L}W^{IJKL}|, \quad (28d)$$

$$w_{\alpha\beta\gamma\delta} := -\frac{1}{\mathcal{N}(\mathcal{N}-1)(\mathcal{N}-2)(\mathcal{N}-3)}\nabla_{(\alpha I}\nabla_{\beta J}\nabla_{\gamma K}\nabla_{\delta)L}W^{IJKL}|, \quad (28e)$$

$$y^{IJKL} := \frac{i}{\mathcal{N}-3}\nabla^{\gamma[I}\nabla_{\gamma P}W^{JKL]P}|, \quad (28f)$$

$$X_{\alpha_1\cdots\alpha_n}{}^{I_1\cdots I_{n+4}} := I(n)\nabla^{[I_1}_{(\alpha_1}\cdots\nabla^{I_n}_{\alpha_n)}W^{I_{n+1}\cdots I_{n+4}]}|. \quad (28g)$$

The factor $I(n)$, which is needed to ensure the fields $X_{\alpha_1\cdots\alpha_n}{}^{I_1\cdots I_{n+4}}$ are real, is defined to be $I(n) = i$, when $n = 1, 2 \pmod 4$ and $I(n) = 1$ with $n = 3, 4 \pmod 4$. The fields defined in (28), when organized by dimension, diagrammatically form the following tower [15, 16]:



The component fields $w_{\alpha\beta}{}^{IJ}$, $w_{\alpha\beta\gamma}{}^{I}$ and $w_{\alpha\beta\gamma\delta}$ are constrained by the geometry to be composite [14].

Although we have only defined the component fields coming from the Cotton tensor for $\mathcal{N} > 3$, the coefficients in Eq. (28) have been chosen to allow one to derive component results for lower $\mathcal{N}$ from the higher ones. We simply switch off the components with more than $\mathcal{N}$ $SO(\mathcal{N})$ indices (independently) and define

$$\varepsilon^{I_1\cdots I_\mathcal{N}} w_{\alpha_1\cdots\alpha_{4-\mathcal{N}}} := w_{\alpha_1\cdots\alpha_{4-\mathcal{N}}}{}^{I_1\cdots I_\mathcal{N}}. \tag{29}$$

For $\mathcal{N} < 5$ the component fields defined by Eq. (28g) are identically zero. The $\mathcal{N} = 1$ components of the super-Cotton tensor are

$$w_{\alpha\beta\gamma} := W_{\alpha\beta\gamma}|, \quad w_{\alpha\beta\gamma\delta} := i\nabla_{(\alpha}W_{\beta\gamma\delta)}|, \tag{30}$$

while for $\mathcal{N} = 2$ they are defined by

$$w_{\alpha\beta} := W_{\alpha\beta}|, \quad w_{\alpha\beta\gamma}{}^{I} := 2\varepsilon^{IJ}\nabla_{(\alpha J}W_{\beta\gamma)}|, \quad w_{\alpha\beta\gamma\delta} := i\varepsilon_{IJ}\nabla^I_{(\alpha}\nabla^J_\beta W_{\gamma\delta)}|, \tag{31}$$

which are all composite. The $\mathcal{N} = 3$ component fields of the super-Cotton tensor are

$$w_\alpha := W_\alpha|, \quad w_{\alpha\beta}{}^{IJ} := -\varepsilon^{IJK}\nabla_{(\alpha K}W_{\beta)}|, \quad w_{\alpha\beta\gamma}{}^{I} := -\varepsilon^{IJK}\nabla_{(\alpha J}\nabla_{\beta K}W_{\gamma)}|, \tag{32a}$$

$$w_{\alpha\beta\gamma\delta} := -\frac{i}{3}\varepsilon_{IJK}\nabla^I_{(\alpha}\nabla^J_\beta\nabla^K_\gamma W_{\delta)}|, \tag{32b}$$

where the only auxiliary field is w_α, and all other components are composite.

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