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EXPANDING $(3+1)$-DIMENSIONAL UNIVERSE FROM THE IIB MATRIX MODEL

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We show that $(3+1)$-dimensional expanding universe emerges in the IIB matrix model, which is conjectured to be a nonperturbative formulation of superstring theory. We also discuss how the Standard Model particles appear in the model.

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INTRODUCTION

Superstring theory is a promising candidate for the unified theory including quantum gravity. Indeed, one can find perturbative vacua, which give rise to the Standard Model with some extra exotic particles. However, there are some serious problems as well. It is known that there actually exist tremendously many perturbative vacua, which is a situation commonly referred to as the string landscape nowadays. Each vacuum gives different space-time dimensionality, gauge group and matter content. Moreover, it is known that the cosmic singularity is not resolved generally within perturbative formulation [1–4].

Of course, all these problems might be simply because superstring theory has been studied essentially in perturbation theory including, at most, some nonperturbative effects represented by the existence of D-branes. Indeed, it is possible that in a fully nonperturbative formulation the true vacuum is determined uniquely and the beginning of universe can be explored by resolving singularity through strong coupling dynamics of quantum gravity.

In this paper, as a nonperturbative formulation of superstring theory, we consider the IIB matrix model [5]. The IIB matrix model has an action

$$S = S_b + S_f, \tag{1}$$

$$S_b = -\frac{1}{4g^2}\,\mathrm{Tr}\left([A_M, A_N]\left[A^M, A^N\right]\right), \tag{2}$$

$$S_f = \frac{1}{2g^2}\,\mathrm{Tr}\left(\bar{\Psi}\Gamma^M[A_M, \Psi]\right), \tag{3}$$

where Γ_M are 32×32 gamma matrices in 10d. The bosonic $N\times N$ matrices A_M $(M = 0,\ldots,9)$ are traceless Hermitian, while the fermionic $N\times N$ matrices Ψ_α $(\alpha = 1,\ldots,32)$

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are Majorana–Weyl fermions in 10d. The action of the model can be formally obtained by dimensionally reducing that of ten-dimensional $\mathcal{N} = 1\, SU(N)$ super Yang–Mills theory to zero dimension. The IIB matrix model has a direct connection to perturbative-type IIB superstring theory, but it is expected to describe the unique nonperturbative theory of superstrings underlying the duality web of various perturbative formulations.

There are various pieces of evidence for the conjecture that the IIB matrix model is a nonperturbative formulation of superstring theory. First of all, the action (1) can be regarded as a matrix regularization of the worldsheet action of type IIB superstring theory in the Schild type [5] [1]. Secondly, the interaction between D-branes is reproduced correctly [5]. Thirdly, under a few reasonable assumptions, the string field Hamiltonian for type IIB superstring theory can be derived from Schwinger–Dyson equations for the Wilson loop operators, which are identified as creation and annihilation operators of strings [6].

In all these connections to type IIB superstring theory, the target space coordinates are identified with the eigenvalues of the matrices A_μ. In particular, this identification is consistent with the supersymmetry algebra of the model, in which the translation that appears from the anticommutator of supersymmetry generators is identified with the shift symmetry $A_\mu \mapsto A_\mu + \alpha_\mu \mathbf{1}$ of the model, where $\alpha_\mu \in \mathbf{R}$. Also, the fact that the model has extended $\mathcal{N} = 2$ supersymmetry in ten dimensions is consistent with the assertion that the model actually includes gravity, since it is known in field theory that $\mathcal{N} = 1$ supersymmetry is the maximal one that can be achieved in ten dimensions without including gravity. Thus, space-time is treated as a part of dynamical degrees of freedom in the bosonic matrices A_μ in the IIB matrix model. It is, therefore, possible that four-dimensional space-time appears dynamically.

For more than fifteen years since its proposal, the IIB matrix model has been studied in its Euclidean version, which can be obtained by making a "Wick rotation" $A_0 = -iA_{10}$. In general, the Wick rotation in gravitational theories is nontrivial unlike that in field theories. The Euclidean version has been studied intensively, nevertheless, because it has a finite partition function [7,8]. See [9,10] for recent work, which suggests that spontaneous breaking of the $SO(10)$ symmetry occurs in the Euclidean matrix model.

In addition to nontriviality of the Wick rotation, it is difficult to derive the real-time dynamics in the Euclidean version of the IIB matrix model, as usual in field theories. It is, of course, necessary to study the real-time dynamics in cosmology. Thus, in this talk, we consider the Lorentzian version of the IIB matrix model. We show that $(3+1)$-dimensional expanding universe emerges in the IIB matrix model. We also discuss how the Standard Model particles appear in the model.

1. DEFINING THE LORENTZIAN VERSION

The IIB matrix model [5] is defined in its Lorentzian version by the partition function [11]:

$$Z = \int dA\, d\Psi\, \mathrm{e}^{i(S_b + S_f)}. \tag{4}$$

[1]This does not imply that the matrix model is merely a formulation for the "first quantization" of superstrings. In fact, multiple worldsheets appear naturally in the matrix model as block-diagonal configurations, where each block represents the embedding of a single worldsheet into the ten-dimensional target space.

In the definition (4), we have replaced the "Boltzmann weight" e^{-S} used in the Euclidean model by e^{iS}. This is theoretically motivated from the connection to the worldsheet theory [5], where we have to make an inverse Wick rotation for the worldsheet coordinates as well as the target-space coordinates.

One finds that the bosonic action is proportional to

$$S_b \propto \mathrm{Tr}\left(F_{MN}F^{MN}\right) = -2\,\mathrm{Tr}\,\left(F_{0i}\right)^2 + \mathrm{Tr}\left(F_{ij}\right)^2 , \tag{5}$$

where $i, j = 1, \ldots, 9$, and we have defined Hermitian matrices $F_{\mu\nu} = i\left[A_\mu, A_\nu\right]$. Therefore, the bosonic action is not positive definite. In order to make the partition function finite, one actually needs to introduce infrared cut-offs

$$\frac{1}{N}\,\mathrm{Tr}\,\left(A_0\right)^2 \leqslant \kappa L^2, \tag{6}$$

$$\frac{1}{N}\,\mathrm{Tr}\,\left(A_i\right)^2 \leqslant L^2 \tag{7}$$

in both temporal and spatial directions. It turned out that these cut-offs can be removed in the large-N limit, and clear scaling behaviors corresponding to the continuum and infinite-volume limits were observed [11]. This implies that the resulting theory has no parameters except the scale parameter, which is an expected property in nonperturbative string theory. In actual simulation, we set $L = 1$ without loss of generality since it only fixes the scale, and choose κ appropriately as a function of N so that both the continuum and infinite-volume limits are taken.

At first sight, it is difficult to simulate the partition function (4) due to the phase factor e^{iS_b}. However, by integrating out the scale factor of the bosonic matrices, one can rewrite the partition function into the form that allows direct Monte Carlo studies,

$$Z = \int dA\,\mathrm{Pf}\,\mathcal{M}\left(A\right)\delta\left(\frac{1}{N}\,\mathrm{Tr}\,\left(F_{MN}F^{MN}\right)\right)\delta\times \times\left(\frac{1}{N}\,\mathrm{Tr}\,\left(A_i\right)^2 - L^2\right)\theta\left(\kappa L^2 - \frac{1}{N}\,\mathrm{Tr}\,\left(A_0\right)^2\right). \tag{8}$$

The Pfaffian $\mathrm{Pf}\,\mathcal{M}(A)$ in (8), which is obtained by integrating out fermionic matrices, is real in the present Lorentzian case, and it does not cause any sign problem [1].

2. EMERGENCE OF $(3+1)$-DIMENSIONAL EXPANDING UNIVERSE

In order to extract the time evolution from configurations generated by (8), we first diagonalize the temporal matrix A_0 as

$$A_0 = \mathrm{diag}\,\left(\alpha_1, \ldots, \alpha_N\right), \quad \text{where} \quad \alpha_1 < \ldots < \alpha_N, \tag{9}$$

[1] For large N, there is no possible sign flip of the Pfaffian.

using the $SU(N)$ symmetry. In such a basis, it turned out that the spatial matrices A_i have a band-diagonal structure; namely, it was found that the off-diagonal elements $(A_i)_{IJ}$ with $|I-J| > n$ are small for some n. This nontrivial dynamical property motivates us to define $n \times n$ matrices

$$\left(\bar{A}_i(t)\right)_{ab} \equiv (A_i)_{\nu+a,\nu+b}\,, \tag{10}$$

where $\nu = 0, 1, \ldots, N-n$, and $a, b = 1, \ldots, n$. We consider that these block matrices represent the states of the universe at time t, where

$$t = \frac{1}{n}\sum_{a=1}^{n} \alpha_{\nu+a}. \tag{11}$$

For instance, the extent of space at time t is defined by

$$R^2(t) = \frac{1}{n}\,\mathrm{tr}\,\left(\bar{A}_i(t)\right)^2. \tag{12}$$

In Fig. 1, we plot the extent of space (12) for $N = 16$ and $n = 4$. Since the result is symmetric under the time reflection $t \to -t$ as a consequence of the symmetry $A_0 \to -A_0$, we only show the results for $t < 0$. There is a critical κ, beyond which the peak at $t = 0$ starts to grow.

Next, we study the spontaneous breaking of the $SO(9)$ symmetry. As an order parameter, we define the 9×9 (positive definite) real symmetric tensor

$$T_{ij}(t) = \frac{1}{n}\,\mathrm{tr}\left\{\bar{A}_i(t)\bar{A}_j(t)\right\}, \tag{13}$$

which is an analog of the moment of inertia tensor. The nine eigenvalues of $T_{ij}(t)$ are plotted against t in Fig. 2 for $\kappa = 4.0$. We find that three largest eigenvalues of $T_{ij}(t)$ start to grow

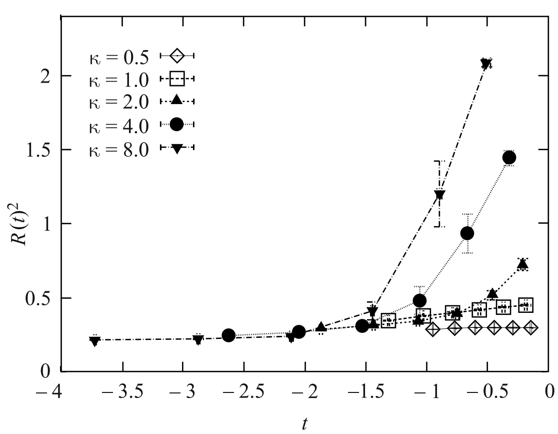

Fig. 1. The extent of space $R(t)^2$ with $N = 16$ and $n = 4$ is plotted as a function of t for five values of κ. The peak at $t = 0$ starts to grow at some critical κ

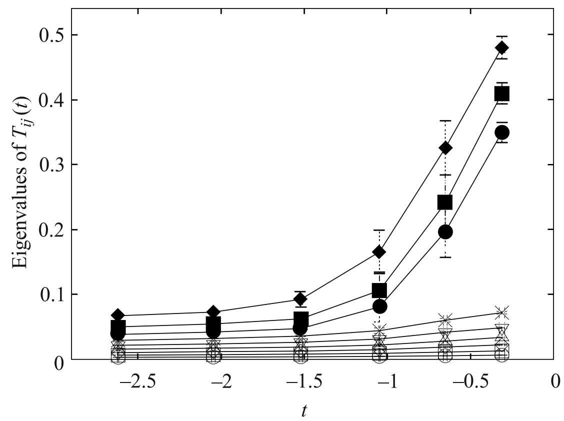

Fig. 2. The nine eigenvalues of $T_{ij}(t)$ with $N = 16$ and $n = 4$ are plotted as a function of t for $\kappa = 4.0$. After the critical time t_c, three eigenvalues become larger, suggesting that the $SO(9)$ symmetry is spontaneously broken down to $SO(3)$

at the critical time t_c, which suggests that the $SO(9)$ symmetry is spontaneously broken down to $SO(3)$ after t_c. Note, that we have not imposed any initial conditions in the simulation. Thus, the above results are unique.

3. REALIZING THE STANDARD MODEL PARTICLES

In the previous section, we have shown that $(3+1)$-dimensional expanding universe appears dynamically and uniquely from the Lorentzian version of the IIB matrix model. Similarly, it is possible that the Standard Model appears uniquely at the electroweak scale from the same model at late times. In this section, we discuss how the Standard Model particles appear in the IIB matrix model. In particular, we are concerned with a constructive definition of the theory, in which we start with finite-N matrices and take the large-N limit afterwards.

Realization of chiral fermions and the Standard Model in the IIB matrix model has been discussed by various authors [12–15]. While these works attempt to realize chiral fermions by modifying the model, [16] proposed to realize chiral fermions in the original IIB matrix model based on the idea of intersecting branes [17–20]. It was shown that chiral fermions indeed appear at the intersections when the branes are given by (hyper)planes, which can be represented by infinite-dimensional matrices in the matrix model. The authors then proposed to replace these branes by fuzzy spheres and other fuzzy manifolds, which can be represented by finite-N matrices. Realization of the Standard Model was also discussed along this line.

Recently, the authors of [21] calculated explicitly the spectrum of the Dirac operator for a finite-N configuration suggested in [16], which represents a 5-brane and a 7-brane intersecting at a point in the extra dimensions. It was confirmed that a chiral zero mode localized at the intersection point indeed appears in the large-N limit. However, one also obtains another chiral zero mode with opposite chirality, which was not anticipated naively from the brane configuration. This result was understood as a consequence of a no-go theorem, which states that chiral fermions cannot be realized in the large-N limit of finite-N IIB matrix model as far as one assumes that space-time is given by a direct product of our four-dimensional space-time and extra six-dimensional space. In fact, the $SO(3,1)$ Lorentz symmetry alone does not imply the direct product structure of space-time, and one generally obtains a warp factor as we will discuss below [21].

As shown in the previous section, an expanding three-dimensional space appears dynamically after some time. At later times, it is speculated that three-dimensional space becomes much larger than the typical scale of the model, and that quantum fluctuations can be neglected at large scales [22, 23]. Furthermore, as far as we do not consider too long time scale, we can neglect the expansion of space, and therefore the space-time has the $SO(3,1)$ Lorentz symmetry. Thus, we are led to consider matrix configurations given by

$$A_\mu = X_\mu \otimes M \quad (\mu = 0, \ldots, 3), \tag{14}$$

$$A_a = \mathbb{1}_n \otimes Y_a \quad (a = 4, \ldots, 9). \tag{15}$$

Here, we assume that the $n \times n$ Hermitian matrices X_μ have the property $O_{\mu\nu} X_\nu = g[O]\, X_\mu\, g[O]^\dagger$, where $O \in SO(3,1)$ and $g[O] \in SU(n)$. Then, (14) and (15) can be regarded as the most general configuration that is $SO(3,1)$ invariant up to $SU(N)$ symmetry.

The Hermitian matrix M in (14) can be regarded as a matrix version of the warp factor. The special case $M = \mathbb{1}$ corresponds to a space-time, which is a direct product of $(3+1)$-dimensional space-time and the extra dimensions. In a generic case with a nontrivial matrix M representing a warp factor, chiral zero modes in extra six dimensions do not automatically correspond to those in our four-dimensional space-time. For the configuration studied in [21], it was found that there are huge degrees of freedom in M, which allows only the desired chiral zero mode to appear in four dimensions [1]. Thus, one can realize a chiral fermion in the large-N limit of finite-N IIB matrix model thanks to the matrix warp factor M.

Taking into account the no-go theorem and the need for introducing a nontrivial M to avoid its consequence, we explore realizing chiral fermions and the Standard Model in the IIB matrix model [24]. We realize chiral fermions from intersecting fuzzy S^2 and fuzzy $S^2 \times S^2$, which can be obtained as classical solutions in the IIB matrix model assuming that a Myers term [25] is induced dynamically. (See [26–28], which discuss the appearance of these fuzzy manifolds in the IIB matrix model due to quantum corrections. Note also that, including the dimensionality of our four-dimensional space-time, fuzzy S^2 and fuzzy $S^2 \times S^2$ correspond to a D5-brane and a D7-brane, respectively, which naturally appear in type IIB superstring theory.) The two types of fuzzy manifold intersect in the six-dimensional space generically at even number of points, which give rise to pairs of chiral fermions with opposite chirality in six dimensions. However, by using the degrees of freedom in the matrix warp factor M, one can obtain only the desired chiral zero modes in four dimensions.

Extending this basic setup, we give an explicit realization of the Standard Model [24]. The $SU(r)$ group can be realized as a subgroup of $U(r)$, which appears naturally from r coinciding

[1] Moreover, a local field in four-dimensional space-time can actually be realized, for instance, by a mechanism discussed in [29].

branes. First, we introduce "$SU(3)$ branes", which consist of three coinciding fuzzy $S^2 \times S^2$, and "$SU(2)$ branes", which consist of two coinciding fuzzy S^2. In addition, we introduce a "lepton brane", which is a single fuzzy $S^2 \times S^2$, and an "up-type brane" and a "down-type brane", which are two separate fuzzy S^2. Thus, we end up with a configuration with five stacks of branes intersecting with each other (see Fig. 3). An important point here is that chiral fermions actually appear only from intersections of fuzzy S^2 and fuzzy $S^2 \times S^2$. This enables us to obtain just the chiral fermions in the Standard Model plus a right-handed neutrino, with the correct gauge interactions. One can also check that the hypercharge can be assigned to the chiral fermions consistently.

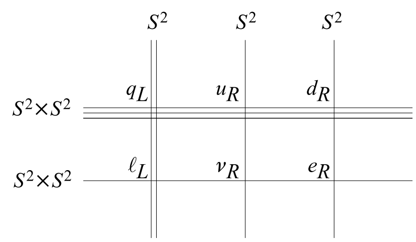

Fig. 3. A schematic view of the configuration with five stacks of branes, which gives rise to the Standard Model fermions and a right-handed neutrino

In fact, we show that the number of intersections of S^2 and $S^2 \times S^2$ in six dimensions cannot exceed four for arbitrary radii, location of the centers and their relative angles. This implies that we can obtain only up to two generations, if we restrict ourselves to such configurations. Three generations can be realized, for instance, by squashing S^2 or $S^2 \times S^2$ that appear in the configuration. We can also discuss how the Higgs field appears from the bosonic matrices, with nontrivial Yukawa couplings to the Standard Model fermion.

4. CONCLUSION AND DISCUSSION

In this paper, we showed that in the Lorentzian version of the IIB matrix model three out of nine spatial directions actually start to expand after a critical time, which may be interpreted as the birth of universe. We also discussed how three generations of the Standard Model particles can be obtained in the model.

If we identify $R(t_{\rm c})$ with the Planck length, we see only the Planckian time interval in the present simulation. While the late-time behaviors are difficult to study by direct Monte Carlo methods because larger N is needed, the classical equations of motion are expected to become more and more valid at later times, since the value of the action increases with the cosmic expansion. One can actually construct a simple solution representing an expanding $(3+1)$-dimensional universe, which naturally solves the cosmological constant problem [22].

There are many classical solutions [22, 23], which is reminiscent of the fact that string theory possesses infinitely many vacua that are perturbatively stable. However, unlike in perturbative string theory, we have the possibility to pick up the unique solution that describes our universe by requiring smooth connection to the behavior at earlier times accessible by Monte Carlo simulation. We expect that the idea of the renormalization group developed in [30] is useful to pursue this possibility.

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