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STRINGY HOLOGRAPHY AT FINITE DENSITY

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We consider an exactly solvable worldsheet string theory in the background of a black brane with a gauge field flux. Holographically, such a system can be interpreted as a field theory with finite number of degrees of freedom at finite temperature and density. We construct closed string vertex operators which holographically represent the $U(1)$ gauge field and the stress energy tensor and compute their two-point functions. At finite density, the system behaves like a sum of two noninteracting fluids.

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INTRODUCTION

In the usual AdS/CFT setting, gauge theory on the boundary has a dual description in terms of closed string theory in the bulk. Most often, a limit of small curvature is taken to yield a low energy theory of strings, supergravity. In the $\mathcal{N} = 4$ supersymmetric Yang–Mills case, this limit implies strong 't Hooft coupling of field theory. A distinct example of nongravitational theory with a holographically dual description is the Little String Theory (LST) [1, 2]. It can be viewed as the theory of N coincident NS5-branes, taken at vanishing string coupling, $g_s = 0$, where the bulk degrees of freedom decouple.

The holographic dual of the little string theory [2, 3] is the theory of closed strings in the background of NS5-branes, with the geometry $R^{5,1} \times R^{\phi} \times SU(2)_N$, the two-form field and the linear dilaton. The CFT on $SU(2)$ is described by WZW action at level N. The bulk physics (in the double scaling limit) can be reformulated as the string theory on $R^5 \times \dfrac{SL(2,R)_N}{U(1)} \times SU(2)_N$ space-time. This is due to the fact that the gauged WZW model on $SL(2,R)_N/U(1)$ gives rise to the classical «cigar» geometry of the two-dimensional black hole with the asymptotically linear dilaton [4]. In the large N limit, the bulk theory reduces to supergravity[3].

Generally one expects that a lot of nontrivial physics drastically simplifies in the limit of infinitely many degrees of freedom (large N limit), both in the boundary field theory and from the dual bulk perspective. For example, one expects the large N physics of a field theory at finite temperature and density to have «classical» nature, resulting, in particular, in the mean field critical exponents.

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[3]The radius of the $SU(2)$ sphere is $R_{\rm sph} = \sqrt{N}\ell_s$. Therefore the large N limit is equivalent to the limit of small $\ell_s/R_{\rm sph}$.

1. SETUP

In our work we study string theory in the background of a direct product of flat space R^{d-1} and the two-dimensional charged black hole [5]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2}, \quad \Phi = \Phi(r), \quad A_t \simeq -\frac{q}{r}. \tag{1}$$

The temperature, chemical potential, and pressure of the QFT_d are given by [6] ($\psi \in [0, \pi/2]$)

$$T = \frac{1}{2\pi\sqrt{N}} \frac{\cos\psi}{\cos^2(\psi/2)}, \quad \mu = \tan(\psi/2), \quad P = 0. \tag{2}$$

The string theory in the two-dimensional charged black hole background is described by the gWZW action with the target space-time [7, 8] $M = \dfrac{SL(2,R) \times U(1)_x}{U(1)}$. Here $U(1)_x$ is a compact circle, which is Kaluza–Klein reduced, and $U(1)$ subgroup of $SL(2,R) \times U(1)_x$ is gauged asymmetrically

$$(g, x_L, x_R) \sim \left(\mathrm{e}^{\tau\cos\psi\sigma_3/\sqrt{N}} g\, \mathrm{e}^{\tau\sigma_3/\sqrt{N}}, x_L + \tau\sin\psi, x_R\right). \tag{3}$$

The left-moving sector of the gauged $U(1)$ is a linear combination of the left-moving sector of the $U(1)_x$ and the left-moving sector of the $U(1)$ subgroup of the $SL(2,R)$. The coefficient of this linear combination determines the charge-to-mass ratio of the resulting black hole. The right-moving sector of the gauged $U(1)_x$ is the right-moving sector of the $U(1)$ subgroup of the $SL(2,R)$.

This bulk system is holographically dual to the boundary quantum field theory at finite temperature and charge density. (One can think of the resulting system as a little string theory at finite density, but we do not study the field theoretic interpretation here in detail.) The finite charge density in the field theory is described holographically by the background $U(1)$ potential in the bulk.

We construct the vertex operators which describe massless closed string excitations in this model, which constitute the NS-NS sector of type-II supergravity. We also construct the gauge field vertex operators, which are obtained by Kaluza–Klein reduction on $U(1)_x$ from graviton and antisymmetric tensor field vertex operators. The graviton in the bulk is dual to the stress-energy tensor on the boundary; the gauge field in the bulk is dual to the charge current on the boundary. We study the low-energy excitations of the system by computing holographically the two-point functions for the charge current and the stress-energy tensor and reading off the dispersion relation from their poles. We find two distinct gapless modes in the shear channel; the dispersion relation of one of them is independent of the charge-to-mass ratio of the black hole. The two modes merge in the limit of vanishing charge. We confirm these results by solving fluctuation equations of the type-II supergravity. The situation in the sound channel is similar.

2. VERTEX OPERATORS AND TWO-POINT FUNCTIONS

The vertex operators of the graviton and the 2-form field are

$$G^{\mu\nu} = (j^{\mu}_{-1}\tilde{j}^{\nu}_{-1} + j^{\nu}_{-1}\tilde{j}^{\mu}_{-1})V_g, \quad B^{\mu\nu} = (j^{\mu}_{-1}\tilde{j}^{\nu}_{-1} - j^{\nu}_{-1}\tilde{j}^{\mu}_{-1})V_g. \tag{4}$$

Taking one polarization along $U(1)_x$ and performing KK reduction, we obtain the gauge fields

$$a^\mu = (j^\mu_{-1}\tilde{j}^x_{-1} + j^x_{-1}\tilde{j}^\mu_{-1})V_g, \quad b^\mu = (j^\mu_{-1}\tilde{j}^x_{-1} - j^x_{-1}\tilde{j}^\mu_{-1})V_g. \tag{5}$$

The ground-state vertex operator of the bosonic string on $SL(2,R)$ is the $SL(2,R)$ Kac–Moody primary field. The two-point function of $SL(2,R)$ primaries is known exactly [9]:

$$\langle V_{j,m,\bar{m}} V_{j,-m,-\bar{m}} \rangle \simeq \frac{\Gamma\left(1 - \dfrac{2j+1}{N}\right)\Gamma(-2j-1)\Gamma(1+j+m)\Gamma(1+j-\bar{m})}{\Gamma\left(1 + \dfrac{2j+1}{N}\right)\Gamma(2j+1)\Gamma(-j+m)\Gamma(-\bar{m}-j)}. \tag{6}$$

Gauging of $U(1)$ from $SL(2,R) \times U(1)_x$ is realized by imposing the BRST conditions

$$j_n|\Psi\rangle = 0, \quad n \geqslant 0, \quad j(z) = k(z) - \partial w(z), \tag{7}$$

where $j(z)$ is a null current, $k(z)$ is $U(1)$ current which we gauge, and $w(z)$ parameterizes auxiliary $U(1)$ circle. Similarly for antiholomorphic sector.

The ground-state vertex operator of the bosonic string on M is

$$\Phi_{jm\bar{m}} = V_{jm\bar{m}}\, \mathrm{e}^{in_L x + in_R \tilde{x}}, \tag{8}$$

subjected to the $U(1)$ BRST condition [8]

$$m\cos\psi + \bar{m} + \sqrt{N} n_L \sin\psi = 0. \tag{9}$$

We have performed covariant quantization of a string to construct physical massless NS-NS states.

Choose X to be direction of propagation of excitations, with momentum p and frequency ω. We have the following groups of vertex operators defined by the spin w.r.t. to the rotations in the transverse noncompact space (with coordinates x^a). Sound channel: the spin is zero, G^{XX}, A^X B^X. Shear channel: the spin is one, G^{Xa}, B^{Xa}, A^a, B^a. Scalar channel: the spin is two, G^{ab}, B^{ab}. Due to the rotational symmetry in the transverse space, vertex operators from different groups are decoupled from each other.

Mass-shell condition of the gravity multiplet states is

$$-\frac{j(j+1)}{N} + \frac{p^2 - \omega^2}{2} = 0, \quad \omega = \frac{2im\cos\psi}{\sqrt{N}}. \tag{10}$$

In the shear channel, we have vertex operators, which we regroup into two decoupled systems,

$$S^{Xa} = \frac{1}{2}(G^{Xa} + B^{Xa}) = j^X \bar{\partial} x^a V_g, \qquad W^a = \frac{1}{2}(A^a + B^a) = j^x \bar{\partial} x^a V_g, \tag{11}$$

$$R^{Xa} = \frac{1}{2}(G^{Xa} - B^{Xa}) = \tilde{j}^X \partial x^a V_g, \qquad U^a = \frac{1}{2}(A^a - B^a) = \bar{\partial} x \partial x^a V_g. \tag{12}$$

3. LOW-ENERGY MODES

We read off the low-energy modes of the QFT_d as poles of two-point functions for the operators of the stress-energy tensor $T_{\mu\nu}$ and charge current J_μ. Holographic correspondence maps these operators to the vertex operators of the fields in the bulk:

$$T_{\mu\nu} \leftrightarrow G_{\mu\nu}, \quad J_\mu \leftrightarrow G_{\mu x} = A_\mu. \tag{13}$$

The holographic prescription for computing the two-point functions is

$$\langle T_{\mu\nu} T_{\lambda\rho} \rangle_{\mathrm{QFT}} = \langle G_{\mu\nu} G_{\lambda\rho} \rangle_{w.s}, \quad \langle T_{\mu\nu} J_\lambda \rangle_{\mathrm{QFT}} = \langle G_{\mu\nu} A_\lambda \rangle_{w.s}, \quad \langle J_\mu J_\lambda \rangle_{\mathrm{QFT}} = \langle A_\mu A_\lambda \rangle_{w.s}. \tag{14}$$

We have computed $\langle T_{Xa} T_{Xa} \rangle$ via string holographic dual and found the poles at

$$\omega = -i \frac{\sqrt{N}}{2} p^2, \quad \omega = -i \cos\psi \frac{\sqrt{N}}{2} p^2. \tag{15}$$

Each mode comes from one of the two decoupled systems (11), (12). From QFT point of view, we therefore have two different noninteracting fluids, each supporting one of these modes. We have verified the dispersion relations (15) by solving fluctuation equations in supergravity approximation.

Finally, recall that all the two-point functions are proportional to ground-state two-point function (6), which in turn is proportional to $\Gamma\left(1 - \frac{2j+1}{N}\right)$. Using the mass-shell condition (10), we obtain singularity at $\omega = 0$ and $p = p_*$, $p_*^2 = \frac{1}{\ell_s^2}\left(N - \frac{1}{N}\right)$ [10]. Measured in units of curvature radius $R = \sqrt{N}\ell_s$, it goes as $(Rp_*)^2 \sim N^2$, when N is large and therefore becomes parametrically large in supergravity approximation. Therefore, $\omega = 0, p = p_*$ singularity is a purely stringy effect.

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