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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

M5s AND n^3

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In this brief communication, based on joint with E. Hatefi and I. Y. Park papers [9, 10], I discuss a criterion for constructing 11D/10D maximal supergravity black-brane solutions with the near-extremal $S \sim n^3T^5$ entropy-temperature dependence and their lower dimensional descents.

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1. MOTIVATION AND OUTLOOK

It is well known, the entropy of n coincident M5 branes scales as n^3 [1]. This dependence on the number of branes in the stack is quite different from the corresponding D-branes scaling. Say, for D3 branes in the near-extremal limit the entropy depends on the temperature as $S \sim n^2T^3$. This result is in agreement with $U(n)$ symmetry of n coincident D3-branes [2]. But for the stack of n near-extremal M5 black branes, the corresponding relation takes $S \sim n^3T^5$. The n^3 entropy growth cannot be associated with any classical Lie group. It should be treated as indication of some novel degrees of freedom of 6D (2,0) theory.

What is the origin of novel hidden constituents in M5-branes physics? Part of them may be related to the self-dual strings [3] on the M5 worldvolume. They contribute to the effective potential of 6D chiral supersymmetric theory via string junctions [4] and instanton transitions [5]. The n^3 contribution of the internal degrees of freedom was also viewed from anomaly inflow in 11D supergravity in the presence of dynamical M5 branes [6], as well as in pure 6D (2,0) superconformal theory [7]. It is also worth mentioning that in the AdS/CFT of holographic hydrodynamics [8] inclusion of M5 branes led to the n^3 behavior of pressure and shear viscosity of 6D strongly coupled supersymmetric plasma at non-zero temperature. So, the n^3 scaling comes from different aspects of M5-theory, but is this result so exclusive for M5 branes? For instance, the near-extremal entropy of nDp type II black branes, $S_{\rm Dp} \sim n^{(7-p)/(5-p)}T^{(9-p)/(5-p)}$, shows that type IIA D4 black branes behave in the same way as M5s. The close relation between 6D effective worldvolume theory of M5 branes and 5D effective worldvolume description of D4 branes goes back to the equivalence of M-theory and the strongly coupled type IIA string theory. This duality allows one to study M5 branes through D4 brane/open string techniques. Following this way, the n^3 entropy growth was obtained in a D0-based SYM setup (D0/D4 setup) by including Myers' terms and subsequently applying the localization technique (see [9] for details).

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Are M5s and D4s so unique in the n^3 entropy growth? Could it be possible to figure out the same near-extremal entropy-temperature dependence for other black branes or their configurations? Main candidates to figure out the answer are various configurations of KKW/M2/M5 branes, since they enter the M-algebra, part of which is $\{Q, Q\} = \gamma^m P_m + \gamma^{mn} Z_{mn} + \gamma^{m_1 \ldots m_5} Z_{m_1 \ldots m_5}$, in a democratic way. And indeed, following this way, the n^3 entropy growth can be recovered for intersections/interpolations of black KKW/M2/M5 branes [10]. An interesting feature we observed consists in thermodynamical equivalence of pertaining configurations of lower-dimensional branes, lying inside or intersecting with M5s, to the stack of coincident M5 branes. It is a manifestation of the Myers–Emparan effect on supergravity side, which results (after the tuning of the parameters) in dissolving the lower-dimensional branes into higher-dimensional ones.

2. 11D BLACK M5s INTERSECTIONS/INTERPOLATIONS AND THEIR THERMODYNAMICS IN THE NEAR-EXTREMAL LIMIT

Let us consider the near-extremal $S(T)$ relation for the M5/KKW bound state. The corresponding solution of D = 11 supergravity equations of motion is [11]

$$ds_{11}^2 = H_5^{2/3}[H_5^{-1}(-K^{-1} f dt^2 + K d\hat{x}_1^2) + H_5^{-1}(dx_2^2 + \cdots + dx_5^2) + f^{-1} dr^2 + r^2 d\Omega_4^2],$$

$$H_5(r) = 1 + n_5 \frac{h_5^3}{r^3}, \quad K(r) = 1 + n_0 \frac{k_0^3}{r^3}, \quad f(r) = 1 - \frac{\mu^3}{r^3}, \quad d\hat{x}_1^2 = [dx_1 + (K^{-1} - 1)dt]^2.$$

Computing the ADM mass, entropy and temperature of the non-extremal configuration, in the near-extremal limit one gets [10]

$$S \sim \sqrt{n_0 n_5} (\delta M)^{1/3}, \quad T \sim (\sqrt{n_0 n_5})^{-1} (\delta M)^{2/3} \to S \sim (n_0 n_5)^{3/4} T^{1/2},$$

and the resulting $S(T)$ dependence is far from the desired $S \sim n^3 T^5$.

But let us take a look at the problem from a different point of view. The non-extremal values of the ADM mass, entropy and temperature of the M5/KKW bound state are

$$M = \frac{l^5 \Omega_4}{16\pi G_{11}} [4\mu^3 + 3n_0 k_0^3 + 3n_5 h_5^3], \quad S = \frac{l^5 \Omega_4}{4G_{11}} \left(1 + n_5 \frac{h_5^3}{\mu^3}\right)^{1/2} \left(1 + n_0 \frac{k_0^3}{\mu^3}\right)^{1/2} \mu^4,$$

$$T = \frac{3}{4\pi} \left(1 + n_5 \frac{h_5^3}{\mu^3}\right)^{-1/2} \left(1 + n_0 \frac{k_0^3}{\mu^3}\right)^{-1/2} \mu^{-1}.$$

One may encounter the symmetry under the interchange of numbers of branes $n_0 \leftrightarrow n_5$ and of their "charges" $h_5 \leftrightarrow k_0$. Now, tuning the parameters as $n_0 k_0^3 \ll 1, n_0 \gg 1$, it is clear that the desired result is recovered in this way, $S_{\mathrm{KKW/M5}} \sim n_5^3 T^5$. The symmetry under the parameters interchange suggests that the bound state is formed with a number of KK-waves in the $n_0 k_0^3 \gg 1$ limit and a number of M5 branes in the $n_5 h_5^3 \ll 1$, $n_5 \gg 1$ limit. With this choice, the thermodynamics of KK-waves models that of the black M5 branes; the near-extremal entropy becomes $S_{\mathrm{KKW/M5}} \sim n_0^3 T^5$. It may be viewed as a manifestation of the Myers effect [12] of polarization of KK-waves (M0 branes) into M5 branes.

The same analysis of M2/M5 intersection solution and boosted M2/M5 intersection corresponding to the KKW/M2/M5 intersection [11] revealed that it is impossible to recover the n^3 entropy growth by tuning the parameters of the problem. We are faced with the question: What is the condition for reproducing of n^3 scaling then?

3. CONDITION OF REPRODUCING THE n^3 SCALING

It is well known that thermodynamics of black branes configurations is completely encoded in the structures of the harmonic functions and of the "blackening" factor(s) that appear in the solution. To reproduce the same thermodynamics of a given system (the M5 brane system here) by another system (say, the boosted M5 branes, or their intersections with KKWs and M2s), it is necessary to assure that the two configurations under consideration have *the same* entropy/temperature dependence on the non-extremality parameter μ in the blackening factor $f(r)$. Comparing $S_{\mathrm{M5}} \sim \mu^4$, $T_{\mathrm{M5}} \sim \mu^{-1}$ with $S_{\mathrm{M5/KKW}} \sim \mu^4$ and $T_{\mathrm{M5/KKW}} \sim \mu^{-1}$, one can see that the entropy/temperature dependences on the extremality parameter μ of two systems are the same. But the entropy of M2/M5 branes intersection, $S_{\mathrm{M2/M5}} \sim \mu^3$, shows different dependence on μ. It leads to a completely different entropy-temperature equation of state in the end, and, consequently, to different thermodynamics.

From this analysis, it becomes clear that compelling candidates for intersecting solutions of M branes that could potentially reproduce (exactly or within some special choice of the parameters) the n^3 growth of the black M5 branes are those whose harmonic functions have $1/r^3$ dependence. Put differently, $d\Omega_4$ *volume element must be present in the solution* [10].

Following this criterion, the boosted M2/M5 intersection (KKW/M2/M5 system) cannot reproduce the n^3 scaling, since the solution is based on $d\Omega_3$ volume element. At the same time, the M2/M5 interpolating solution and its boosted version [13] both fall into the drawn criterion.

4. OTHER BLACK BRANES CONFIGURATIONS WITH n^3

Consider the following solution that interpolates between M2 and M5 branes [13]:

$$ds_{11}^2 = (H\tilde{H})^{1/3}\Big[H^{-1}(-f dt^2 + dx_1^2 + dx_2^2) + \tilde{H}^{-1}(dx_3^2 + dx_4^2 + dx_5^2) + f^{-1}dr^2 + r^2 d\Omega_4^2\Big],$$

$$\hat{F}_4 = \frac{1}{2}\cos\zeta * dH + \frac{1}{2}\sin\zeta\, dH^{-1}\epsilon_3 + \frac{3}{2}\sin 2\zeta\, H^{-2}\, dH\, \bar{\epsilon}_3,$$

$$H(r) = 1 + n\frac{h^3}{r^3}, \qquad \tilde{H} = \sin^2\zeta + H\cos^2\zeta, \qquad f(r) = 1 - \frac{\mu^3}{r^3}.$$

Here ϵ_3 and $\bar{\epsilon}_3$ are volume forms on M^3 and E^3 manifolds parameterized respectively by (t, x_1, x_2) and (x_3, x_4, x_5); $*$ is the Hodge dual of E^5 that is transverse to the M5 branes.

The interpolating solution describes nM2 branes entirely lying within nM5 branes and is completely different from the standard intersection of M2/M5. Due to the single harmonic function $H(r)$ entering the solution the entropy-temperature dependence is exactly the same as for the stack of black M5s, $S \sim n^3 T^5$. The boosted version of the M2/M5 interpolating solution [11] is thermodynamically quite similar to the boosted M5s solution. Hence, in this case one may tune the parameters in such a way that $S \sim n_0^3 T^5$ with n_0 to be the number of KKWs. Therefore, the KKW/M2/M5 interpolating solution becomes thermodynamically equivalent to the stack of n_0 black M5 branes. We interpret this phenomenon as a realization of the Myers effect of polarization of branes.

Dimensional reduction of the obtained solutions with $S \sim n^3 T^5$ near-extremal entropy behavior leads to other configurations with n^3 scaling. They are: Type IIA n coincident

D4 black branes; type IIA D0/D4 bound state when D0 branes may get expanded into D4 branes; type IIA D0/D2/D4 interpolating solution, which inherits features of the KKW/M2/M5 solution; 9D D3 coincident black branes; 8D D2s; 7D D1s; 6D D0 black branes [10].

5. SUMMARY AND CONCLUSIONS

We have constructed examples of black brane solutions of 11D/10D IIA supergravities that exhibit the n^3 near-extremal entropy behavior (with appropriate tuning of the parameters). They include the boosted M5 black branes (M5/KKW bound state) and the boosted M2/M5 interpolating black branes. We have proposed a simple criterion for a solution to have the n^3 near-extremal entropy. The compelling candidates are those whose harmonic functions depend on five coordinates transverse to the host brane worldvolume. In the spherical coordinate system it means that the harmonic functions will have $1/r^3$-dependence. Implementing dimensional reduction on the type IIA black D4 branes solution and taking the criterion as a guideline, we have established that black-brane solutions corresponding to 9D D3, 8D D2, 7D D1 and 6D D0 satisfy the criterion and all share the n^3 entropy growth. Some of these solutions were previously known, e.g., the solution of 8D D2 black branes is similar to the purely magnetic D2 brane solution of [13]. Although some of the black-brane solutions constructed here share the n^3 entropy with the M5 solution of Klebanov–Tseytlin, the former differ from the latter in the following aspect. As noted in [1], the equations of state of non-dilatonic black branes in the near-extremal limit can be reproduced (with an exception of the M2 case) in terms of a weakly interacting ideal gas of massless particles that are associated with massless excitation of p-brane modes. This is not the case for the descents of black M5 brane solutions constructed here, starting with the D4 black branes of 10D IIA. These descents are *dilatonic* branes; nonperturbative corrections become important in $r \to 0$ limit. One novelty of the KKW/M2/M5 solution is thermodynamics “duality”; there are two ways to scale the parameters such that the thermodynamics of the two scalings toggles under the interchange of $n_5 \leftrightarrow n_0$. Perhaps the “duality” is an indication toward the supergravity analogue of the fact that the SYM account of D-branes physics admits “dual” description, one in terms of the lower dimensional branes and the other in terms of higher (see, e.g., [9]).

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