Письма в ЭЧАЯ. 2014. Т. 11, № 7(191). С. 1583–1590

FUZZY TOPOLOGY AND GEOMETRIC QUANTUM FORMALISM

S. N. Mayburov[1]

Lebedev Institute of Physics, Moscow

Dodson–Zeeman fuzzy topology is considered as the possible mathematical framework of geometric quantization. In such a formalism the states of massive particle m correspond to elements of fuzzy manifold called fuzzy points. Due to their weak (partial) ordering, m space coordinate x acquires principal uncertainty σ_x. It is shown that m evolution with minimal number of additional assumptions obeys Schrödinger or Dirac formalisms in norelativistic and relativistic cases correspondingly. It is argued that particle's interactions on such a fuzzy manifold should be gauge-invariant.

PACS: 03.65.Ca; 02.10.Ab; 02.40.-k; 11.10.Cd

INTRODUCTION

Importance of geometric methods in Quantum Physics is duly acknowledged now. The popular example of it is Connes noncommutative geometry which attempts to describe the fundamental interactions at Planck distances [1]. It is also worth mentioning the extensive studies of noncommutative fuzzy spaces with finite (sphere, tori) and infinite discrete structure [2, 3]. The general feature of such theories is that the space coordinates turn out to be principally fuzzy, the reason of that is the noncommutativity of coordinate observables $x_{1,2,3}$. Meanwhile, it was shown that similar fuzzy properties can be obtained for the spaces equipped with dedicated fuzzy topology (FT) [4,5]. In our previous papers it was shown that in its framework the quantization procedure by itself can be defined as the transition from the classical phase space to fuzzy one. Therefore, the quantum properties of particles and fields can be deduced directly from the geometry of phase space induced by underlying FT and do not need to be postulated separately of it [6,7]. I was also shown that FT equipped geometry induces the geometrodynamics which is equivalent to quantum mechanics (QM) [6, 7]. Earlier some phenomenological assumptions were used by the author; here the new and simple formalism which permits him to drop them is described. As an example, the quantization of massive particles will be considered. It will also be argued that the interactions on such a fuzzy manifold possess the local gauge invariance and under simple assumptions would correspond to Yang–Mills theory [7].

Here we shall describe briefly only the most important steps in construction of mechanics on fuzzy manifold called fuzzy mechanics (FM); the details can be found in [6,7]. To illustrate

[1]E-mail: mayburov@sci.lebedev.ru

FT formalism, let us describe it first for the simple discrete structure and consider some set A of n elements $\{a_i\}$. If it is the ordered set, then the ordering relation between all element pairs $a_k \leqslant a_l$ (or vice versa) is fulfilled. As an example, we choose A for which $\forall i,\ a_i \leqslant a_{i+1}$. But if A is the partial-ordered set (Poset), then some of its element pairs can enjoy the incomparability (equivalence) relations (IR) between them: $a_j \sim a_k$. If this is the case, then both $a_j \leqslant a_k$ and $a_k \leqslant a_j$ propositions are false. To illustrate its meaning, suppose that the element a_{n+1} is added to A, for which $a_{n+1} \sim a_j$, $\forall j$; $i \leqslant j \leqslant l$ and $a_{i-1} \leqslant a_{n+1} \leqslant a_{l+1}$. In this case a_{n+1} is "smeared" over $\{a_i, a_l\}$ subset (interval), which is rough analogue of a_{n+1} coordinate uncertainty relative to A "coordinate axis".

It is possible to detalize such a smearing by introducing the fuzzy relations, for that purpose one can put in correspondence to each a_{n+1}, a_j pair the weight $w_j \geqslant 0$ with the norm $\sum_j w_j = 1$. In this case A is fuzzy ordered set (Foset), a_{n+1} called the fuzzy point (FP) [4,5]. The continuous 1-dimensional Foset C^F is defined analogously: $C^F = A^P \cup X$, where A^P is the discrete subset of incomparable elements a'_j, X is the continuous ordered subset, which is equivalent to R^1 axis of real numbers. Correspondingly, fuzzy relations between elements a'_j, x are described by real function $w^j(x) \geqslant 0$ with the norm $\int w^j dx = 1$. In 1-dimensional Euclidean geometry, the elements of its manifold X are the points x_a which constitute the ordered continuum set. Yet in 1-dimensional FT equipped geometry the position of fuzzy point a'_j becomes the positive normalized function $w^j(x)$ on X; w^j dispersion σ_x characterizes a'_j coordinate uncertainty on X. Note that in such a geometry $w^j(x)$ does not have any probabilistic meaning but only the algebraic one, characterizing the properties of fuzzy values $\tilde{x}$. To describe the distinction between the fuzzy structure and probabilistic one, the weight correlation $K_f(x, x')$ defined over w_j support can be introduced; thus, if $w(x_{1,2}) \neq 0$, then $\forall x_1, x_2$; $K_f(x_1, x_2) = 1$ for FP a'_j and $K_f(x_1, x_2) = 0$ for probabilistic a'_j distribution. Thus, a'_j "state" G on X is described by two functions $G = \{w(x), K_f(x, x')\}$ which characterize the fuzzy value $\tilde{x}_a$.

1. LINEAR MODEL OF FUZZY DYNAMICS

In the described terms the massive particle of 1-dimensional classical mechanics corresponds to the ordered point $x_a(t) \in X$. By analogy, we suppose that in 1-dimensional FM the particle m corresponds to fuzzy point $a(t)$ in C^F characterized by normalized positive density $w(x,t)$. Beside $w(x)$, m fuzzy state $|g\}$ can also depend on other m degrees of freedom (DFs), i.e., $|g\}$ parameters characterizing its evolution. The temptative candidate for that is m average velocity $\bar{v}$:

$$\bar{v} = \frac{\partial}{\partial t} \int_{-\infty}^{\infty} x w(x)\, dx = \int_{-\infty}^{\infty} x \frac{\partial w}{\partial t}(x,t)\, dx. \quad (1)$$

It is sensible to expect that $\bar{v}(t)$ is independent of $w(x,t)$. Below we shall look for such DFs in form of real functions $q_{1,\dots,n}(x,t)$. We shall start from considering m free evolution, and suppose that in FM it is local, i.e.,

$$\frac{\partial w}{\partial t}(x,t) = -\Phi(x,t), \quad (2)$$

where Φ is an arbitrary function which depends on $x, w(x,t)$, $q_{1,\ldots,n}(x,t)$ only. From w norm conservation:

$$\int_{-\infty}^{\infty} \Phi(x,t)\,dx = -\int_{-\infty}^{\infty} \frac{\partial w}{\partial t}(x,t)\,dx = -\frac{\partial}{\partial t}\int_{-\infty}^{\infty} w(x,t)\,dx = 0. \tag{3}$$

Since w free evolution should possess x, t-shift invariance, Φ cannot depend on x, t directly, but only on $w(x,t)$ and $q_i(x,t)$. If $\Phi = \partial J/\partial x$ is substituted for some $J(x)$, then Eq. (3) demands:

$$J(\infty,t) - J(-\infty,t) = 0. \tag{4}$$

If it is supposed that it is fulfilled, $J(x)$ can be regarded as w flow (current), so Eq. (2) can be transformed to the 1-dimensional flow continuity equation [8]:

$$\frac{\partial w}{\partial t} = -\frac{\partial J}{\partial x}. \tag{5}$$

$J(x)$ can be decomposed formally as $J = w(x)v(x)$, where $v(x)$ corresponds to 1-dimensional w flow velocity. In these terms w evolution equation transforms to

$$\frac{\partial w}{\partial t} = -v\frac{\partial w}{\partial x} - \frac{\partial v}{\partial x}w. \tag{6}$$

Note that for normalized density $w(x,t)$ the relation (4) is rather obvious, in particular, it is fulfilled, if w flow $J(x,t)$ from/to $x = \pm\infty$ is negligible. FM will be constructed here as the minimal theory in a sense that at every step we shall choose its variant with minimal number of DFs and theory constants. We shall consider here only the pure fuzzy states which are not the probabilistic mixture of several different states.

It is sensible to suppose that $v(x,t)$ can be considered as $|g\}$ free parameter, yet we shall use the related parameter $\gamma(x)$ defined as

$$\gamma(x,t) = \mu \int_{-\infty}^{x} v(\xi,t)\,d\xi + c_\gamma, \tag{7}$$

where μ is theory constant; c_γ is an arbitrary real value. If it is assumed that m state $|g\}$ does not depend on any other DFs, i.e., $|g\} = \{w(x), \gamma(x)\}$, then it can also be expressed as some complex function $g_a(x)$, the simplest example is $g_a(x) = w(x) + i\gamma(x)$. Alternatively, one can suppose that $|g\} = \left\{w(x), \gamma(x), \ldots, \frac{\partial^n \gamma}{\partial x^n}\right\}$ regarding formally each γ derivative as independent DF, yet below it will be shown that such an assumption is excessive. Moreover, FM premises restrict essentially the possible $|g\}$ complex ansatz $g_a(x)$. It is sensible to admit that m evolution as a whole can be characterized by m velocity u with expectation value $\bar{u}(t)$. Yet in FM, alike m coordinate x, u can also be considered as fuzzy value $\tilde{u}$ with the corresponding distribution $w_u(u,t)$. Plainly, $\bar{u}(t)$ coincides with $\bar{v}(t)$ of (1), hence it can be written as

$$\bar{u} = \frac{1}{\mu}\int_{-\infty}^{\infty} \frac{\partial \gamma}{\partial x} w\,dx. \tag{8}$$

Yet w_u should be expressed as function of given $|g\}$ DFs w, γ, since it was supposed that they describe m state completely. If so, then w_u can be expressed via arbitrary w, γ X-Fourier transform. To calculate it, let us introduce the auxiliary function $\varphi(u)$ and its X-Fourier decomposition:

$$\varphi(u) = w_u^{1/2}(u)\,\mathrm{e}^{i\beta(u)} = \int_{-\infty}^{\infty} f(x)\,\mathrm{e}^{i\lambda(x)+iux}\,dx. \tag{9}$$

Here $\beta(u)$ is some real function, f, λ are supposed to be the real functions of w, γ in a given point x. It follows then:

$$\int_{-\infty}^{\infty} f^2(x)\,dx = \int_{-\infty}^{\infty} \varphi(u)\varphi^*(u)\,du = \int_{-\infty}^{\infty} w_u(u)\,du = 1.$$

The calculation of $\delta\gamma, \delta w$ variations for f^2 integral gives $f(w,\gamma) = \pm w^{1/2}(x)$. Then $\bar{u}$ can be calculated anew from Fourier analysis [9]:

$$\bar{u} = \int_{-\infty}^{\infty} u\varphi(u)\,\varphi^*(u)\,du = \int_{-\infty}^{\infty} \frac{\partial\lambda}{\partial x} f^2(x)\,dx. \tag{10}$$

From the comparison with Eq. (8) it follows that

$$\lambda(x) = \frac{1}{\mu}[\chi(w) + \gamma(x)],$$

where $\chi(w)$ is an arbitrary real function which obeys the condition

$$\int_{-\infty}^{\infty} w\frac{\partial\chi}{\partial x}\,dx = 0.$$

Admitting that $f = w^{1/2}(x)$, w_u and $\beta(u)$ can be found now from Eq. (9) as functions of χ. $\beta(u)$ is the counterpart of $\gamma(x)$ for u observable. Below we shall also use the observable $p = \mu u$, if one substitutes in the integral of Eq. (9) ux by px and $f\,\mathrm{e}^{i\lambda}$ by the function

$$\eta(x) = w^{1/2}(x)\,\mathrm{e}^{i\chi(w)+i\gamma} \tag{11}$$

correspondingly, then the resulting X-Fourier transform will be equal to $\varphi(p)$. $\eta(x)$ is the vector (ray) of complex Hilbert space $\mathcal{H}$. In this framework the observable u corresponds to the operator $\hat{u} = \dfrac{i}{\mu}\dfrac{\partial}{\partial x}$ acting on η. By analogy, we postulate that all m observables are the linear, self-adjoint operators on $\mathcal{H}$. Then η is the temptative candidate for $|g\}$ state ansatz in X-representation, since the expectation values of all observables are expressed via η bilinear forms. Note that $\eta = \mathrm{e}^{i\chi}g$, where $g(x,t)$ is standard QM wave function, so that $\eta(x,t)$ is its trivial map. Thus, one can consider in detail only $g(x,t)$ evolution and derive $\eta(x,t)$ properties from obtained results. Evolution equation for g is supposed to be of the first order in time, i.e.,

$$i\frac{\partial g}{\partial t} = \hat{H}g. \tag{12}$$

In general, $\hat{H}$ is nonlinear operator; for simplicity we shall consider first the linear case and nonlinear one below. The free m evolution is invariant relative to x space shifts performed by the operator $\hat{W}(a) = \exp\left(a\frac{\partial}{\partial x}\right)$. Because of it, $\hat{H}$ should commute with $\hat{W}(a)$ for the arbitrary a, i.e., $\left[\hat{H}, \frac{\partial}{\partial x}\right] = 0$. It holds only if $\hat{H}$ is differential polynom, which can be written as

$$\hat{H} = -\sum_{l=1}^{n} b_l \frac{\partial^l}{\partial x^l}, \tag{13}$$

where b_l are arbitrary real constants, $n \geqslant 1$. From X reflection invariance $b_l = 0$ for noneven l. Let us rewrite Eq. (12) as follows:

$$i\frac{\partial g}{\partial t} = \left(i\frac{\partial w^{1/2}}{\partial t} - w^{1/2}\frac{\partial \gamma}{\partial t}\right) \mathrm{e}^{i\gamma} = \mathrm{e}^{i\gamma}\hat{F}g, \tag{14}$$

where $\hat{F} = \mathrm{e}^{-i\gamma}\hat{H}$. Hence,

$$\frac{\partial w^{1/2}}{\partial t} = \mathrm{im}\,(\hat{F}g).$$

Yet if one substitutes $v(x)$ by $\gamma(x)$ in Eq. (6) and transforms it to $w^{1/2}$ time derivative, then

$$\frac{\partial w^{1/2}}{\partial t} = -\frac{1}{\mu}\frac{\partial w^{1/2}}{\partial x}\frac{\partial \gamma}{\partial x} - \frac{1}{2\mu}w^{1/2}\frac{\partial^2 \gamma}{\partial x^2}. \tag{15}$$

Plainly, this expression and $\mathrm{im}\,(\hat{F}g)$ coincide, then $\hat{H}$ can be obtained from their comparison term by term. In particular, the imaginary part of $\hat{F}g$ includes the highest γ derivative as the term $b_n w^{1/2}\frac{\partial^n \gamma}{\partial x^n}$, yet for Eq. (15) the highest γ derivative is proportional to $w^{1/2}\frac{\partial^2 \gamma}{\partial x^2}$. Hence, for all $l > 2$ it should be that $b_l = 0$; only in this case both expressions for $\frac{\partial w^{1/2}}{\partial t}$ can coincide. The same is true for other $\hat{F}g$ terms. Thus, g free evolution is described by the only $\hat{H}$ term with $b_2 = 1/2\mu$, it corresponds to free Schrödinger equation for particle with mass μ. Note that in standard QM the evolution equation is postulated $ad\,hoc$. The obtained ansatz also gives $J(\pm\infty, t) = 0$ for w flow of Eq. (3), in accordance with our expectations. Plainly, $\gamma(x)$ corresponds to $|g\}$ quantum phase, so that

$$\Delta(x, x') = \gamma(x) - \gamma(x')$$

describes the dynamical or phase correlation between the state components in x, x'.

2. GENERAL FUZZY DYNAMICS

In the previous section 1-dimensional FM formalism was derived from FT premises assuming that $|g\}$ evolution is linear and $g(x)$ coincides with QM wave function $\psi(x)$. Here we shall drop both these assumptions one by one. Concerning nonlinear evolution, the conditions of QM dynamics linearity were reconsidered by Jordan, and turn out to be essentially weaker than Wigner theorem asserts [10]. In particular, it was proved that if the evolution maps the set of all pure states one to one onto itself, and for arbitrary mixture of orthogonal states $\rho(t) = \sum P_i(t)\rho_i(t)$ all P_i are independent of time, then such evolution is linear. Here $\rho_i(x, x', t) = g_i(x, t)g_i^*(x, t)$ are the density matrices of orthogonal pure states g_i. Yet for the considered FM formalism the first condition is, in fact, generic: no mixed (i.e., probabilistic) state can appear in the evolution of pure fuzzy state. The second condition involves the probabilistic mixture of such orthogonal states and also seems to be rather weak assumption.

Now let us return to $\eta(x)$ ansatz of (11) and demonstrate that the Jordan theorem demands that $\chi(w) = 0$. For the obtained sets of m states, if $\langle g_i|g_j\rangle = \delta_{ij}$, then $\langle \eta_i|\eta_j\rangle = \delta_{ij}$ and vice versa. As was argued above, in FM any pure state $g(t_0)$ should evolve to pure state $g(t)$ for arbitrary t, so the same should be true for any $\eta(t_0)$. Now the Jordan theorem can be applied to η evolution, for that let us rewrite g evolution equation for η:

$$i\frac{\partial g}{\partial t} = i\frac{\partial}{\partial t}(\eta\,\mathrm{e}^{-i\chi}) = i\frac{\partial \eta}{\partial t}\,\mathrm{e}^{-i\chi} + \eta\frac{\partial \chi}{\partial w}\frac{\partial w}{\partial t}\,\mathrm{e}^{-i\chi} = \hat{H}(\eta\,\mathrm{e}^{-i\chi}). \tag{16}$$

From it one can come to the equation for η, the term containing $\partial w/\partial t$ can be rewritten according to (15). As a result, it gives

$$i\frac{\partial \eta}{\partial t} = \mathrm{e}^{i\chi}\hat{H}(\eta\mathrm{e}^{-i\chi}) + \frac{\eta}{\mu}\mathrm{e}^{i\chi}\frac{\partial \chi}{\partial w}\frac{\partial}{\partial x}\left(w\frac{\partial \gamma}{\partial x}\right). \tag{17}$$

The resulting equation for η is also of first time order, but is openly nonlinear. Therefore, for arbitrary $\chi(w)$, given the initial $\eta(x, t_0)$, the resulting $\eta(x, t)$ is equivalence class of $g(x, t)$ which evolved linearly from $g(x, t_0) = \eta(x, t_0)\,\mathrm{e}^{-i\chi}$.

In fact, 3-dimensional FM does not demand any principal modifications. In this case FT fundamental set will be $C^F = A^p \cup R^3$; hence, for any fuzzy point $a'_j \in A^p$ its ordering properties should be defined relative to X, Y, Z coordinate axes separately. Assuming FM rotational invariance, it follows that a'_j fuzzy properties can be described by the positive function $w^j(\mathbf{r})$ with norm $\int w^j\,d^3r = 1$. Then the particle m corresponds to the fuzzy point $a(t)$ characterized by $w(\mathbf{r}, t)$. Analogously to Sec. 2, given w evolution depends on local parameters only, it can be expressed as 3-dimensional flow continuity equation:

$$\frac{\partial w}{\partial t} = -\mathrm{div}\,\mathbf{J}. \tag{18}$$

Then one can decompose formally $\mathbf{J} = w\mathbf{v}$ and regard w flow velocity $\mathbf{v}(\mathbf{r})$ as independent $|g\}$ parameter. $|g\}$ phase $\gamma(\mathbf{r})$ is related to it via the equality $\mu\mathbf{v} = \mathrm{grad}\,(\gamma)$. To guarantee the formalism consistency, we assume that for m evolution the phase correlation $\Delta(\mathbf{r}, \mathbf{r}')$ is independent of the path l which connects $\mathbf{r}$, $\mathbf{r}'$ where

$$\Delta(\mathbf{r}, \mathbf{r}') = \gamma(\mathbf{r}) - \gamma(\mathbf{r}') = \int_{\mathbf{r}'}^{\mathbf{r}} \mathrm{grad}\,(\gamma)dl.$$

From a similar sequence of calculations, as for 1-dimensional case, free Schrödinger equation can be derived for 3-dimensional geometry. Like in 1-dimensional case, we assume first that g evolution equation is of the first order in time, and g evolution operator $\hat{H}$ is linear. Then for free m evolution it should be polynom of the form

$$\hat{H} = -\sum_{l=1}^{n} b_{2l} \frac{\partial^{2l}}{\partial \mathbf{r}^{2l}}. \tag{19}$$

From $\partial g/\partial t$ the term $\partial w^{1/2}/\partial t$ can be extracted and expressed via w, γ $\mathbf{r}$-derivatives. From their comparison with corresponding $\hat{H}g$ derivatives it follows $b_2 = 1/2\mu$ and $b_{2l} = 0$ for $l > 1$, i.e., free Schrödinger equation is obtained for 3-dimensional case. The applicability of the Jordan theorem to 3-dimensional $\hat{H}$ is obvious, because its proof of $\hat{H}$ linearity does not depend on the dimensionality of coordinate space. The same is true for the proof of uniqueness of $g(\mathbf{r}, t)$ ansatz, i.e., that $\chi(w) = 0$ for 3 dimensions as well.

All m states $g(\mathbf{r}, t)$ belong to $\mathcal{H}$; hence, the superposition principle also holds true in FM. In our approach the state space is defined by geometry and corresponding dynamics, i.e., is derivable concept. For pure states of free nonrelativistic particle m it is obtained to be equivalent to $\mathcal{H}$, but, in principle, it can be different for other systems. Similar features possess the formalism of algebraic quantum mechanics where the state space is defined by the observable algebra and system dynamics [11]. The Planck constant $\hbar = 1$ in our FM ansatz, but the same value ascribed to it in relativistic unit system together with velocity of light $c = 1$; in FM framework $\hbar$ only connects x, p geometric scales and does not have any other meaning.

In our derivation of evolution equation we did not assume Galilean invariance of FM, rather in our approach it follows itself from the obtained evolution equation, if the observer reference frame (RF) is regarded as the physical object with mass $\mu \to \infty$ [6]. For the transition to relativistic FM the important condition is that m density $w(\mathbf{r}, t)$ ansatz exists and is normalized and nonnegative in any RF. Then it follows that for massive particle m the simplest extension of FM state $|g\}$ is 4-spinor $\psi_i(\mathbf{r}, t)$; $i = 1, 4$; its evolution is described by Dirac equation for spin-1/2, and $w(\mathbf{r}) = \psi_i^+(\mathbf{r})\psi_i(\mathbf{r})$.

Now we shall consider the particle interactions in nonrelativstic FM. As follows from Eqs. (13)–(15), m free dynamics can be described by the system of two equations which define $\partial w^{1/2}/\partial t$ and $\partial\gamma/\partial t$ which for 3-dimensions can be written as

$$\begin{aligned} \frac{\partial w^{1/2}}{\partial t} &= -\frac{1}{\mu}\frac{\partial w^{1/2}}{\partial \mathbf{r}}\frac{\partial \gamma}{\partial \mathbf{r}} - \frac{1}{2\mu} w^{1/2} \frac{\partial^2 \gamma}{\partial \mathbf{r}^2}, \\ \frac{\partial \gamma}{\partial t} &= -\frac{1}{2\mu}\left[\left(\frac{\partial \gamma}{\partial \mathbf{r}}\right)^2 - \frac{1}{w^{1/2}}\frac{\partial^2 w^{1/2}}{\partial \mathbf{r}^2}\right]. \end{aligned} \tag{20}$$

Yet the first of them is equivalent to Eq. (18) which describes just $w(\mathbf{r})$ balance and so is, in fact, kinematical one and cannot depend on any interactions directly. Namely, under some external influence the values of w, γ variables can change, but no new terms can appear in the equation. Hence, m interactions can be accounted for only via modification of the second equation of this system. In the minimal case, assuming that the evolution terms are additive, it gives

$$\frac{\partial \gamma}{\partial t} = -\frac{1}{2\mu}\left[\left(\frac{\partial \gamma}{\partial \mathbf{r}}\right)^2 - \frac{1}{w^{1/2}}\frac{\partial^2 w^{1/2}}{\partial \mathbf{r}^2}\right] + \hat{H}_{\text{int}}, \tag{21}$$

where $\hat{H}_{\rm int}$ is the interaction term which is nontrivial if $\partial \hat{H}_{\rm int}/\partial \mathbf{r} \neq 0$. If for the interaction of two particles $\hat{H}_{\rm int} = F(r_{12})$, then it corresponds to the classical potential. Since γ corresponds to the quantum phase, it supposes that in FM m interactions can possess some form of local gauge invariance [13]. In our previous paper the toy-model of Abelian gauge interactions on fuzzy manifold was formulated which in the main aspects is similar to QED [7]. Despite the fact that the fermion state is described by several quantum phases, the same invariance is fulfilled for it and can also be extended to relativistic case. Preliminary results for interactions of fermion multiplets show that their interactions can also possess local $SU(n)$ gauge invariance and be transferred by corresponding Yang–Mills fields.

In conclusion, we have shown that the quantization of elementary systems can be derived directly from axiomatic of Set theory and topology together with the natural assumptions about system evolution. It allows us to suppose that the quantization phenomenon has its roots in foundations of mathematics [11]. At the same time, the considered fuzzy manifold describes the possible variant of fundamental pregeometry which is basic component of some quantum gravity theories [2]. It is worth noticing that in such a formalism the commutation relation $[\hat{x}, \hat{p}_x] = i$ results, in fact, from the geometry and topology of fuzzy manifold. The main aim of our theory, as well as of other studies of fuzzy spaces, is the construction of nonlocal QFT (or other more general theory) [12]. In this vein, FM provides the interesting opportunities, being generically nonlocal theory which, at the same time, can possess Lorentz covariance and local gauge invariance.

REFERENCES

1. *Connes A.* Noncommutative Geometry. New York: Acad. Press, 1994.
2. *Balachandran A. P., Kurkcuoglu S., Vaidia S.* Lectures on Fuzzy and Fuzzy SUSY Physics. Singapore: World Sci., 2007.
3. *Madore J.* Fuzzy Sphere // Class. Quant. Grav. 1992. V. 9. P. 69–83.
4. *Zeeman C.* Topology of Brain and Visual Perception // Topology of 3-Manifolds. Prentice-Hall, New Jersey, 1961. P. 240–248.
5. *Dodson C. T.* Tangent Structures for Hazy Space // J. London Math. Soc. 1975. V. 2. P. 465–474.
6. *Mayburov S.* Fuzzy Geometry of Phase Space and Quantization of Massive Fields // J. Phys. A. 2007. V. 41. P. 164071–164080.
7. *Mayburov S.* Fuzzy Space-Time Geometry and Particle's Dynamics // Intern. J. Theor. Phys. 2010. V. 49. P. 3192–3198.
8. *Landau L. D., Lifshitz E. M.* Mechanics of Continuous Media. Oxford; New York: Pergamon Press, 1976.
9. *Vladimirov V. S.* Equations of Mathematical Physics. M.: Nauka, 1971.
10. *Jordan T.* Assumptions That Imply Quantum Mechanics Is Linear // Phys. Rev. A. 2006. V. 73. P. 022101.
11. *Jauch J. M.* Foundations of Quantum Mechanics. Reading: Addison-Wesley, 1968.
12. *Blokhintsev D. I.* Space-Time in the Microworld. Berlin: Springer, 1973.
13. *Cheng T., Li L.* Gauge Theory of Elementary Particles. Oxford: Claredon, 1984.