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THE TWO-PHOTON EXCHANGE AMPLITUDE IN ep AND $e\mu$ ELASTIC SCATTERING: A COMPARISON

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In this note we give arguments in favor of the statement that the contribution of the box diagram calculated for electron–muon elastic scattering can be considered an upper limit to electron–proton scattering. As an exact QED calculation can be performed, this statement is useful for constraining model calculations involving the proton structure.

Показано, что вклад от диаграмм Фейнмана с двухфотонным обменом в амплитуду упругого электрон-мюонного рассеяния можно рассматривать как верхнюю границу для соответствующей амплитуды электрон-протонного рассеяния. Поскольку для первого случая расчет может быть выполнен явно в рамках квантовой электродинамики, это утверждение полезно для расчетов в рамках моделей, описывающих структуру протона во втором случае.

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The problem of the two-photon exchange amplitude (TPE) contribution to elastic electron–proton scattering amplitude has been widely discussed in the past, and the main attention was devoted to single-particle polarization effects [1]. This amplitude has in principle a complex nature. Experimentally its real part, more exactly the real part of the interference between one- and two-photon exchange, can be obtained from electron–proton and positron–proton scattering in the same kinematical conditions. A similar information in the annihilation channel (electron–positron annihilation into proton–antiproton and in the reversal process) can be obtained from the measurement of the forward-backward asymmetry in the angular distribution of one of the emitted particles in the reaction center-of-mass (CMS) system.

Recently, a lot of attention was devoted to the two-photon exchange amplitude (TPE) in electron–proton elastic scattering as a possible solution to a discrepancy between polarized and unpolarized measurements devoted to the determination of the proton form factors [2]. Whereas no experimental evidence has been found on the presence of TPE effects (real part) in non-linearities of the Rosenbluth fit, for example [3], the imaginary part is responsible for beam transverse single-spin asymmetry, which although very small, of the order

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10^{-5}–10^{-6} (ppm), but has been experimentally measured by experimental collaborations, and firstly reported in [4].

These results triggered new theoretical work and the asymmetry at first order of perturbation theory was calculated by several groups, see, for instance, [5]. Inelastic intermediate states of the TPE amplitude give rise to contributions containing the square of large logarithm (logarithm of the ratio of the four momentum squared to the electron mass squared). At higher order of perturbation theory, such contributions were calculated in [6] where it was shown that they cannot be neglected.

The contribution of the interference of the Born amplitude with the imaginary part of TPE, which is responsible for one-spin asymmetry, is proportional to the electron mass. Therefore, its presence does not contradict the Kinoshita–Lee–Nauenberg theorem [7] about cancellation of mass singularities, since the corresponding cross sections are suppressed by the lepton mass.

The theoretical description of TPE amplitude is strongly model-dependent, as it involves modeling of the proton and of its excited states, but it is still possible to derive rigorous results and predict exact properties of the two-photon box: model-independent statements based on symmetry properties of the strong and electromagnetic interaction have been suggested in [8, 9]. It has been proved that, due to C-parity conservation, the amplitude for $e^+ + e^- \to p + \bar{p}$, taking into account the interference between one- and two-photon exchange, is an odd function of $\cos\theta$, where θ is the angle of the emitted proton in the CMS of the reaction.

For the scattering channel, the interference between one- and two-photon exchange amplitudes induces a kinematical term $\sqrt{(1+\epsilon)/(1-\epsilon)}$ in the «reduced cross section», $\epsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta_e/2)$, where θ_e is the electron scattering angle, in the lab. system. This term violates the commonly accepted linear behavior of the reduced cross section $\sigma_{\rm red} = a + b\epsilon$. This property must be satisfied by all model calculations.

A second possibility is to do an exact calculation of the box diagram, which is possible for electron–electron and electron–muon scattering, and in the crossed channel (i.e., replacing the proton with a lepton) [10], where the muon can be considered a structureless proton. Even if such a calculation is not rigorous when applied to the interaction on proton, the interest of a pure QED calculation is that the results should be considered as an upper limit for any calculation involving protons, as it will be discussed in this work.

The discussion of TPE box diagram in ep scattering cannot be restricted to one-proton intermediate state, but inelastic amplitudes should be consistently taken into account. Concerning the real part, the contributions to the amplitude from the proton on one side and from the inelastic intermediate states on the other side, are not gauge-invariant, if considered separately. Only their sum is gauge-invariant: the Ward–Takahashi identities relate the vertex function and the nucleon Green function [11]. On the contrary, for the imaginary part, these contributions are separately gauge-invariant, as the intermediate nucleon is on shell, as well as the external nucleons, therefore they must have comparable values.

Analyticity arguments lead to a (almost complete) compensation of elastic and inelastic contributions in the whole amplitude. This statement is rigorous in QED [12], and has been recently extended to electron–hadron scattering at small scattering angles. Moreover, based on such a statement, sum rules which relate peripheral cross section and elastic form factors have been derived in QCD and their validity verified on experimental data (see [13] and refs. therein). Therefore, one can state that elastic and inelastic contributions are of the same order of magnitude, which is sufficient for our aim here.

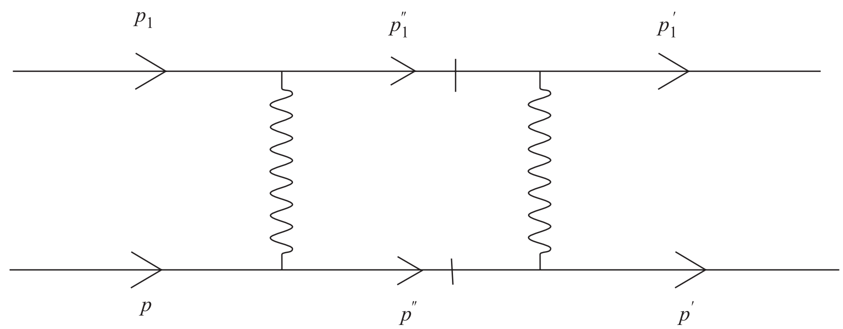

Fig. 1. s-channel discontinuity of the Feynman box amplitude for $e\mu$ scattering

The notion of «nucleon form factors (FF)» cannot be applied to the two-photon exchange amplitude (TPE) since one of the nucleons is off mass shell. Nevertheless, the s-channel imaginary part of TPE, which corresponds to a single on mass shell nucleon and on mass shell electron in the intermediate state, can be analyzed in terms of FFs. Moreover, it provides the gauge-invariant contribution to the imaginary part of the whole TPE. We build a simple model, calculating the $e\mu$ box Feynman diagram with one muon (nucleon) in the intermediate state. For the proton case, the muon mass is taken equal to the proton mass, and the proton structure is described by form factors.

We can neglect the spin dependence and we calculate scalar four-dimensional integrals with point-like particles (in case of $e\mu$ scattering), and including proton form factors (for ep scattering). A complete calculation was performed in [14] where similar scalar Feynman integrals with three and four denominators are involved.

Our aim is not to do a complete calculation of the box diagram, but to find an upper limit of this term: in every step, one should compare the relevant integrals. The purpose of this note is to prove that modeling of the proton by Q^2 decreasing form factors leads to a smaller contribution of the box diagram, compared to the QED case. We will prove this statement for the imaginary part of the amplitude corresponding to the box diagram with one proton line connecting two γpp vertexes and the validity for the relevant part of the full amplitude, $\mathcal{A}$, can be inferred through dispersion relations:

$$\mathcal{A}(s,t) = \frac{1}{\pi}\int \frac{ds' \,\mathrm{Im}\,\mathcal{A}(s',t)}{(s'-s-i\epsilon)}. \tag{1}$$

Let us consider the cases where the target T is a proton or a muon (Fig. 1) with the following convention for the particle four momenta:

$$e(p_1)+T(p)\to e(p_1'')+T(p'')\to e(p_1')+T(p'). \tag{2}$$

The following kinematical relations hold in the center-of-mass frame:

$$p_1+p=p_1'+p', q=p_1-p_1', p_1'+p'=p_1''+p'', q_1=p_1-p_1'', q_2=p_1''-p_1',$$
$$Q_1^2=-q_1^2=-(p_1-p_1'')^2=2(\mathbf{p})^2(1-c_1),$$
$$Q_2^2=-q_2^2=-(p_1''-p_1')^2=2(\mathbf{p})^2(1-c_2),$$
$$Q^2=-q^2=-(p_1-p_1'')^2=2(\mathbf{p})^2(1-c),$$

where $c_1 = \cos\theta_1$, $c_2 = \cos\theta_2$, $c = \cos\theta$, and $\theta_1 = \widehat{\mathbf{p}_1\mathbf{p}_1''}$, $\theta_2 = \widehat{\mathbf{p}_1''\mathbf{p}_1'}$, and $\theta = \widehat{\mathbf{p}_1\mathbf{p}_1'}$. The momenta carried by the virtual photons are $q_1 = k$ and $q_2 = q - k$.

The contribution to the Feynman amplitude corresponding to the diagram of Fig. 1 can be written as

$$\mathcal{M} = \frac{1}{(2\pi)^2}\int \frac{\mathcal{N} d\Gamma}{(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}, \tag{3}$$

where $\mathcal{N} = F(Q_1^2)F(Q_2^2)(\mathcal{N}=1)$ for $ep(e\mu)$ scattering, λ is a fictitious photon mass and $d\Gamma$ is the phase volume of the loop intermediate state. The notation $F(Q_1^2)F(Q_2^2)$ indicates the bilinear combination of the Dirac F_1 and Pauli F_2 nucleon form factors.

Taking into account the fact that the intermediate particles are on shell, one can write for the proton case:

$$d\Gamma = d^4p_1''\delta(p_1''^2 - m^2)\delta(p''^2 - M^2)d^4p''\delta^4(p_1 + p - p_1'' - p'') =$$
$$= \frac{d^3p_1''}{2\epsilon_1''}\frac{d^3p''}{2\epsilon''}\delta^4(p_1+p-p_1''-p'') = \frac{d^3p_1''}{4\epsilon_1''\epsilon''}\delta(\sqrt{s}-\epsilon_1''-\epsilon''),$$
$$\epsilon_1'' = \frac{s-M^2}{2\sqrt{s}}, \quad \epsilon'' = \frac{s+M^2}{2\sqrt{s}}. \tag{4}$$

Finally, one can write

$$d\Gamma = \frac{s-M^2}{8s}dO_1'', \tag{5}$$

where dO_1'' is the solid angle of the electron in the intermediate state, which can be expressed as a function of the angles defined above as

$$dO_1'' = \frac{2dQ_1^2\,dQ_2^2}{\sqrt{\mathcal{D}_1 Q_0^2}}, \quad \mathcal{D}_1 = 2(Q_1^2+Q_2^2)Q^2Q_0^2 - 2Q^2Q_1^2Q_2^2 - (Q_1^2-Q_2^2)^2Q_0^2 - (Q^2)^2Q_0^2, \tag{6}$$

with the relation $Q_0^2 = 2\mathbf{p}^2 = (s-M^2)^2/(2s)$. The positivity of the function $\mathcal{D}$ defines the solid angle kinematically available for the reaction.

Therefore, one can write the contributions corresponding to the «QED» diagram in Fig. 1, in case of a muon target:

$$\mathcal{M}_\mu = \frac{1}{\sqrt{8s}}\int \frac{dQ_1^2\,dQ_2^2}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}. \tag{7}$$

Introducing a generalized form factor for the proton, one finds for the «QCD» diagram of Fig. 1, in case of a proton target:

$$\mathcal{M}_p = \frac{1}{\sqrt{8s}}\int \frac{dQ_1^2\,dQ_2^2\,F(Q_1^2)\,F(Q_2^2)}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}. \tag{8}$$

We imply that both amplitudes are similarly infrared regularized, which, for the purpose of our paper, is equivalent to consider λ as a finite quantity. Therefore, the condition $F(Q_1^2)\,F(Q_2^2) < 1$ is equivalent to the statement that the value of the electron–muon scattering amplitude can be considered an upper estimation of the amplitude for electron–proton scattering.

Nucleon form factors are functions which are rapidly decreasing with Q^2. The Pauli and Dirac form factors, F_1 and F_2, are related to the Sachs form factors by

$$F_1(Q^2) = \frac{\tau G_M(Q^2) + G_E(Q^2)}{\tau + 1}, \quad F_2(Q^2) = \frac{G_M(Q^2) - G_E(Q^2)}{\tau + 1}, \quad \tau = \frac{Q^2}{4M^2}, \tag{9}$$

with the following normalization: $F_1(0) = 1$, $F_2(0) = \mu_p - 1 = 1.79$, where μ_p is the magnetic moment of the proton in units of Born magneton.

Let us consider the dipole approximation as a good approximation at least for the magnetic proton form factor G_M, although it has been shown that the electric form factor G_E deviates from the dipole form [2]. In any case, any parametrization closer to the data will give even lower values as compared to the dipole form. In this approximation, we have

$$F_1^D(Q^2) = \frac{(\tau\mu_p + 1)G_D(Q^2)}{\tau + 1}, \quad F_2^D(Q^2) = \frac{(\mu_p - 1)G_D(Q^2)}{\tau + 1}, \tag{10}$$
$$G_D(Q^2) = [1 + Q^2(\mathrm{GeV})^2/0.71]^{-2}.$$

In Fig. 2 we show $F_1(Q^2)$ (solid line), $F_2(Q^2)$ (dashed line), which are smaller than unity practically overall the Q^2 range. The product $F_1(Q_1^2)F_1(Q_2^2)$ is shown in Fig. 3 as a bidimensional plot, and in Fig. 4, as a projection on the Q_1^2 axis for $Q_2^2 = 0.05$ GeV^2 (solid line), $Q_2^2 = 1.2$ GeV^2 (dashed line), $Q_2^2 = 2$ GeV^2 (dotted line).

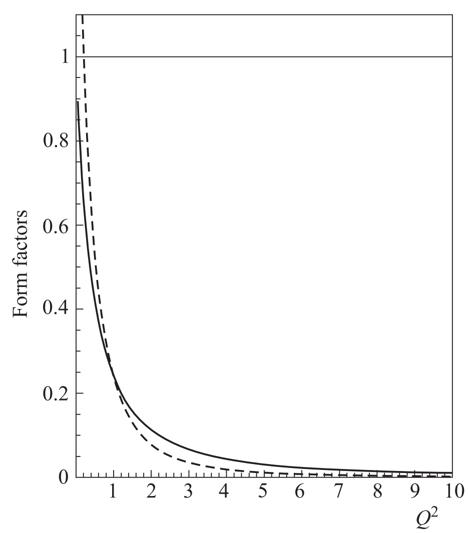

Fig. 2. Form factors as a function of Q^2: solid line — $F_1(Q^2)$, dashed line — $F_2(Q^2)$

One can see that the condition $F(Q_1^2)F(Q_2^2) < 1$ is satisfied, starting from very low values of Q^2. Let us stress that $F_1(Q^2)$ is normalized to 1 and decreases with Q^2, being therefore smaller than unity; in the expression of the hadronic current, $F_2(Q^2)$ is multiplied by q_μ, which lowers its contribution at small Q^2, whereas at larger Q^2 it does not compensate the steep Q^{-6} behavior of this form factor, as expected from quark counting rules [16]. This is the reason why we can replace the bilinear combination $F(Q_1^2)F(Q_2^2)$ by $F_1(Q_1^2)F_1(Q_2^2)$.

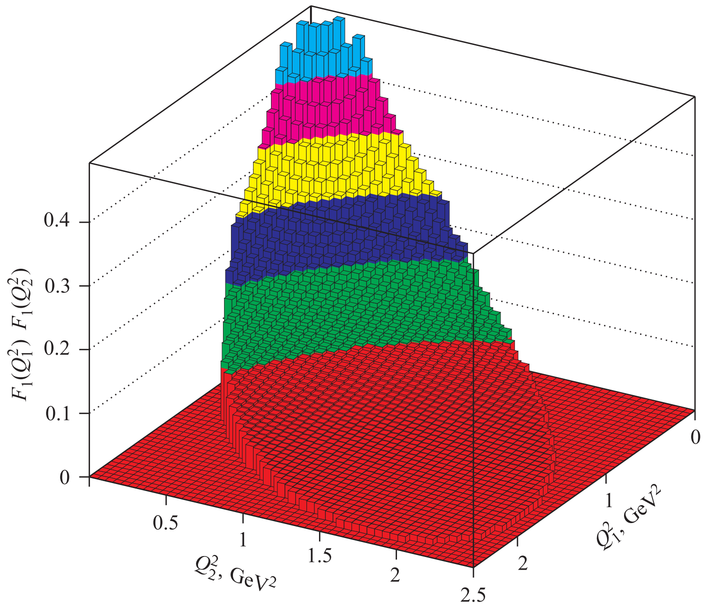

Fig. 3. Bidimensional plot of $F_1(Q_1^2)F_1(Q_2^2)$ as a function of Q_1^2 and Q_2^2

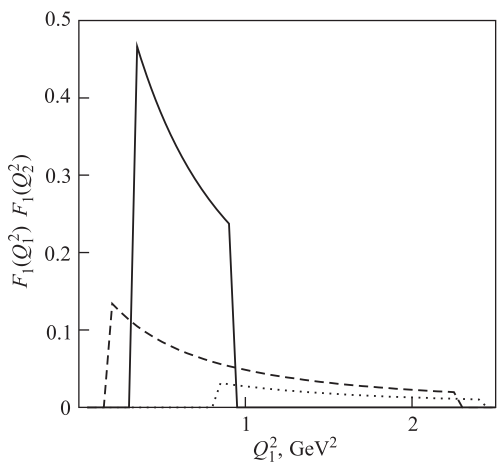

Fig. 4. Projection on $F_1(Q_1^2)F_1(Q_2^2)$ on the Q_1^2 axis for $Q_2^2 = 0.05$ GeV^2 (solid line), $Q_2^2 = 1.2$ GeV^2 (dashed line), $Q_2^2 = 2$ GeV^2 (dotted line)

Furthermore, we note that a destructive interference of the contributions of the single proton and its excited states takes place in (1), which results in additional suppression of ep amplitude compared to $e\mu$. A mutual compensation of the amplitudes for «elastic» proton intermediate state with the excited hadronic states exists, and the reason lies in the superconvergent character of the dispersion relation (1), where the total amplitude is implied. Indeed, considering the amplitude for virtual Compton scattering in the complex s plane, closing the integration contour to the right-hand singularities (which correspond to the proton intermediate state (pole) and to excited hadron states (cuts)) a compensation takes place, up to the small contribution of the left-hand cut. Details are given in [13]. Therefore, all model calculations for ep elastic scattering as [15] should result in smaller contribution of the two-photon amplitude, as compared to QED calculations [10].

In conclusion, let us note that any (quantitative) application of our considerations to polarization phenomena is outside the purpose of this paper. Our statements were done for the imaginary part of the scattering amplitude, and extended to the full amplitude with the help of dispersion relations. As far as the real amplitude of ep scattering is concerned, the whole TPE amplitude is smaller than the contribution of the one-proton intermediate state of the TPE amplitude, due to a compensation of elastic and inelastic states. Reasons in favor of this cancellation were given in the literature [13]. But, even if we neglect the compensation effects, the QED $e\mu$ amplitude dominates the relevant one-proton real ep TPE amplitude, due to the steep falling of form factors. Our statement, about the QED dominance, is relevant to the whole ep amplitude. This is the main conclusion of the present work.

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