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MATHEMATICAL SIMULATION OF ELECTROMAGNETIC FIELDS INSIDE AN ANECHOIC CHAMBER OF RECTANGULAR SHAPE

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A mathematical model for computing electromagnetic fields inside anechoic chambers of rectangular shape with walls covered with impedance coating has been developed for an operating frequency range from several kilohertz to tens of gigahertz. Numerical analysis is based on the solution of the corresponding difference Helmholtz problem. In the region of lower and medium frequencies, the solution is carried out using the method of variable directions, and, in the region of higher frequencies, this method uses multiprocessor systems with distributed memory for parallel computations. The computing results are given as well.

Разработана математическая модель для расчета электромагнитных полей внутри безэховой камеры прямоугольной формы с импедансным покрытием стенок в диапазоне частот от единиц килогерц до десятков гигагерц. Численный анализ основан на решении соответствующей разностной задачи Гельмгольца. В области низких и средних частот решение осуществляется методом переменных направлений, а в высокочастотной области — параллельной реализацией метода на многопроцессорных системах с распределенной памятью. Приведены результаты расчетов.

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INTRODUCTION

One of the main features of an anechoic chamber is the ability to maintain the level of reflected (interfering) signals below a value specified for a certain region inside the chamber rather than for its entire volume. This region is called the homogeneous (or anechoic) zone. Geometry of the homogeneous zone (its shape and size) depends on the geometry of anechoic chamber itself, quality of material covering the walls of the chamber, arrangement of radiation sources inside the chamber, and radiation frequency. Currently, both domestic and industrial electronic devices are usually capable of operating in a wide range of frequencies, so anechoic chambers must address quite stringent requirements for broadband capability (up to 40:1 and more). The requirements for operating (or designing) an anechoic chamber can be securely derived only through the use of numerical simulation techniques. Traditionally, the numerical simulation of fields inside an anechoic chamber is carried out on the basis of techniques usually employed in geometrical optics [1]. Since current anechoic chambers can be operated over a wide range of frequencies, in order to analyze the characteristics of an anechoic chamber and solve the corresponding Helmholtz equation, we propose a method based on the standard (for lower and medium frequencies) and parallel (for higher frequencies) implementations of the algorithm realizing the method of variable directions.

1. PROBLEM FORMULATION

Let us consider the case of E-polarization where the vector $\mathbf{E}$ is in parallel with the side walls of an anechoic chamber, i.e., the situation when the quality of coating material has the most pronounced effect. If the floor and the ceiling of the chamber are ideal conductors and a source generates the z-polarized electric field, the problem is reduced to its two-dimensional version. In this case the source is considered as isotropic, which does not impairs the generality of the results obtained, since the directivity of the source in the XY-plane can be taken into consideration by using the standard principle of superposition. The side wall coating is taken into account by the impedance boundary conditions

$$(\mathbf{E} \times \mathbf{n}_j) = Z_j(\mathbf{n}_j \times (\mathbf{n}_j \times \mathbf{H})), \quad j = 1, \dots, 4, \tag{1}$$

where Z_j is the impedance of the jth wall and $\mathbf{n}_j$ is the outward-directed normal to the jth wall.

Now consider a rectangular area with the sides $y = \pm b/2$ ($j = 1, 3$), $x = \pm a/2$ ($j = 2, 4$). The coordinates of the source are (x_0, y_0). Assuming that

$$Z_j = iZ_0W_j, \quad Z_0 = \sqrt{\mu_0/\varepsilon_0}, \tag{2}$$

we can rewrite the conditions (1) as

$$E_z = iZ_0W_j(n_{jy}H_x - n_{jx}H_y), \quad j = 1, \dots, 4. \tag{1a}$$

The z-component of the electric field strength vector satisfies within this area the following inhomogeneous Helmholtz equation:

$$\Delta E_z + k^2 E_z = i\omega\mu_a\delta(x - x_0, y - y_0), \tag{3}$$

where $k = \omega\sqrt{\mu_a\varepsilon_a}$ is the wave number; ω is the radiation frequency of the source; μ_a is the absolute permeability of the medium in the interior of the anechoic chamber, and ε_a is the absolute permittivity of the medium in the interior of the chamber.

2. PROBLEM SOLUTION

We denote $E_z/(-i\omega\mu_a) \equiv U \equiv U^0 + U^1$, where

$$U^0 = \frac{1}{4i}H_0^{(2)}(k\sqrt{(x - x_0)^2 + (y - y_0)^2}).$$

In accordance with the above physical definition, the calculation of the field in the interior of the anechoic chamber reduces to solving the following equation satisfied with the z-component of the secondary electric field strength $U^1(x, y)$:

$$\Delta U^1 + k^2 U^1 = 0. \tag{4}$$

Throughout the rest of the paper, we assume that $\varepsilon_a = \varepsilon_0, \mu_a = \mu_0$ (the medium in the interior of the chamber is vacuum), i.e., $k = k_0 = \omega\sqrt{\varepsilon_0\mu_0}$, where ε_0, μ_0 are the permittivity and permeability of vacuum, respectively.

Let us introduce dimensionless variables

$$\bar{x} = k_0 x, \quad \bar{y} = k_0 y \tag{5}$$

and a function $\overline{U}^1$, such that

$$\overline{U}^1 = 4iU^1.$$

Then (4) takes the form

$$\Delta_{\bar{x},\bar{y}}\overline{U}^1 + \overline{U}^1 = 0. \tag{6}$$

The boundary condition (1a) can be conveniently rewritten in the following manner:

$$\overline{U}_j^1 = W_j \left[(\mathbf{n}_j, \operatorname{grad}(\overline{U}^1 - H_0^{(2)}(\bar{r}_s))) + H_0^{(2)}(\bar{r}_s) \right] \Big|_{S_j}, \tag{7}$$

where $\bar{r}_s = \sqrt{(\bar{x}-\bar{x}_0)^2 + (\bar{y}-\bar{y}_0)^2}$, and S_j is the jth wall. The point (x, y) belongs to the homogeneous zone if, at this point, the following relation is correct:

$$20 \left| \log_{10} \left| 1 - \overline{U}^1 / H_0^{(2)}(\bar{r}_s) \right| \right| < N(dB), \tag{8}$$

where N is the anechoic parameter. Numerical solution of the problem (6), (7) can be found by using the difference communication theory and the method of variable directions [2]. The cross-sectional area $\overline{XY}$ of the chamber is covered by a rectangular array with the step size of h_x along the $\overline{X}$ axis and h_y along the $\overline{Y}$ axis. Equation (6) is approximated by the difference equation

$$(L_1 + L_2)V = 0, \tag{9}$$

where

$$L_1 V = \frac{V_{i-1,j} - 2V_{i,j} + V_{i+1,j}}{h_x^2} + \frac{1}{2}V_{i,j}, \; L_2 V = \frac{V_{i,j-1} - 2V_{i,j} + V_{i,j+1}}{h_y^2} + \frac{1}{2}V_{i,j}, \tag{10}$$

$1 \leqslant i \leqslant n_x - 1$, $1 \leqslant j \leqslant n_y - 1$, n_x is the number of partitions along the $\overline{X}$ axis, n_y is the number of partitions along the $\overline{Y}$ axis, and

$$V_{i,j} \equiv \overline{U}^1(\bar{x}_i, \bar{y}_j), \quad \bar{x}_i = ih_x, \quad \bar{y}_j = jh_y.$$

This equation, combined with the difference approximation of boundary $(i = 0, n_x; j = 0, n_y)$ conditions (7), leads to the boundary problem similar to the problem (6), (7). The above-mentioned difference boundary problem can be solved by the method of variable directions (iterative method). In this case the equations are to be written for every inner node of the array (the procedure in its simplest form):

$$\begin{aligned} \frac{V^{n+1/2} - V^n}{0,5\tau} &= L_1 V^{n+1/2} + L_2 V^n, \\ \frac{V^{n+1} - V^{n+1/2}}{0,5\tau} &= L_1 V^{n+1/2} + L_2 V^{n+1}, \; n = 0, 1, 2, \ldots, \end{aligned} \tag{11}$$

where τ is the iteration parameter, $V^n = V(t_n)$; $V^{n+1/2} = V(t_n + 0,5\tau)$; $V^{n+1} = V(t_n + \tau) = V(t_{n+1})$; t_n is the formal parameter whose value t_0 is used to specify the initial distribution V^0. Taking into account the boundary conditions and the initial distribution V^0, the values $V^{n+1/2}, V^{n+1}, n = 0, 1, 2, ...$ can be found by the standard sweep method. Increasing the radiation frequency generates a need for employing a more finely divided spatial array with a large quantity of nodes, which requires much more time for total simulation procedure — work beyond the power of one-processor PCs. Currently, a widespread use has been made of multiprocessor systems with distributed memory. Among these are simple cluster systems — PCs incorporated into Ethernet-type local networks — as well as specialized multiprocessor systems such as Parsytec CC systems (utilized in our work) based on Power PC processors that are connected together by HS-Link high throughput channels. Unlike common memory systems, distributed memory systems can be readily scaled; that is, they allow us to increase computational power practically without limit, with a considerable gain in cost and capacity. Widespread use of Message Passing Interface (MPI) makes it possible to develop portable applications software which can be exploited with equal advantage by a wide variety of computing systems under the control of various operational systems (Windows, UNIX, FreeBsd, AIX, Linux). However, gains in scaling capability and low cost go along with a penalty in terms of complex concurrent algorithms. In many cases conventional applications software fails when it is adapted to distributed memory systems. We are thus led to modify the algorithms significantly or to develop fundamentally new ones. In advantageous situation are algorithms for simulating physical process, since these algorithms are based on explicit difference schemes or simple iteration methods. Multisequencing of such algorithms can be easy realized by using the domain decomposition method. This method involves a uniform partitioning of the nodes of computation array according to the number of processors (see Fig. 1). The approach automatically ensures that the amount of computation involved is uniformly distributed among the processors, and the amount of data transmitted between processors during computation is minimized. In this case the algorithms employed are highly effective.

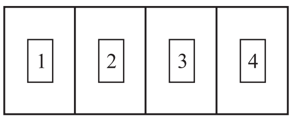

Fig. 1

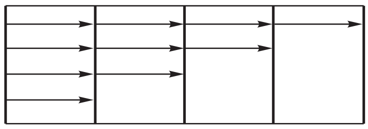

Fig. 2

Implicit difference equations (11) can be solved with vertical sweeps when all processors are simultaneously involved independently of one another, being practically one hundred per cent effective. With horizontal sweeps the data are transmitted between processors from left to right with the direct sweep and from right to left with the reverse sweep. In this case the simultaneous operation of the processors is not possible when an equation is solved. Taking into account the fact that the system (11) involves a great number of equations we can use the pipelined sweep operation. As with any pipeline parallel scheme, only the first processor is operated at the first stage, performing the direct sweep for the first equation (see Fig. 2). It determines the coefficients of the direct sweep for the points of the first equation and transmits the corresponding data to the second processor. At the second stage, the first processor performs the direct sweep for the second equation and, simultaneously, the second processor performs the direct sweep for the first equation. The third step introduces the third processor and so on. Therefore, beginning with a certain step, all processors simultaneously process «their own» parts of different equations. The method described is effective when the number of processors is small compared with the total number of equations. Taking into consideration the fact that the data time includes a significant initial delay independent of the amount of transmitted data, a number of equations are processed simultaneously and information about them is transferred in a single block, which results in a decrease in both the number and the time of exchanges. This allows us to increase the total effectiveness in spite of some increase in down time for certain processors at the first and the conclusive stages of computations.

When the number of processors is great, the other approach is effective. It is based on the use of an «$\alpha - \beta$»-iteration method [4] for solving five-point difference equations (12). Equations (12) correspond to the difference approximation of elliptical-type equations (6):

$$B_{i,j}V_{i,j-1} + K_{i,j}V_{i-1,j} - C_{i,j}V_{i,j} + \\ + E_{i,j}V_{i,j+1} + D_{i,j}V_{i+1,j} + F_{i,j} = 0,\ i = 0,1,2,\ldots;\ j = 0,1,2,... \quad (12)$$

Assuming that the solution of the system (12) satisfies the relations

$$V_{i,j} = \alpha_{i+1,j}V_{i+1,j} + \beta_{i+1,j}, \qquad V_{i,j} = \gamma_{i-1,j}V_{i-1,j} + \delta_{i-1,j},$$
$$V_{i,j} = \bar{\alpha}_{i,j+1}V_{i,j+1} + \bar{\beta}_{i,j+1}, \qquad V_{i,j} = \bar{\gamma}_{i,j-1}V_{i,j-1} + \bar{\delta}_{i,j-1},$$

where $\alpha, \gamma\, \bar{\alpha}, \bar{\gamma}, \beta, \delta, \bar{\beta}, \bar{\delta}$ are unknown sweep coefficients; these coefficients can be determined by creating a parallel algorithm (see, for example, [5]) whose effectiveness is $\sim 60–65\%$ when 40 processors are employed. Mathematical simulation was carried out for an anechoic chamber whose metal walls were covered with ferrite absorbing plates. The walls had identical coating, that is, $W_j = W, j = 1, \ldots, 4$, with

$$W_j = W = \sqrt{\mu_1/\varepsilon_1}\,\mathrm{tg}\,(k_1 d),$$

where μ_1, ε_1 are the relative permeability and permittivity of the covering material, respectively, $k_1 = k_0\sqrt{\varepsilon_1\mu_1}$, $k_0 = 2\pi/\lambda$; and d is the thickness of the coating.

Figure 3 shows the results of computing the ratio $|U/U^0|$ at a point separated from the radiation source by 3 m. Both methods described above were used for a chamber whose dimensions were $a \times b = 12.5 \times 9$ m. Cover thickness was 0.0075 m. Operating frequency range was from 30 to 100 MHz. Coordinates of the point were $x_0 = 1.75$ m and $y_0 = 0$ m. The values of W for these frequencies were obtained according to [3].

Figure 4 shows the results of numerical evaluation of field nonuniformity ($|U/U^0|$) over the cross-sectional area of a chamber (along the Y axis) at a distance of 3 m from the source. The dimensions of the chamber were $a \times b = 8.4 \times 7.2$ m; cover thickness 0.05 m; operating frequency $f = 200$ MHz ($W = -0.1747 - i0.6412$). Taking into account the symmetry (coordinates of the source are $x_0 = -0.6$ m, $y_0 = 0$ m), we reproduced here only a portion of the curve for $0 \leqslant y \leqslant 3.4$ m.

Figure 5 shows the results of field simulation inside the anechoic chamber having dimensions $a \times b = 12.5 \times 9$ m, cover thickness 0.0065 m, over a frequency range from 0.1 to

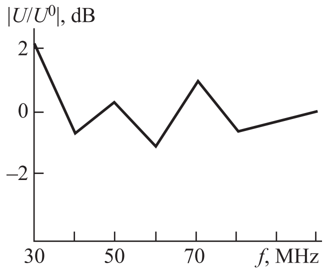

Fig. 3

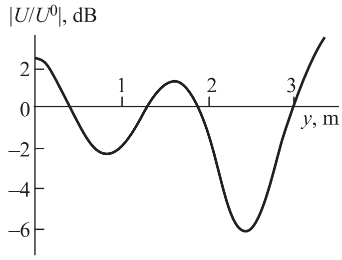

Fig. 4

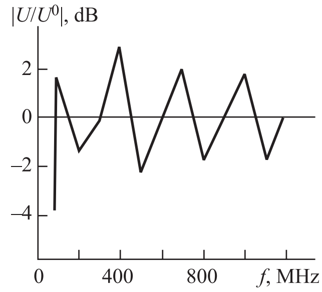

Fig. 5

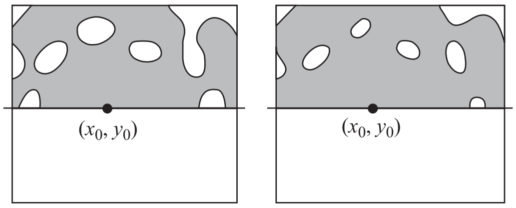

Fig. 6

1.2 GHz in a 0.1 GHz increments. The point was taken on the X axis at a distance of 2 m from the source ($x_0 = 1.25$ m, $y_0 = 0$ m).

Figure 6 shows the results of simulating the dependence of homogeneous zone geometry on the values of N for a) $N = 3$ dB and b) $N = 4$ dB ($a = 8.4$ m, $b = 7.2$ m, $x_0 = -0.6$ m, $y_0 = 0$ m, $f = 100$ MHz, $d = 0.005$ m, $W = 0.071 - i0.668$). The points belonging to the homogeneous zone are in the hatched region.

CONCLUSIONS

A software complex for numerical simulation of E-polarized electromagnetic field characteristics inside an anechoic chamber of rectangular shape with walls covered with impedance coating has been developed. The complex can be used for a frequency range from several kilohertz to tens of gigahertz. The computational error does not exceed 0.5–1.0 dB (depending on the quality of coating). The mathematical model can be modified for the cases of H-polarization and three-dimensional representation as well as for anechoic chambers with piecewise homogeneous coating.

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