Письма в ЭЧАЯ. 2009. Т. 6, № 4(153). С. 472–484

MODIFIED ACTION FOR LIGHT QUARKS IN THE INSTANTON VACUUM

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The procedure of averaging in an instanton medium on quarks with any number of flavors is discussed. It is shown that the effect of the instanton medium is equivalent to an interaction of light quarks with dynamically generated mass (four-quark interaction $N_f = 2$) and massless bosonic spinor fields (ghosts). The fact that the instanton liquid is dilute makes it possible to use perturbation theory.

Предложена процедура усреднения в инстантонной жидкости по кваркам для любого числа ароматов. Показано, что эффект инстантонной среды эквивалентен введению эффективного взаимодействия легких кварков с динамической массой (четырехфермионное взаимодействие) и безмассовых бозонных спинорных полей (духов). Применение теории возмущений оправдано благодаря разреженности инстантонной жидкости.

PACS: 11.10.Lm; 11.10.St; 11.15.Ex; 11.15.Pg; 11.15.Tk

INTRODUCTION

Important progress was achieved in understanding of the mechanism of spontaneous breaking of chiral invariance in [1–5] and, consequently, in understanding of the physics of particles of the pseudoscalar meson octet [5–7]. The constructed model, a vacuum instanton liquid, made it possible to find the low-energy characteristics of π, K mesons, in good agreement with the experimental data [5, 8–10]. The method of calculation of the correlation functions in an instanton medium was proposed by Diakonov and Petrov. In the framework of this method the superposition of instanton–anti-instanton (pseudoparticle) pairs is considered as an external classical field. Correlation functions calculated in this field depend on characteristics of all pseudoparticles, i.e., their sizes ρ_I, orientations U, and centers z. Averaging over a statistical ensemble of instantons with a distribution function of instanton sizes gives the exact correlation function in the instanton vacuum. This distribution function, which is a probability that a fluctuation with given parameters occurs, is determined exclusively by the interaction of the pseudoparticles and does not contain corrections from the quark–pseudoparticle interaction, i.e., in the quenched approximation $N_f/N_c \ll 1$, where N_f and N_c are the numbers

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of flavors and colors, respectively. The averaging simplifies substantially because of the following approximations:

a) when N_c is large, the distribution function of the instanton sizes $d(\rho_I)$ is very narrow and tends to a δ-function shape as $N_c \to \infty$. It means that in the leading order in $1/N_c$ the instanton sizes ρ_I can be replaced by their average value ρ [4],

b) the packing parameter of the instanton liquid $\eta = \rho/R$, where R is the average distance between pseudoparticles, is small, $\eta \approx 1/3$, and therefore the pseudoparticles can be considered as uncorrelated ones,

c) the Hilbert space of quarks is projected into the space of fermion zero modes. This approximation is valid for long wave vacuum fluctuations (scale > 0.3 fm) and is based on the fact that the spectrum of the Dirac operator in an instanton field is characterized by a gap of the order of $\rho^{-1} \approx 600$ MeV [5].

A diagrammatic averaging procedure was developed and two-point correlation functions were calculated. However, it is problematic to calculate three-point and multipoint correlation functions by using this method. The difficulty is due to the absence of any method for treating the coupled ladder diagrams in different directions and finding the solutions of coupled equations of the Bethe–Salpeter type. Therefore, there is a problem of finding a realistic algorithm to calculate other Green functions.

The idea how to overcome this difficulty is to start with the multi-pseudoparticle partition function and by averaging to obtain more general effective action which is equivalent to the previous model. Several attempts have been made to construct such an action [11–15]. However, none of these actions is equivalent to the previous one (see, for instance, [14]). From our point of view, this is connected with subtleties of the averaging procedure that was not properly taken into account. Namely, the effect of quarks on the distribution function $d(\rho_I)$ (which implies an elimination of zero modes and ultraviolet divergences from the fermion determinant) was neglected, but this nonregularized determinant was included in the definition of the correlation function [16]. In this paper we investigate the problem of averaging more accurately. In Sec. 1 we obtain the multi-pseudoparticle partition function from the QCD Lagrangian in the approximations (a-c) in the case $N_f = 1$. This effective action already contains a nontrivial interaction of quarks in contrast to the previous results [11–15]. Case $N_f = 2$ we investigate in Sec. 2. Section 3 is devoted to the vector channel.

1. MULTIPARTICLE PARTITION FUNCTION

The partition function of QCD in Euclidean space is

$$Z_{\mathrm{QCD}} = \int \mathrm{e}^{-S(\psi,\psi^+,A)} D\psi D\psi^+ DA_\mu. \tag{1}$$

The main assumptions of the model are that the integration over the gauge field A_μ is equivalent to averaging over a statistical ensemble of pseudoparticles with known distribution function for a given configuration and the field A_μ in the interaction term is replaced by a superposition of instanton–anti-instanton configurations $A_\mu = \sum\limits_I A_\mu^I + \sum\limits_{\bar I} A_\mu^{\bar I}$

$$Z' = \int \left\langle\!\!\left\langle \exp\Big\{ \int [\psi^+(i\hat\nabla + im)\psi + \eta^+\psi + \psi^+\eta] d^4x \Big\} \right\rangle\!\!\right\rangle D\psi D\psi^+, \tag{2}$$

where $\hat{\nabla} = \hat{\partial} + \hat{A}$ and $\langle\langle \ldots \rangle\rangle$ stands for an average over the collective coordinates: an integration with respect to the positions z_i of the pseudoparticles and their orientations U_i (following approximation (a) we must put all the sizes of pseudoparticles equal to $\rho \approx (600\ \mathrm{MeV})^{-1}$). However, Z' turns out not equivalent to the previous model [4,5,17] since $\langle\langle AB \rangle\rangle \neq \langle\langle A \rangle\rangle \langle\langle B \rangle\rangle$. Under definition (2) the quark propagator is

$$S' = -\frac{\langle\langle (i\hat{\nabla} + im)^{-1} \mathrm{Det}\,(i\hat{\nabla} + im) \rangle\rangle}{\langle\langle \mathrm{Det}\,(i\hat{\nabla} + im) \rangle\rangle}. \tag{3}$$

The determinant in the denominator of (3) contains ultraviolet and infrared ($m \to 0$) divergences which must be regularized in order to make S' to be unambiguous (see [5]). In turn, this requires allowance for the back reaction effect of the fermions on the instanton medium (which is beyond the framework of the quenched approximation considered) and a modification of the distribution function. As a result, the propagator (3) is not equal to that found in [5]:

$$S = -\langle\langle (i\hat{\nabla} + im)^{-1} \rangle\rangle. \tag{4}$$

In order to solve this problem and construct a theory with quark propagator identical to (4), we use the following trick. Let us introduce additional boson spinor fields (ghosts) χ and χ^+ in the effective action to compensate the contributions from the quark degrees of freedom

$$Z = \int \left\langle\left\langle \exp\Big\{ \int [\psi^+ (i\hat{\nabla} + im)\psi + \chi^+ (i\hat{\nabla} + im)\chi + \eta^+\psi + \psi^+\eta] d^4x \Big\} \right\rangle\right\rangle \times \\ \times D\psi D\psi^+ D\chi D\chi^+. \tag{5}$$

In this way the whole averaging process is not modified. We recall that the distribution function was obtained in the quenched approximation neglecting the contributions from quark fields. This approximation corresponds to neglecting dynamical quark loops included into the determinant. In order to keep the distribution function obtained in the quenched approximation untouched, the determinant term is compensated by introduction of additional ghost fields. It is this point that makes our approach essentially different from those proposed previously. Following approximation (c) we must replace the exact quark propagator in the field of a single instanton by the model propagator

$$S_I(x, y) = S_0(x, y) - \frac{\phi_I(x)\phi_I^+(y)}{im}, \tag{6}$$

where $S_0(x, y)$ is the free propagator, and $\phi_I(x)$ is the quark zero mode in the field of Ith pseudoparticle: a right-handed (left-handed) Weyl spinor for an (anti)instanton. This model propagator interpolates the correct behavior at small momenta $\rho p \ll 1$ (the region where spontaneous breaking of chiral invariance occurs) and at high momenta where (6) becomes the free quark propagator. Using $S_0^{-1} = -(i\hat{\partial} + im)$, $S_I^{-1} - S_0^{-1} = -i\hat{A}_I$, we have $-i\hat{A} = \sum\limits_I (S_I^{-1} - S_0^{-1}) + \sum\limits_{\bar{I}} (S_{\bar{I}}^{-1} - S_0^{-1})$

$$S = \left[S_0^{-1} + \sum_I (S_I^{-1} - S_0^{-1}) + \sum (I \to \bar{I}) \right]^{-1}. \tag{7}$$

Equations (6) and (7) give

$$i\hat{\nabla}+im=i\hat{\partial}+im+\sum_{I}\left[1+(i\hat{\partial}+im)\frac{\phi_I\phi_I^+}{im}-1\right](i\hat{\partial}+im)+[I\rightarrow\bar{I}]. \qquad (8)$$

Using the chiral properties and the normalization conditions for the zero modes $\langle\phi_I^+\mid i\hat{\partial}+im\mid\phi_I\rangle=im$,

$$\left[1+i\hat{\partial}\frac{\phi_I\phi_I^+}{im}\right]^{-1}=1-i\hat{\partial}\frac{\phi_I\phi_I^+}{im},$$

$$\left[1+(i\hat{\partial}+im)\frac{\phi_I\phi_I^+}{im}\right]^{-1}=1-(i\hat{\partial}+im)\frac{\phi_I\phi_I^+}{2im},$$

we obtain from (5) in the chiral limit

$$Z=\int D\psi D\psi^+D\chi D\chi^+\exp\left\{\int[\psi^+(i\hat{\partial})\psi+\chi^+(i\hat{\partial})\chi+\eta^+\psi+\psi^+\eta]d^4x\right\}\times$$
$$\times\left\langle\!\!\left\langle\prod_{I,\bar{I}}\exp\left\{\frac{i}{m}\int\psi^+i\hat{\partial}\phi_Id^4x\int\phi_I^+i\hat{\partial}\psi d^4y+\frac{i}{m}\int\chi^+i\hat{\partial}\phi_Id^4x\int\phi_I^+i\hat{\partial}\chi d^4y\right\}\right\rangle\!\!\right\rangle. \qquad (9)$$

Using the properties of Grassmann variables ψ,ψ^+, we get for $Z(\eta,\eta^+)$

$$Z\left(\eta,\eta^+\right)=\int D\psi D\psi^+D\chi D\chi^+\exp\left\{\int[\psi^+(i\hat{\partial})\psi+\chi^+(i\hat{\partial})\chi+\eta^+\psi+\psi^+\eta]d^4x\right\}\times$$
$$\times\left\langle\!\!\left\langle\prod_{I,\bar{I}}\exp\left\{\frac{i}{m}\int\chi^+i\hat{\partial}\phi_Id^4x\int\phi_I^+i\hat{\partial}\chi d^4y\right\}\left(1-\frac{1}{im}\int\psi^+i\hat{\partial}\phi_Id^4x\int\phi_I^+i\hat{\partial}\psi d^4y\right)\right\rangle\!\!\right\rangle. \qquad (10)$$

We shall average Z over the positions and orientations of the pseudoparticles using the density matrix of fermion zero modes [5]

$$\phi_{ia}^I(x)\phi_{jb}^{I+}(y)=\int\frac{d^4kd^4p}{(2\pi)^8}\,\mathrm{e}^{ik(x-z_I)-iq(y-z_I)}\frac{\phi(k)\phi(q)}{8\mid k\mid\mid q\mid}\left(\hat{k}\gamma_\mu\gamma_\nu\hat{q}\frac{1-\gamma_5}{2}\right)_{ij}\left(U_I\tau_\mu^-\tau_\nu^+U_I^+\right)_{ab}. \qquad (11)$$

Here, $a,b(i,j)$ are color (spinor) indices, $\tau_\mu^\pm$ are $N_c\times N_c$ matrices with $(\bar{\tau},\mp i)$ in the upper left corner (the other elements are zero), τ are the Pauli matrices and $\hat{k}=k_\mu\gamma_\mu$. The function $\phi(k)$ is related to the Fourier transform of the zero modes:

$$\phi(k)=\pi\rho^2\frac{d}{dz}\left[I_0(z)K_0(z)-I_1(z)K_1(z)\right]_{z=\frac{|k|\rho}{2}}=\left\{\begin{array}{l}\dfrac{-2\pi\rho}{\mid k\mid},\mid k\mid\rho\ll1\\[2mm]\dfrac{-12\pi}{k^4\rho^2},\mid k\mid\rho\gg1\end{array}\right\}. \qquad (12)$$

For anti-instantons one has $\gamma_5 \to -\gamma_5$ and $\tau^{\pm}_{\mu} \to \tau^{\mp}_{\mu}$, and in averaging over the orientations with the Haar measure normalized to unity the relations are used:

$$\int dU = 1, \quad \int U_{mi} U^{+}_{jn} dU = \frac{1}{N_c} \delta_{ij} \delta_{mn},$$
$$\int U_{kp} U_{lq} U^{+}_{mr} U^{+}_{ns} dU = \frac{1}{N_c^2 - 1} \left[\delta_{kr} \delta_{ls} \left(\delta_{mp} \delta_{nq} - \frac{1}{N_c} \delta_{np} \delta_{mq} \right) + \begin{pmatrix} m \to n \\ r \to s \end{pmatrix} \right]. \tag{13}$$

In the leading in $1/N_c$ approximation only planar terms are kept when integration over the orientation matrices U_I is performed. We denote the integrals considered below by $I^{'}$ and I^{ab}_{ij} $(\lambda \equiv i/m)$

$$I^{'} = \int dU \exp \left[\lambda \int \chi^{+} i \hat{\partial} \phi_I d^4 x \int \phi^{+}_I i \hat{\partial} \chi d^4 y \right],$$
$$I^{ab}_{ij}(k,q) = \int dU \left[\hat{k} \phi_I(k) \right]^a_i \left[\phi^{+}_I(q) \hat{q} \right]^b_j \exp \left[\lambda \int \chi^{+} i \hat{\partial} \phi_I d^4 x \int \phi^{+}_I i \hat{\partial} \chi d^4 y \right]. \tag{14}$$

From the explicit expression (12) it follows that $I^{'}$ can contain only scalar and tensor products of left-handed components. However, we note that the tensor terms are suppressed in the limit $N_c \to \infty$. Thus, $I^{'}$ is a function of variable t, i.e., $I^{'}(t)$, with

$$t = \frac{\lambda}{N_c} \left\{ \chi^{+}_L \chi_L \right\},$$

where brackets denote the convolution

$$\left\{ A^{+a}_i B^b_j \right\}(z) = \int A^{+a}_i(k) B^b_j(q) \exp\left[iz(k-q) \right] a(k) a(q) \frac{d^4 k d^4 q}{(2\pi)^8}, \quad a(k) = \mid k \mid \phi(k).$$

We note that $I^{'}$ and I^{ab}_{ij} are related by

$$\lambda^2 \frac{\partial I^{ab}_{ij}(k,q)}{\partial \lambda} + \lambda I^{ab}_{ij}(k,q) = \frac{\delta^2 I^{'}}{\delta \chi^{+a}_i(k) \delta \chi^b_j(q)}.$$

Solving this differential equation we find

$$I^{ab}_{ij}(k,q) = \frac{1}{N_c} \frac{I^{'}(t) - I^{'}(0)}{t} \delta^{ab} \left(\frac{1+\gamma_5}{2} \right)_{ij} a(k) a(q) \exp\left[iz(k-q) \right] +$$
$$+ \frac{\lambda}{N_c^2} \frac{1}{t} \left(\frac{d}{dt} I^{'}(t) - \frac{I^{'}(t) - I^{'}(0)}{t} \right) a(k) a(q) \exp\left[iz(k-q) \right] \left\{ \chi^{+b}_j \chi_i \right\}. \tag{15}$$

From the observation [18] that an integration over the group is equivalent to a projection of the tensor product of the fundamental representation onto the singlets of the group (see, for instance, Eqs. (13) and also [14]) we obtain

$$I^{'} = \frac{1}{1 - \dfrac{\lambda}{N_c} \left\{ \chi^{+}_L \chi_L \right\}}. \tag{16}$$

Substituting (15) into (10) and integrating over the positions of pseudoparticles we find for $N_f = 1$ [16]

$$Z(\eta, \eta^+) =$$
$$= \int D\psi D\psi^+ D\chi D\chi^+ \exp[-J_0 + (\eta^+\psi)(\psi^+\eta)] \left[\frac{1}{V}\int d^4K_+\right]^{N_+} \left[\frac{1}{V}\int d^4K_-\right]^{N_-}, \quad (17)$$

where

$$J_0 = -\int \left[\psi^+ i\hat{\partial}\psi + \chi^+ i\hat{\partial}\chi\right] d^4x, \qquad K_- = K_+(L \to R),$$

and

$$K_+ = I^{'}(t) + \frac{i}{mN_c}\left(\frac{I^{'}(t) - I^{'}(0)}{t}\right)\left\{\psi_L^+\psi_L\right\} +$$
$$+ \left(\frac{i}{mN_c}\right)^2 \frac{1}{t}\left(\frac{d}{dt}I^{'}(t) - \frac{I^{'}(t) - I^{'}(0)}{t}\right)\left\{\psi_L^+\chi_L\right\}\left\{\chi_L^+\psi_L\right\}.$$

Note that this expression is finite in m and the chiral limit $m \to 0$ is valid. The action contains the dynamical mass $M(k) \sim (N_c)^0$ and the interaction potential is $\sim N_c^{-1}$ (number of instantons, N, is proportional to the number of colors, N_c). Retention of the potential is justified by the fact that it leads to results which are stable in N_c for correlation functions of colorless currents. The N_c^{-1} suppression of the potential is compensated by extra powers of N_c from fermion-ghost loops [16]. Note that the dependence of the product $\{\chi^+\chi\}(z)$ on z can be neglected. This approximation, which is valid in the chiral limit, is based on the fact that the function $a^2(k)$ decreases rapidly (see also [19]) for average values of the ghost fields $\{\chi^+\chi\} \sim 1/k^2$ and the largest contribution to integral comes from the region $k \sim q \sim 0$. In the chiral limit $m \to 0$ the dependence on z vanishes. We recall that our Z is normalized to unity and that any suppression is compensated by the analogous contribution of the determinant from ψ^+, ψ, i.e., the dependence on z in $I^{'}(t)$ can be neglected. It must be emphasized that when mass corrections are taken into account, the integration over the fields χ and χ^+ can be carried out exactly, and the result in the limit $m \to 0$ is the same as the approximate one [16]. The advantage of this approximation is that the interaction potential can be calculated. Beyond the chiral limit it is very hard to find. In accordance with what we have said above, we shall write down the action keeping the terms $O(1/N_c)$ and making replacement $\left\{\chi_L^+\chi_L\right\}(z) \to \left\{\chi_L^+\chi_L\right\}(0)$:

$$J = J_0 - N_+ \ln\left[1 + \frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0) + \frac{i}{mVN_c}\int \psi_L^+\psi_L(k)a^2(k)\frac{d^4k}{(2\pi)^4}\right] +$$
$$+ \begin{pmatrix} L \to R \\ N_+ \to N_- \end{pmatrix}. \quad (18)$$

In the thermodynamic limit, when $N \to \infty, V \to \infty$, we have $N_+ = N_- = N/2$

$$J = J_0 - \frac{N}{2}\ln\left[1 + \frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0)\right] - \frac{N}{2mVN_c}\frac{\int \psi_L^+\psi_L(k)a^2(k)\frac{d^4k}{(2\pi)^4}}{1 + \frac{iN_c}{m}\left\{\chi_L^+\chi_L\right\}(0)} - (L \to R) \quad (19)$$

and $Z(0,0)$ can be written as

$$Z(0,0) = \int D\psi D\psi^+ D\chi D\chi^+ \frac{d\omega_+}{2\pi}\frac{d\mu_+}{\mu_+^2}\frac{d\omega_-}{2\pi}\frac{d\mu_-}{\mu_-^2} \exp\left[-J_0 - \frac{N}{2}\ln\mu_+ + \frac{iN\mu_+}{2mVN_c} \times \right.$$
$$\left. \times \int \psi_L^+\psi_L(k)a^2(k)\frac{d^4k}{(2\pi)^4} + i\omega_+\left(\frac{1}{\mu_+} - 1 - \frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0)\right) + \begin{pmatrix} L \to R \\ (\omega)\mu_+ \to (\omega)\mu_- \end{pmatrix}\right]. \tag{20}$$

After integration over $\psi, \psi^+, \chi, \chi^+$ [12, 13, 16] we have

$$Z(0,0) = \int d\mu_+ d\mu_- \frac{g(\mu_+\mu_-)}{\mu_+^2\mu_-^2} \exp\left[-\frac{N}{2}\ln\mu_+ - \frac{N}{2}\ln\mu_- + \right.$$
$$\left. +2VN_c \int \ln\left(1 + \frac{\mu_+\mu_-}{m^2}\left(\frac{N}{2VN_c}\right)^2 \frac{a^4(k)}{k^2}\right)\frac{d^4(k)}{(2\pi)^4}\right], \tag{21}$$

where

$$g(\mu_+,\mu_-) = \int \frac{d\omega_+}{(2\pi)}\frac{d\omega_-}{(2\pi)}\left(1 + \frac{\omega_+ + \omega_-}{4N_c}\right)^{-2N_c} \exp\left[i\omega_+\left(\frac{1}{\mu_+} - 1\right) + i\omega_-\left(\frac{1}{\mu_-} - 1\right)\right].$$

In the thermodynamic limit $\ln\left[\frac{g(\mu_+,\mu_-)}{\mu_+^2\mu_-^2}\right]$ should be neglected in comparison with terms proportional to V or N. Integration over $\mu_\pm$ can be carried out by the saddle-point method, since $\triangle\mu_\pm$ is of the order of $1/\sqrt{N_c}$. We obtain $\mu_+ = \mu_- = m\epsilon$, where $\epsilon \approx (100 \quad \mathrm{MeV})^{-1}$ is determined by the self-consistency condition introduced in [5] naturally arising in the integration over $\mu_\pm$:

$$1 = \frac{4VN_c}{N}\int \frac{M^2(p)}{p^2 + M^2(p)}\frac{d^4p}{(2\pi)^4}, \qquad M(k) = \frac{N\epsilon}{2VN_c}a^2(k). \tag{22}$$

In accordance with the results of [5, 12, 13] at this order of $1/N_c$ we have obtained a theory of noninteracting quarks with effective masses $M(k)$:

$$Z(0,0) = \mathrm{const}\int D\psi D\psi^+ \exp\int \psi^+[-\hat{k} + iM(k))]\psi\frac{d^4(k)}{(2\pi)^4}. \tag{23}$$

From Eqs. (19) and (23) it can be seen that the average value $\langle\left\{\chi_L^+\chi_L\right\}(0)\rangle$ is determined by the relation

$$\left[1 + \frac{i}{mN_c}\langle\left\{\chi_L^+\chi_L\right\}(0)\rangle\right]^{-1} = m\epsilon. \tag{24}$$

We shall now consider the action in the order $1/N_c$:

$$J = J_0 - [N_+\ln Q] - \begin{bmatrix} L \to R \\ N_+ \to N_- \end{bmatrix},$$

where

$$Q = 1 + \frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0) + \left(\frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0)\right)^2 + \\ + \left(\frac{i}{mVN_c}\right)\int \psi_L^+\psi_L(k)a^2(k)\frac{d^4k}{(2\pi)^4}\left(1 + \frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0)\right) + \\ + \left(\frac{i}{mN_c}\right)^2\int \frac{d^4z}{V}\left\{\psi_L^+\chi_L\right\}(z)\left\{\chi_L^+\psi_L\right\}(z). \quad (25)$$

Substituting the average value (24) for colorless products

$$\left(\frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0)\right)^2 \to \left(\left\langle\frac{i}{mN_c}\left\{\chi_L^+\chi_L\right\}(0)\right\rangle\right)^2,$$

one gets in the chiral limit $m \to 0$, $N_c \to \infty$

$$J = \int \left(\psi^+[\hat{k} - iM(k)]\psi + \chi^+\hat{k}\chi\right)\frac{d^4k}{(2\pi)^4} + 2\frac{V}{N}\int (M(k)M(q)M(p)M(l))^{1/2} \times \\ \times \delta^4(k+p-q-l)\left(\psi_L^+(k)\chi_L(q)\right)\left(\chi_L^+(p)\psi_L(l)\right)\frac{d^4k\,d^4p\,d^4q\,d^4l}{(2\pi)^{12}} + (L \to R). \quad (26)$$

Thus, in the case $N_f = 1$, in contrast with the results of [12, 13, 20] we obtain the nonlinear theory with nonlocal interaction. This action has three important advantages: (a) perturbation theory can be applied since the vertex function is parametrically small $\left(-\frac{N}{2VN_c}\frac{1}{N_c}\right)$, (b) the model is superconvergent, and (c) the potential retains massless ghost fields [16].

2. ACTION FOR LIGHT QUARKS IN INSTANTON VACUUM MODEL FOR $N_f = 2$

The partition function of QCD in Euclidean space with any number of flavors has the form

$$Z(\eta\eta^+) = \int D\psi D\psi^+ D\chi D\chi^+ \exp\left(\int \psi^{+f} I_{ff'} i\hat{\partial}\psi^{f'}\,d^4x + \chi^{+f} I_{ff'} i\hat{\partial}\chi^{f'}\,d^4x\right) \times \\ \times \left\langle\!\left\langle \exp\left(I_{ff'}\frac{i}{m}\int \chi^{+f} i\hat{\partial}\phi_I \int \phi_I^+ i\hat{\partial}\chi^{f'}\right)\prod_{f,f'}^{N_+}\left[I_{ff'} - \frac{1}{im}\int \psi_f^+ i\hat{\partial}\phi_I d^4x \int \phi_I^+ i\hat{\partial}\psi_{f'} I_{ff'}\right]\right\rangle\!\right\rangle, \quad (27)$$

where $I_{ff'} = \delta_{ff'}$, $\psi^f = \begin{pmatrix}\psi_1\\ \psi_2\\ \vdots\end{pmatrix}$ $(f, f' = 1, 2)$. In the case $N_f = 2$, repeating the steps which bring us to (10) we obtain in the leading order in $1/N_c$

$$Z = Z_1^* + Z_2^*,$$

$$Z_1^* = \int d^4 z_{I(\bar I)} \int dU_I D\psi D\psi^+ D\chi D\chi^+ \exp \int \left[\psi^{+f} i\hat\partial \psi^f + \chi^{+f} i\hat\partial \chi^f\right] d^4x \times$$
$$\times \exp \frac{i}{m} \int \chi^{+f} i\hat\partial \phi_{I(\bar I)}\, d^4x \int \phi^+_{I(\bar I)} i\hat\partial \chi^f d^4y \left(1 - \frac{1}{im}\int \psi_f^+ i\hat\partial \phi_{I(\bar I)}\, d^4x \int \phi^+_{I(\bar I)} i\hat\partial \psi_f\, d^4y\right), \tag{28}$$

$$Z_2^* = \int \frac{(-1)^2}{m^2} d^4 z_{I(\bar I)}\, dU_I D\psi D\psi^+ \left(\exp \int \psi_f^+ i\hat\partial \psi_f\right) \times$$
$$\times \int \psi_1^+ i\hat\partial \phi_{I(\bar I)}\, d^4x_1 \int \phi^+_{I(\bar I)} i\hat\partial \psi_1\, d^4y_1 \int \psi_2^+ i\hat\partial \phi_{I(\bar I)}\, d^4x_2 \int \phi^+_{I(\bar I)} i\hat\partial \psi_2\, d^4y_2,$$

where we omit in (28) the nonleading in $1/N_c$ terms χ, χ^+ in the exponent. Here Z_1^* coincides with our result (26) in the case $N_f = 1$ and Z_2^* is exactly the expression first investigated in [12, 13]. Using the zero-mode density matrix (11), relations (13) and $\gamma_5[\gamma_\alpha, \gamma_\beta] = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}[\gamma_\mu, \gamma_\nu]$, and Fierz transformations we obtain

$$Z_2^* = \left(\frac{2V}{mN\epsilon}\right)^2 \int d^4x \det W_\pm(x), \tag{29}$$

where $W_\pm(x)$ are 2×2 flavor matrices

$$W_\pm(x)_{fg} = \int \frac{d^4k d^4l}{(2\pi)^8}\, \mathrm{e}^{i(k-l,x)} \sqrt{M(k)M(l)}\psi_f^+(k) \frac{1\pm\gamma_5}{2}\psi_g(l). \tag{30}$$

The 4-fermion interaction is reminiscent of the famous 't Hooft interaction [1]. In the instanton liquid model it becomes nonlocal owing to the diagonalization of the would be zero modes of individual pseudoparticles in the instanton medium [12, 13]. Strength of interaction (29) in our model is unambiguously fixed by the characteristics of the instanton medium and the averaging procedure. Collecting (25) and (29) we obtain the effective action

$$J = J_0 - [N_+ \ln Q^*] - \begin{bmatrix} L \to R \\ N_+ \to N_- \end{bmatrix},$$

$$Q^* = 1 + \frac{i}{mN_c}\left\{\chi^+_{fL}\chi_{fL}\right\}(0) + \left(\frac{i}{mN_c}\left\{\chi^+_{fL}\chi_{fL}\right\}(0)\right)^2 +$$
$$+ \left(\frac{i}{mVN_c}\right) \int \psi^+_{fL}\psi_{fL}(k) a^2(k) \frac{d^4k}{(2\pi)^4}\left(1 + \frac{i}{mN_c}\left\{\chi^+_{fL}\chi_{fL}\right\}(0)\right) +$$
$$+ \left(\frac{i}{mN_c}\right)^2 \int \frac{d^4z}{V}\left\{\psi^+_{fL}\chi_{fL}\right\}(z)\left\{\chi^+_{fL}\psi_{fL}\right\}(z) + \left(\frac{2V}{mN\epsilon}\right)^2 \int d^4x \det W_L(x). \tag{31}$$

Expanding the ln function, using (24), $\lim P_{m\to 0} \to \dfrac{1}{(m\epsilon)^2}$

$$P = 1 + \frac{i}{mN_c}\left\{\chi^+_{fL}\chi_{fL}\right\}(0) + \left(\frac{i}{mN_c}\left\{\chi^+_{fL}\chi_{fL}\right\}(0)\right)^2$$

and definition (22) for $M(k)$, we finally obtain in the case $N_f = 2$

$$J = \int \left(\psi^+[\hat{k} - iM(k)]\psi + \chi^+\hat{k}\chi\right) \frac{d^4k}{(2\pi)^4} + 2\frac{V}{N}\times$$
$$\times\int (M(k)M(q)M(p)M(l))^{1/2}\delta^4(k+p-q-l)\frac{d^4k\, d^4p\, d^4q\, d^4l}{(2\pi)^{12}}\Big\{\big(\psi_L^+(k)\chi_L(q)\big)\big(\chi_L^+(p)\psi_L(l)\big)+$$
$$+\frac{2V}{N}\left[\left(\psi_{1L}^+(k)\psi_{1L}(q)\right)\left(\psi_{2L}^+(p)\psi_{2L}(l)\right) - \left(\psi_{1L}^+(k)\psi_{2L}(q)\right)\left(\psi_{2L}^+(p)\psi_{1L}(l)\right)\right]\Big\} + (L \to R)\,. \tag{32}$$

Note that the four-quark coupling in our approach differs from the result [12, 13]. Besides we have significant contribution from quark interaction with massless bosonic spinor fields.

3. ROLE OF THE INSTANTON VACUUM IN THE VECTOR CHANNEL

Now let us demonstrate that instanton fluctuations of the QCD vacuum not only make it possible to describe the mechanism of spontaneous breaking of chiral invariance (SBCI), but also can play a decisive role in the formation of bound states in the vector channel. It should be emphasized that the vector channel can be described using a unified approach based on quark interaction generated by instanton. Using common notation: S for action (32) and J for the currents, we can define the connected part of the current correlation function as

$$\Pi^\Gamma(p) = \int \frac{d^4k_1 d^4k_2}{(2\pi)^4}\delta(k_1 - k_2 + p)J^\Gamma(k_1)J^\Gamma(k_2)\,\mathrm{e}^{-S},$$

where $J^\Gamma(k) = \psi^+(k)\Gamma\psi(k)$ with $\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$. We will consider two types of diagrams contributing to the connected part of the correlation function in different channels: a) Figure 1 presents the connected part of the correlation function in the pseudoscalar channel ($\Gamma = \gamma_5$). The vertex $\pi \to q\bar{q}$ as $\Gamma_\pm^\Gamma(p)$ is defined as (all factors N_c and N/V are included in the correlation function)

$$\Gamma_\pm^{\gamma_5}(p) = \int \frac{d^4k}{(2\pi)^4}\, Sp\left[\gamma_5(\hat{k} + p/2 + iM_1)\frac{1 \pm \gamma_5}{2}\,(\hat{k} - p/2 + iM_2)\right]\times$$
$$\times\,\frac{\sqrt{M_1M_2}}{[(k+p/2)^2 + M_1^2]\,[(k-p/2)^2 + M_2^2]} \simeq \mp 2\int \frac{d^4k}{(2\pi)^2}\frac{M(k)}{k^2 + M^2} = \pm\frac{\langle\bar{\psi}\psi\rangle}{2N_c},$$

where 4-fermion vertex is the second term in (32) and gives major inclusion in Fig. 2, while the first term in (32) related to the ghost field includes the suppression factor (ρ/R) density of instanton liquid. Results of calculation bring us to value $f_\pi = 142$ MeV which coincides

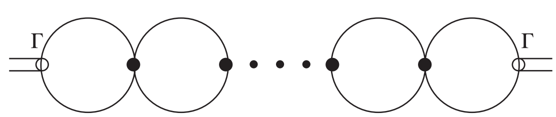

Fig. 1. Connected part of the current correlation function. Solid thin lines correspond to quark fields

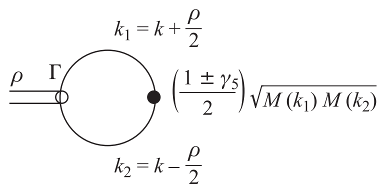

Fig. 2. The vertex $\pi \to q\overline{q}$

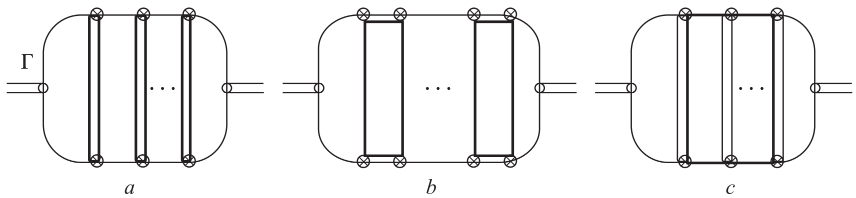

Fig. 3. Rescattering part of the current correlation function. The crosses correspond to the first term in (32), interaction of quarks with massless bosonic spinor fields (thin lines are quarks, thick lines are bosonic spinor fields)

with the previous result [12, 13]. From symmetry considerations it follows that the vertex $\Gamma^{\Gamma}_{\pm}(p)$ vanishes in the vector ($\Gamma = \gamma_\mu$) and tensor ($\Gamma = \sigma_{\mu\nu}$) channels as it occurred in the Diakonov–Petrov model [12, 13].

b) In the vector channel: Fig. 3 shows a type of diagrams that describe the effects of permanent scattering in continuous media.

If we isolate only one-meson states in the correlation function by imposing the condition that any structure obtained by cutting an arbitrary diagram contains only two quarks and that no colorless subsystems are formed by ghost legs (in other words, if we recall the general rule that only planar diagrams and minimum number of quark loops survive in the limit $N_c \to \infty$) [21], the entire class of diagrams of the type presented in Fig. 3, c can be discarded. The coupling part of the correlation function is thus determined by the diagrams in Fig. 3, a, b which can be summed in the standard way by using the Fredholm equation. It brings us to the results $f_\rho = 193$ MeV, $m_\rho = 797$ MeV. Detailed calculations in the pseudoscalar and vector channels will be represented elsewhere, where we will also consider test related to axial-anomaly low-energy theorems (LET).

CONCLUSIONS

Thus, our procedure of averaging the total QCD Lagrangian density in statistical ensemble of pseudoparticles (rather than to average individual correlation functions as in [4, 5, 12, 13]) leads to the action which even in the case $N_f = 1$ contains the interaction potential of quarks in an instanton medium and makes it possible to calculate colorless correlation functions.

We want to stress again that the self-consistent quenched approximation is the base of our approach. It corresponds to neglecting dynamic quark loops containing in the quark determinant. To compensate the quark determinant we introduce the additional scalar ghost fields with spin $1/2$.

The physical pattern is as follows: the instanton vacuum medium is treated as a discrete system of $I\bar{I}$ pairs, and passage to the continuous limit was performed only at the final stage of calculations. This approach is correct for a quark propagating in the instanton vacuum because deceleration effects that are due to medium (and which lead to emergence of the effective quark mass $M(p)$) do not depend on the way in which the thermodynamic limit appears $N \to \infty$, $V \to \infty$, $N/V = \text{const}$. The same approach can be used to describe a Goldstone mode (pion) for which the quark interaction with vacuum plays a leading role, while the exchange of momenta between quarks is insignificant. However, for other (non Goldstone) modes such as vector particles, the continuity of medium plays a decisive role because quarks exchange momenta only through a continuous medium and can form bound state. The problem of taking correctly into account both of these effects simultaneously (quark interactions with instanton fluctuations of the vacuum and interactions between quarks) can be solved by the method discussed in this paper. Quark interaction in the instanton medium is mediated by spin-1/2 boson fields χ and χ^+ arising in this approach and corresponding to the physical degrees of freedom of the continuous medium. This effective (superconvergent) interaction has far-reaching consequences, including the possibility of describing the vector particles in the model of instanton liquid. Thus, this approach enables us to construct a model describing physically observable particles.

Acknowledgements. A. E. D. acknowledges partial support from the Scientific School Grant No. 195.2008.2.

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Received on November 18, 2008.