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MIXING OF NEUTRAL ATOMS AND LEPTON NUMBER OSCILLATIONS

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We discuss oscillations of two neutral atoms which proceed with the violation of lepton number. One of the neutral atoms is stable, the other one represents a quasistationary state subjected to electromagnetic deexcitation. The system of neutral atoms exhibits oscillations similar to those of the system of neutral kaons and neutron–antineutron oscillations in the nuclear medium. The underlying mechanism is a transition of two protons and two bound electrons to two neutrons $p + p + e^-_b + e^-_b \leftrightarrow n + n$. A signature of the oscillations might be an electromagnetic deexcitation of the involved unstable nucleus and atomic shell with the electron holes. A resonant enhancement of the neutrinoless double electron capture takes place when the atomic masses tend to be degenerate. Qualitative estimates show that in searches for lepton number violation oscillations of atoms might be a possible alternative to the conventional mechanism of the neutrinoless double β decay process with emission of two electrons.

Обсуждаются осцилляции двух нейтральных атомов, которые приводят к нарушению сохранения лептонного заряда. Один из нейтральных атомов стабилен, другой представляет собой квазистационарное состояние, распадающееся за счет электромагнитного излучения. Система нейтральных атомов испытывает осцилляции, аналогичные осцилляциям в системе нейтральных каонов и нейтрон-антинейтронным осцилляциям в ядерной среде. Основным механизмом является переход двух протонов и двух связанных электронов в два нейтрона $p + p + e^-_b + e^-_b \leftrightarrow n + n$. Осцилляции могут наблюдаться как электромагнитное излучение нестабильного конечного ядра и атомной оболочки, содержащей электронные дырки. Резонансное усиление безнейтринного двойного захвата электронов происходит, когда массы атомов близки к вырождению. Качественные оценки показывают, что в поисках нарушения сохранения лептонного числа осцилляции атомов могут быть альтернативой обычного процесса безнейтринного двойного бета-распада с рождением двух свободных электронов.

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INTRODUCTION

After the discoveries of oscillations of atmospheric [1], solar [2], reactor [3] neutrinos and neutrinos from accelerators [4, 5], the physics community worldwide is embarking on the next challenging problem, finding whether neutrinos are indeed Majorana particles (i.e.,

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identical to its own antiparticle) as many particle models suggest or Dirac particles (i.e., is different from its antiparticle). This problem is directly related with the issue of the total lepton number conservation.

Lepton number (LN) conservation is one of the most obscure sides of the Standard Model (SM) not supported by an underlying principle. It follows from an accidental interplay between gauge symmetry and the field content. However, nonzero neutrino masses, as indicated by the recent neutrino oscillations experiments, have proved that the success of the SM should be viewed as that of a low-energy effective theory.

It is not unreasonable to expect that in some extensions of the SM, LN conservation may not hold. More specifically, once lepton number is broken, neutrinos are not protected from getting nonzero Majorana masses after electroweak symmetry breaking. Indeed, a Majorana mass term for the neutrinos violates total lepton number. A viable scenario is that the neutrino masses are generated at some high-energy scale. This is well motivated by the observed properties of the light neutrinos including tiny masses, large mixings, and the fact that neutrinos are the only electrically neutral fundamental fermions.

A Majorana type of the neutrino mass matrix induces a class of lepton number violating processes [6] like neutrino–antineutrino oscillations [7, 8], semileptonic decays of mesons, muon-to-positron conversion in nuclei [9], neutrinoless double β decay, muonic analogue of the neutrinoless double β decay [10], etc. Probabilities and decay rates of these processes are given in terms of the neutrino mass matrix elements, and a semi-realistic event rate has been estimated.

Searches for LN violation are an important and active area of research. The lepton number violating process has been sought in many experiments. Over the years the possibility of LN nonconservation has been attracting a great deal of theoretical and experimental efforts since any positive experimental lepton number violating signal would request physics beyond the SM. In addition, it would also show that neutrinos are Majorana particles.

The aim of this paper is to propose oscillations of neutral atoms

$$(\mathcal{A}, \mathcal{Z}) \leftrightarrow (\mathcal{A}, \mathcal{Z}+2)^{**}, \quad (\mathcal{A}, \mathcal{Z}) \leftrightarrow (\mathcal{A}, \mathcal{Z}-2)^{**}, \qquad (1)$$

one of which is metastable, as a possible consequence of LN violation, and discuss perspectives of experimental investigation of the oscillations. Throughout this paper, we shall use the notation $(\mathcal{A}, \mathcal{Z})$ for atoms and (A, Z) for nuclei. The double star index indicates a possibility of the double nuclear and atomic excitations.

1. A NEW TOOL FOR STUDY OF LEPTON NUMBER VIOLATION

The neutrinoless double β decay ($0\nu\beta\beta$ decay) which violates the total lepton number,

$$(A, Z) \to (A, Z+2) + 2e^-, \qquad (2)$$

is currently the most powerful tool to clarify if the neutrino is a Dirac or a Majorana particle. It is due to the fact that a large amount of double β decaying isotopes is placed in the experiment. Since the $0\nu\beta\beta$ decay gives practically the only possibility of distinguishing between Majorana and Dirac neutrinos, much effort has been devoted to the problem of $0\nu\beta\beta$ decay (for reviews see [11]).

The $0\nu\beta\beta$ decay has not yet been confirmed. The most stringent lower bound on the half-live of the $0\nu\beta\beta$ decay was measured for ^{76}Ge in the Heidelberg–Moscow experiment [12]:

$$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) \geqslant 1.9 \cdot 10^{25}\ \mathrm{y}. \tag{3}$$

We note that some authors of the Heidelberg–Moscow (HM) collaboration have claimed the experimental observation of the $0\nu\beta\beta$ decay of ^{76}Ge with half-lives $T^{0\nu}_{1/2} = (0.8{-}18.3)\cdot 10^{25}$ y (best-fit value of $1.5 \cdot 10^{25}$ y) [13]. The disproof or the confirmation of the claim will come from future experiments. In the near future this Heidelberg–Moscow limit is expected to be improved in the GERDA experiment [14] by 1–2 orders of magnitude. For the $0\nu\beta\beta$ decays of ^{100}Mo and ^{130}Te in the two running experiments, NEMO3 [15] and CUORICINO [16], the following sensitivities have been achieved:

$$\begin{aligned} T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) &\geqslant 5.8 \cdot 10^{23}\ \mathrm{y}, \\ T^{0\nu}_{1/2}(^{130}\mathrm{Te}) &\geqslant 3.0 \cdot 10^{24}\ \mathrm{y}. \end{aligned} \tag{4}$$

Recently, there is an increased interest to the neutrinoless double electron capture with emission of one real photon,

$$2e^-_b + (A, Z) \to (A, Z-2) + \gamma, \tag{5}$$

in spite of the fact that this mode has to be calculated at least by the third-order perturbation theory. Sujkowski and Wycech suggested [17] that the decay rate of this process can be enhanced due to some resonance condition associated with the $2P{-}1S$ atomic level difference. This idea has been extended by Frekers [18] for a transition to excited nuclear states

$$2e^-_b + (A, Z) \to (A, Z-2)^* + \gamma \tag{6}$$

without presenting some theoretical background. The nuclear transition $^{74}\mathrm{Se} \to {}^{74}\mathrm{Ge}^{**}$ was proposed due to a small energy difference of initial and final atoms. The first experimental measurement of this process established a half-live limit of $T^{0\nu ECEC\gamma}_{1/2} \geqslant 5.5 \cdot 10^{18}$ y [19]. A subject of interest has been also another transition $^{112}\mathrm{Sn} \to {}^{112}\mathrm{Cd}^{**}$ with the measured half-live limit of $9.2 \cdot 10^{19}$ y [20].

A new phenomenon of oscillation plus deexcitations of atoms has origin in a mixing of a pair of neutral atoms, $(\mathcal{A}, \mathcal{Z})$ and $(\mathcal{A}, \mathcal{Z} \pm 2)^{**}$, which lepton numbers differ by two units. It is also assumed that there is a small energy difference between both atomic states. The first atom $(\mathcal{A}, \mathcal{Z})$ is in a ground state and the second atom $(\mathcal{A}, \mathcal{Z} \pm 2)^*$ might be in an excited state in respect of both atomic and nuclear structure. The $(\mathcal{A}, \mathcal{Z})$ atom is unstable only due to LN violating decay channels. The atom $(\mathcal{A}, \mathcal{Z} \pm 2)^*$ deexcites due to electromagnetic interaction by emission of X-rays (atomic structure) and/or gamma rays (nuclear structure). The major issue here is the much shorter time of electromagnetic deexcitations for the $(\mathcal{A}, \mathcal{Z} \pm 2)$ atom. Thus, the width of $(\mathcal{A}, \mathcal{Z})$ atomic state is negligibly small in comparison with the width of $(\mathcal{A}, \mathcal{Z} \pm 2)^{**}$ state and for a sake of simplicity will be neglected.

The studied system of neutral atoms has some features in common with systems of neutrinos, neutral kaons or B mesons for which oscillations are known to occur in nature.

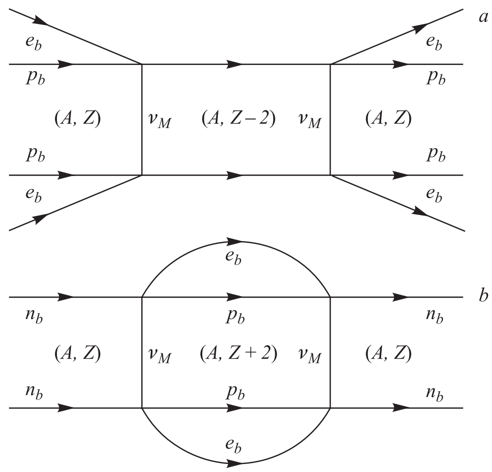

Fig. 1. Two different neutral atoms, carrying different lepton number, can turn from one into another through the LN violating weak interactions

The observation of oscillations in these systems has yielded valuable information on various aspects of physics (lepton flavor violation, neutrino mass, strangeness, CP and T-invariance violation) that are not accessible using less sensitive techniques. As a consequence of mixing of atoms, we expect that the oscillation plus decay can occur. A possibility of oscillation of neutral atoms due to lepton number violation is presented in Fig. 1. In the context of strong and electromagnetic interactions the neutral atoms are most naturally described in terms of eigenstates of lepton and baryon numbers. As soon as we turn on the LN violating weak interactions the neutral atoms can mix; i.e., the total lepton number is no longer a conserved quantum number. We distinguish two possibilities:

i) The initially pure atomic state with Z protons and electrons is transformed to the neutral atom with $Z-2$ protons and electrons by the neutrinoless double electron capture:

$$2e_b^- + (A, Z) \to (A, Z-2)^*. \tag{7}$$

The most favorable is the capture of K-shell electrons.

ii) The initially pure atomic state with Z protons and electrons is transformed to the neutral atom with $Z+2$ protons and electrons via the process of the $0\nu\beta\beta$ decay of two nucleons in a nucleus. The two electrons remain in the atom and are located at non-occupied higher energy atomic shells:

$$(A, Z) \to (A, Z+2)^* + 2e_b^-. \tag{8}$$

In what follows we shall discuss the phenomenon of oscillation plus deexcitation of neutral atoms in analogy with oscillation of neutrons to antineutrons in the presence of nuclei [21].

2. OSCILLATIONS OF NEUTRAL ATOMS

In this section, we present a phenomenological framework for the description of oscillations of stable and metastable neutral atoms.

If lepton number is not conserved the mixing can occur between a pair of neutral atoms, $(\mathcal{A},\mathcal{Z})$ and $(\mathcal{A},\pm 2)$. The initial atom $(\mathcal{A},\mathcal{Z})$ is assumed to be in the ground state and the second atom might be in the ground state $(\mathcal{A},\mathcal{Z}\pm 2)$, excited atomic state $(\mathcal{A},\mathcal{Z}\pm 2)^*$ or in excited both atomic and nuclear states $(\mathcal{A},\mathcal{Z}\pm 2)^{**}$. The decay channel of $(\mathcal{A},\mathcal{Z})$ is the LN violating process of the $0\nu\beta\beta$ decay, the width of which is much smaller as compared to the width of $(\mathcal{A},\mathcal{Z}\pm 2)^{**}$ atom of the electromagnetic origin. Therefore, we shall neglect it in our analysis.

2.1. Oscillations in the System of Stable and Metastable Atoms. Lepton number violating interactions induce transitions $(\mathcal{A},\mathcal{Z}) \to (\mathcal{A},\mathcal{Z}\pm 2)^{**}$ which can be described phenomenologically by 2×2 Hamiltonian matrix

$$H_{\rm eff} = \begin{pmatrix} M_i & V \\ V & M_f - \dfrac{i}{2}\Gamma \end{pmatrix}. \tag{9}$$

Here M_i and M_f are atomic masses in the initial and final states; Γ is width of the final state. The off-diagonal matrix elements of $H_{\rm eff}$ are complex conjugate. They can always be made real by phase rotation of one of the states, so V can be taken real. The diagonal elements of the Hamiltonian are determined by strong and electromagnetic interactions which conserve lepton number. The off-diagonal elements mix neutral atoms with the violation of lepton number by two units due to the weak interactions of massive Majorana neutrinos.

Using the Pauli matrices, the Hamiltonian can be written as follows:

$$H_{\rm eff} = b_+ + a\sigma_1 + b_-\sigma_3, \tag{10}$$

with

$$a = V, \tag{11}$$

$$b_\pm = \frac{M_i \pm M_f}{2} \mp \frac{i}{4}\Gamma. \tag{12}$$

The evolution operator $\mathrm{e}^{-iH_{\rm eff}t}$ can be expanded over the Pauli matrices as follows:

$$\mathrm{e}^{-iH_{\rm eff}t} = \mathrm{e}^{-ib_+t}\Big(\cos(\Omega t) - i\frac{a\sigma_1 + b_-\sigma_3}{\Omega}\sin(\Omega t)\Big), \tag{13}$$

where $\Omega = +\sqrt{a^2 + b_-^2}$. One can see that all components of the evolution matrix behave like $\mathrm{e}^{-i\lambda_\pm t}$, with $\lambda_\pm = b_+ \pm \Omega$ being the eigenvalues of $H_{\rm eff}$. The corresponding eigenstates are denoted by $|\pm\rangle$.

The LN violating potential V is significantly smaller than Γ and $M_i - M_f$. To the lowest order in V, we obtain

$$\lambda_+ = M_i + \Delta M - \frac{i}{2}\Gamma_1, \tag{14}$$

$$\lambda_- = M_f - \frac{i}{2}\Gamma - \Delta M + \frac{i}{2}\Gamma_1, \tag{15}$$

where

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}, \tag{16}$$

$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}. \tag{17}$$

The amplitude to find the initial atom t seconds after its preparation in the same initial state is determined by the diagonal matrix element:

$$\langle i|\mathrm{e}^{-iH_{\mathrm{eff}}t}|i\rangle = \mathrm{e}^{-i\lambda_- t}\frac{a^2}{4b_-^2} + \mathrm{e}^{-i\lambda_+ t}\left(1 - \frac{a^2}{4b_-^2}\right). \tag{18}$$

The off-diagonal term V produces in the initial state an admixture $\sim V^2$ of the diagonal state $|-\rangle$. This admixture oscillates with the frequency $\approx M_f$ and decays with the rate $\Gamma - \Gamma_1 \approx \Gamma$, as prescribed by the first term in Eq. (18). The second term oscillates with the frequency $\approx M_i$ and decays with the rate of Γ_1. The initial state is dominated by the diagonal state $|+\rangle$.

The formalism described above applies to oscillations of neutral kaons. In the basis of CP eigenstates, the effective Hamiltonian is diagonal, $H_{\mathrm{eff}} = \mathrm{diag}(M_1 - i\Gamma_1/2, M_2 - i\Gamma_2/2)$. In the basis of kaons with definite strangeness, $|K^0\rangle$ and $|\bar{K}^0\rangle$, the corresponding evolution operator has the form of Eq. (13) with $a = (M_1 - M_2)/2 - i(\Gamma_1 - \Gamma_2)/4$, $b_+ = (M_1 + M_2)/2 - i(\Gamma_1 + \Gamma_2)/4$, and $b_- = 0$. The difference between oscillations of atoms and kaons is also due to the mixing, which is small for atoms and maximal for kaons.

The Hamiltonian (9) with $M_i = M_f \equiv M$ describes neutron–antineutron oscillations in the nuclear medium, with M being the neutron mass and Γ the in-medium antineutron collision width connected to the annihilation processes on the surrounding nucleons. Our formalism is similar to the formalism of neutron–antineutron oscillations [21–23].

2.2. Oscillations in the System of Two Stable Atoms. The lepton number violating potential V is significantly smaller in comparison with $M_i - M_f$. In addition, the final atom is in the ground state, i.e., $\Gamma = 0$, and so $\Omega = (M_i - M_f)/2 > 0$. The transition probability of the initial atom may therefore be written as

$$\left|\langle f|\mathrm{e}^{-iH_{\mathrm{eff}}t}|i\rangle\right|^2 = \frac{a^2}{\Omega^2}\sin^2(\Omega t). \tag{19}$$

This is just the case of oscillations of the two-level system described, e.g., in [24]. In the case of $\Omega t \ll 1$, the transition probability $\sim V^2 t^2$ is determined by the potential V only.

However, in the realistic case of atoms and an experiment with exposure time of about one year, one has $\Omega t \gg 1$. By taking the average over one period, we end up with

$$\left|\langle f|\mathrm{e}^{-iH_{\mathrm{eff}}t}|i\rangle\right|^2 \approx \frac{2V^2}{(M_i - M_f)^2}. \tag{20}$$

The mass difference of the involved atoms is typically of order of a few MeV. The potential V is smaller by about 30 orders of magnitude (see Eq. (23)). In transition ${}^{164}_{68}\mathrm{Er} \to {}^{164}_{66}\mathrm{Dy}$ with the smallest mass difference $M_i - M_f = 24.1$ keV, one finds $|\langle f|\mathrm{e}^{-iH_{\mathrm{eff}}t}|i\rangle|^2 \sim 10^{-55}$. Even for 1 ton of $^{164}_{68}$Er the oscillations cannot be observed.

The small probability prohibits the use of the ground-to-ground state oscillations in searches for LN violation. We thus turn to the case of metastable atoms.

3. ESTIMATE OF COUNTING RATE

The LN violating potential V allowing mixing of the neutral atoms is associated with the elementary processes $n + n \leftrightarrow p + p + e_b^- + e_b^-$ (see Fig. 1). The corresponding potential has the form [25]

$$V \simeq m_{\beta\beta}\; G_F^2 \frac{1}{4\pi R} \mathcal{M} \Psi_1(0)\Psi_2(0), \tag{21}$$

where $m_{\beta\beta}$ is the effective mass of Majorana neutrinos (current limit $m_{\beta\beta} \leqslant 0.5$ eV); G_F is the Fermi constant; $R = 1.2A^{1/3}$ fm is the nuclear radius; $\mathcal{M}$ is the nuclear matrix element, and $\Psi_i(0)$ are the electron wave functions at the origin:

$$|\Psi_i(0)|^2 = \frac{1}{\pi n_i^3 a_B^3} \delta_{l_i 0}. \tag{22}$$

Here, $a_B = 1/(\alpha Z m_e)$ is the Bohr radius; n_i and l_i are the principal and orbital quantum numbers of the ith electron. We neglect spin–isospin structure of the nuclear transition matrix element and set $\mathcal{M} \approx 6$, which is a typical value for the $0^+ \to 0^+$ transitions. The Fermi statistics of electrons, finite nuclear size, relativistic corrections to the electron wave functions, and the overlap factor of the electron shell wave functions in $(\mathcal{A}, \mathcal{Z})$ and $(\mathcal{A}, \mathcal{Z} - 2)^{**}$ atoms are also neglected.

For a medium-heavy nucleus with $Z = 30$ and K-shell electrons and $m_{\beta\beta} = 0.5$ eV, one gets

$$V \sim 10^{-24} \text{ eV}. \tag{23}$$

The capture of electrons from higher electron shells causes V to decrease like $1/(n_1 n_2)^{3/2}$. An additional suppression comes from the screening of the Coulomb potential and decrease of the effective charge Z. The electron capture in ^{30}Zr from the outer electron shell is suppressed, e.g., by a factor of $1/(n_1 n_2)^{3/2}/Z^3 \sim 10^{-4}$. In the process $(\mathcal{A}, \mathcal{Z}) \to (\mathcal{A}, \mathcal{Z} + 2)^{**}$ shown in Fig. 1, *b*, electrons occupy free bound states in the final atom. As compared to the amplitude $(\mathcal{A}, \mathcal{Z}) \to (\mathcal{A}, \mathcal{Z} - 2)^{**}$, the corresponding amplitude is suppressed, respectively.

The capture from the $l_i \neq 0$ states is possible due to finite nuclear size, but suppressed by a factor of $(R/a_B)^{l_1 + l_2}$ where $R/a_B \sim 2 \cdot 10^{-3}$ in ^{30}Zr.

In the transitions of nuclei $(A, Z) \to (A, Z - 2)^*$ from 0^+ to J^P states with $P = +1$ or $P = -1$ due to non-conservation of parity the captured electrons must be in the state J^P. The $0^+ \to 2^+$ transition is possible provided the electrons occupy, e.g., $1s_{1/2}$ and $3d_{3/2}$ or $2p_{1/2}$ and $2p_{3/2}$ states. The corresponding potential V is suppressed by a factor of $(R/a_B)^2 \sim 4 \cdot 10^{-6}$.

The exposure time of atoms in double β decay experiments (days and months) is much greater than electromagnetic widths of atoms in the final quasistationary state. The lepton number violating width Γ_1 is however small. Under conditions $\Gamma_1 t \ll 1$ and $\Gamma t \gg 1$, the second term in the amplitude (18) survives.

Equation (17) gives decay rate of the initial atom in agreement with the Breit–Wigner formula. The final excited atom plays the role of a resonance in the decay amplitude.

The resonance states enhance decay probabilities provided masses of the initial and final states tend to be degenerate and the resonance is narrow. This is the case of $n\bar{n}$ oscillations in the medium, where degeneracy of the masses leads to the expression $\Gamma_1 = 4V^2/\Gamma$ and the maximum enhancement at the unitary limit, accordingly.

The radiative widths of excited nuclei are typically smaller than those of the excited electron shells [26, 27]. The dominant mechanism corresponds to electric dipole transitions of the electrons from higher orbitals to fill the holes formed by the double electron capture.

The electric dipole transition $2P \to 1S$ has the probability [28]

$$\Gamma \approx 4 \cdot 10^{-7} Z^4 \text{ eV}. \tag{24}$$

For $Z = 30$, one has $\Gamma = 0.3$ eV. The electromagnetic $(n_i+1)P \to n_i S$ widths scale with the principal quantum number like $\Gamma \sim 1/n_i^5$, since $\Gamma \sim \omega^3 d^2$ where $\omega \sim 1/n_i^3$ is the transition energy and $d \sim n_i^2$ is the electric dipole transition moment. The estimate (24) does not take into account screening of the nuclear Coulomb potential, finite nuclear size, relativistic corrections to the electron wave functions, and other transitions, e.g., $(n_i + 2)D \to n_i S$.

We come closely to the estimate of the number of events which can be observed at the unitary limit $M_i = M_f$. By assuming one ton of the source material, the counting rate can be found to be

$$R_{\max} = \frac{1 \text{ t}}{M_i} \frac{4V^2}{\Gamma} \sim 10^4 \text{ y}^{-1}, \tag{25}$$

which is equivalent to ~ 10 events per day. Here, we set $Z = 30$ and used the above estimates of V and Γ. $R_{\max}$ scales with the principal quantum numbers roughly like $\sim 1/n$ for $n \equiv n_1 \sim n_2$. For $M_i - M_f \gg \Gamma$, $R_{\max}$ scales like $\Gamma^2/(M_i - M_f)^2$. The half-live unitary limit is $T_{1/2}^{\min} \sim 10^{24}$ y.

Obviously, there exist rich combinatorial possibilities for the optimal choice of the isotopes, their excitation levels, and excitations of the atomic electron shells to match the condition $M_i - M_f = 0$ with high precision and approach the unitary limit.

The electron binding energies in the inner atomic shells vary from ~ 10 eV in light nuclei to ~ 100 keV in heavy nuclei. The outer electrons are bound with an energy of ~ 10 eV in both low- and high-Z nuclei. The interaction energy of two electron holes is expected to be $\sim 1/Z$ of the binding energy. In heavy elements it can reach a value of ~ 1 keV. The energy of two-hole excitations can be calculated with a 10 eV accuracy.

The main problem is the poor experimental knowledge of masses of the ground-state nuclei, which are measured within a few keV only. The excitation energies relative to ground states of nuclei are known better, $\Delta(M^* - M) \sim 100$ eV.

The experimental errors in the mass difference $M_i - M_f$ hamper the rate predictions and restrict R to

$$R \gtrsim R_{\max} \frac{\Gamma^2}{(M_i - M_f)^2} \sim 10^{-3} \text{ y}^{-1}. \tag{26}$$

Further experimental progress in the measurement of the ground- and excited-state masses of nuclei can make the neutrinoless double electron capture competitive with the standard $0\nu\beta\beta$ decay.

The radiative neutrinoless double electron capture is studied in [17] within the standard framework of the perturbation theory. The authors derive an equation similar to our Eq. (17) and observe a resonance enhancement of the process under the condition of degeneracy in energies of the initial state and the final state of nucleus and photon. Despite our formalisms are fairly different, numerical estimates are in qualitative agreement. After finishing this work our attention has been called to Ref. [29], where mixing of atoms is discussed within the standard framework of the perturbation theory.

CONCLUSION

In summary, we have discussed a mixing of neutral atoms due to lepton number violating weak interactions. In our considerations we restricted ourselves to the case of two atoms with the lowest mass difference by taking into account also a possibility of excitations of atomic shells and of the involved nuclei. This concept was discussed in the framework of oscillations of two-level system. A phenomenological analysis brought us to a resonant enhancement of the double electron capture, that has a Breit–Wigner form.

It was manifested that it is reasonable to hope that a search for oscillation plus deexcitation of atoms, which are sufficiently long-lived to conduct a practical experiment, may uncover processes with lepton number violation. For that purpose, systems of two atoms with the smallest mass difference have to be found. The corresponding work based on the available information about masses of stable atoms and their atomic and nuclear level structure is in progress. It goes without saying that experimental effort for re-measuring of atomic masses with high accuracy (up to eV level) is of great importance. We note that there is a significant advantage for experimental study of the double electron capture to excited atomic states; namely, there is practically no background from the $2\nu ECEC$ decay mode.

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