Письма в ЭЧАЯ. 2009. Т. 6, № 4(153). С. 519–527

QUANTUM ELECTRODYNAMICS AND EXPERIMENT DEMONSTRATE THE NONRETARDED NATURE OF ELECTRODYNAMICAL FORCE FIELDS

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In quantum electrodynamics, the quantitatively most successful theory in the history of science, intercharge forces obeying the inverse square law are due to the exchange of space-like virtual photons. The fundamental quantum process underlying applications as diverse as the gyromagnetic ratio of the electron and electrical machinery is then Møller scattering $ee \to ee$. Analysis of the quantum amplitude for this process shows that the corresponding intercharge force acts instantaneously. This prediction has been verified in a recent experiment.

В квантовой электродинамике, являющейся наиболее успешной теорией в истории науки, силы взаимодействия зарядов подчиняются закону обратного квадрата и обусловлены обменом пространственноподобных фотонов. Вследствие этого мёллеровское рассеяние $ee \to ee$ является фундаментальным квантовым процессом, основная область применения которого простирается от гиромагнитного отношения электрона до электротехнической индустрии. Анализ квантовой амплитуды этого процесса показывает, что соответствующие силы между зарядами действуют мгновенно. Это предсказание было недавно подтверждено экспериментально.

PACS: 12.20.-m; 03.50.De

One of the most remarkable developments of physical science during the 20th century was the advent of quantum electrodynamics (QED) [1] born of the fusion of quantum mechanics and special relativity theory. QED has two important aspects; the first is the remarkable quantitative success of the theory — an essentially perfect description of nature within its domain of applicabilty. The second is its role as a «model theory» from which the standard model of particle physics was later developed by attempting to describe the weak and strong interactions by quantum field theories in a similar way as the electromagnetic interaction is described by QED. A recent illustration of the former aspect of QED is the remeasurement at Harvard University of the gyromagnetic ratio of the electron [2]:

$$\frac{g_e^{\mathrm{exp}}}{2} = 1.00115965218085(76).$$

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The measurement uncertainty in the last two figures is indicated in parentheses. The accuracy of the measurement is 7.6 parts in 10^{13}, a six-fold improvement on the previous best measurement performed at the University of Washington in 1987 [3] for which Hans Dehmelt was awarded a part of the 1989 Nobel Prize in Physics. The electron gyromagnetic ratio is the most precisely measured physical quantity, to date, in the history of science. The QED prediction for g_e is [4]

$$\frac{g_e^{\mathrm{th}}}{2} = 1.00115965218875(766),$$

where the measured value of the fine structure constant: $\alpha = 1/137.036\ldots$ derived from spectroscopy of the Rubidium atom [5], is used in the prediction. Theory and experiment for g_e agree at about one part in 10^{11} [4]:

$$\frac{g_e^{\mathrm{exp}}}{2} - \frac{g_e^{\mathrm{th}}}{2} = -7.9(7.7) \cdot 10^{-12}.$$

Assuming the correctness of the QED prediction for g_e, the Harvard measurement determines the value of α with ten times better accuracy than given by Rubidium spectroscopy, which yields the second most accurate experimental measurement of this constant.

The calculational problems which must be surmounted in order to obtain the QED prediction for g_e at the accuracy required by the latest experimental measurement, and so obtain a quantitatively meaningful comparison of theory and experiment, are formidable. However, as pointed out by Feynman, the basic physical concepts on which the calculations are based are extremely simple [6]. Three fundamental quantum mechanical amplitudes suffice to calculate the amplitude of any QED space–time process, no matter how complicated, involving only structureless charged particles and photons. These are the amplitudes for the processes:

a photon goes from place to place;

a charged particle goes from place to place;

a charged particle emits or absorbs a photon.

Combination of these three amplitudes enables the quantum mechanical probability amplitude for any space–time process in QED — one Feynman diagram per amplitude — to be written down. Depending on the accuracy with which it is desired to obtain the prediction for the value of some measured observable, quantum mechanical superposition must be used to add the amplitudes corresponding to one or more Feynman diagrams. The accuracy of the latest Harvard measurement of g_e requires the evaluation of the amplitudes of 891 distinct Feynman diagrams containing up to five virtual photon lines.

The lowest order diagram contributing to the «anomaly» $a_e \equiv g_e/2 - 1$ is shown in Fig. 1, a. The physical observable corresponding to this diagram is the angular frequency with which the spin vector of the electron rotates relative to the direction of its momentum vector: $\omega_a = a_e eB/m_e$, where m_e and e are the mass and electric charge of an electron. The diagram of Fig. 1, a gives the prediction $a_e = \alpha/\pi$ [7]. The left diagram in Fig. 1, a shows the conventional way of drawing the diagram, where the vertical virtual photon line is identified with the (classical) uniform magnetic field B produced, in the Harvard experiment, by the solenoidal magnet of a Penning Trap (PT). In this experiment it is the gyromagnetic ratio of a single electron, e_{PT}, initially in the lowest lying cyclotron level of the PT, which is measured. In the QED description, the classical magnetic field, B, is a manifestation of the exchange of virtual photons between the conduction electrons, e_I, of the solenoidal magnet

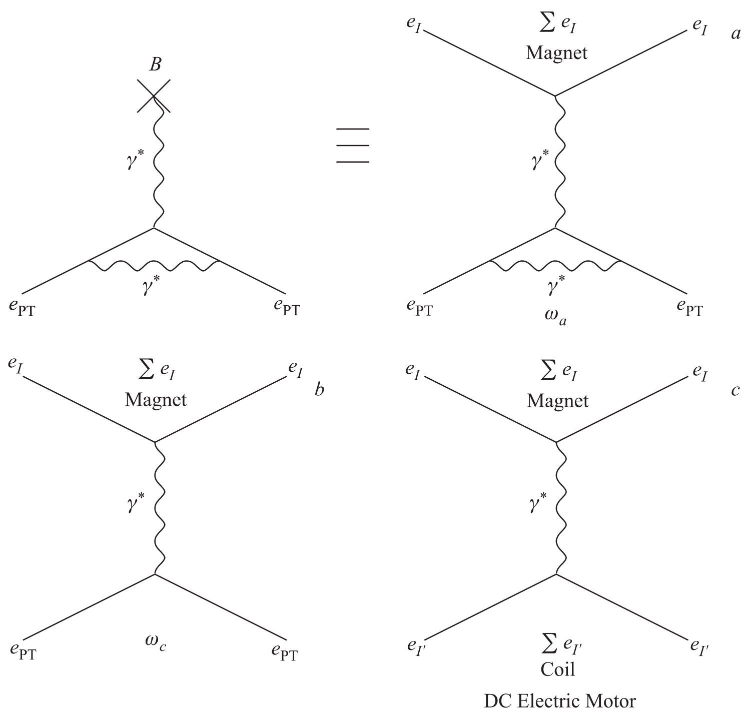

Fig. 1. Lowest order Feynman diagrams contributing to: *a*) $a_e = g_e/2 - 1$ or the spin precession frequency ω_a; *b*) the cyclotron frequency ω_c in the Penning Trap of [2]; *c*) the operation of a direct current electric motor. See text for discussion

and $e_{\rm PT}$. This is shown in the diagram on the right in Fig. 1, a. The field B is the result, in QED, of the superposition of the amplitudes corresponding to each conduction electron in the magnet. Under the influence of the transverse Lorentz force generated by B, $e_{\rm PT}$ undergoes periodic cyclotron motion in the PT with angular frequency $\omega_c = eB/m_e$. The corresponding diagram in the QED description is shown in Fig. 1, b. The Lorentz force is produced by the exchange of virtual photons between $e_{\rm PT}$ and all the conduction electrons in the magnet. Using the technique of «Quantum-Jump Spectroscopy» [8], the Harvard experiment measures separately ω_a and ω_c and derives a_e, and hence g_e, from the relation $a_e = \omega_a/\omega_c$.

The force on $e_{\rm PT}$ in Fig. 1, b, produced by virtual photon exchange, that causes the cyclotron motion of the trapped electron has the same origin, in QED, as the force which causes the coil of a DC electric motor to rotate (Fig. 1, c). The electron $e_{\rm PT}$ is simply replaced by a conduction electron $e_{I'}$ in the rotating coil of the motor. Each pair of conduction electrons, one in the magnet, one in the coil, gives a contribution to the turning force of the motor.

The fundamental QED process underlying cyclotron motion of $e_{\rm PT}$ in the PT of the Harvard experiment, an electric motor, and indeed all interelectron forces obeying an inverse square law in electrodynamics, including those operating in the experiment to be described below in the present paper, is then Møller scattering: $ee \rightarrow ee$ as in Fig. 1, b and c. The lowest order Feynman diagram for the process shown involves the exchange of a single «space-like» virtual photon. The meaning of this appellation is explained below.

In momentum space, the invariant amplitude for Møller scattering $e_A e_B \to e_A e_B$ is [9]

$$T_{fi} = -i \int \frac{\mathcal{J}^A(x_A)\mathcal{J}^B(x_A)}{q^2} d^4 x_A. \tag{1}$$

The 4-vector current $\mathcal{J}^A$ is defined in terms of plane-wave solutions, u_i, u_f of the Dirac equation for the incoming (i) and outgoing (f) electron e_A as

$$\mathcal{J}^A_\mu(x_A) \equiv -e\overline{u}^A_f \gamma_\mu u^A_i \exp\left[i\left(p^A_f - p^A_i\right)\cdot x_A\right]. \tag{2}$$

The incoming electron of 4-vector momentum p^A_i emits the virtual photon at the space–time point x_A and scatters with 4-vector momentum p^A_f. The factor $1/q^2$ in (1) is the amplitude, in momentum space, for the virtual photon to propagate between points on the trajectories of e_A and e_B, q being the 4-vector momentum of the virtual photon. The integral over d^4x_A in (1) in conjunction with the exponential functions in the currents (equivalent to Dirac δ functions) ensures energy-momentum conservation in the scattering process. In the overall center-of-mass (CM) frame, energy-momentum conservation requires that the energy, but, in general, not the momentum, of the virtual photon vanishes. This implies that in the CM frame

$$q^2 = q_0^2 - |\mathbf{q}|^2 \to -|\mathbf{q}|^2. \tag{3}$$

The virtual photon is then «space-like» because its 4-momentum squared is negative. If an object has positive 4-vector momentum squared, its 4-vector momentum is «time-like» and a positive mass may be assigned to it. All ponderable physical objects in the real world have such time-like 4-vector momenta. As pointed out by Einstein in 1905 [10], the speed of an object with a time-like 4-vector momentum is less than that of light. However, as just shown, the virtual photons exchanged in Møller scattering are not time-like. Using (3), (1) may be written, in the CM frame, as

$$T_{fi} = i \int \frac{\mathcal{J}^A(x_A)\mathcal{J}^B(x_A)}{|\mathbf{q}|^2} d^4 x_A. \tag{4}$$

The Fourier transform

$$\frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi}\int \frac{d^3x\, \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}}}{|\mathbf{x}|} \tag{5}$$

enables the invariant amplitude to be written as a space–time integral [11]:

$$T_{fi} = \frac{i}{4\pi}\int dt_A \int d^3x_A \int d^3x_B \frac{\mathcal{J}^A(\mathbf{x}_A, t_A)\mathcal{J}^B(\mathbf{x}_B, t_A)}{|\mathbf{x}_B - \mathbf{x}_A|}. \tag{6}$$

It can be seen that the momentum-space photon propagator $1/|\mathbf{q}|^2$, the value of which is fixed by the electron scattering angle, corresponds, in space–time, to the exchange of an infinite number of virtual photons travelling between all spatial positions $\mathbf{x}_A$ and $\mathbf{x}_B$ on the trajectories of the electrons e_A and e_B. Equation (6) shows that *each virtual photon is both emitted and absorbed at the same instant, so that the corresponding force is transmitted instantaneously*. The same conclusion follows from considerations of relativistic kinematics.

The magnitude of the velocity of an object with momentum p and energy E is $v = pc^2/E$. In the CM frame of Møller scattering, the virtual photon has $p > 0$ for any nonzero scattering angle, but always $E = 0$. Therefore, v is infinite and the corresponding intercharge interaction is instantaneous.

It has been previously noticed that the «Coulomb interaction» associated with the temporal components of the currents $\mathcal{J}^A$ and $\mathcal{J}^B$ (exchange of «longitudinal» virtual photons) is instantaneous, but it is usually stated that the interaction mediated by the spatial components of the currents (exchange of «transverse» virtual photons) is transmitted at the speed of light [12]. The arguments presented above show instead that the whole intercharge interaction is instantaneous in the CM frame. Indeed the argument given above, leading to Eq. (6), is identical to that presented by Feynman in [12] except that the kinematical condition (3), valid in the CM frame, is applied. To a very good approximation, the CM frame of the charges, is the one in which intercharge forces are calculated in the classical nonrelativistic limit where $\beta \ll 1$.

QED provides a simple explanation for the Coulomb inverse square law of force, which, in this theory, is mediated by the exchange of virtual photons. Geometrical considerations and conservation of particle number imply that, if a force is proportional to the number of some exchanged particles, it must decrease as the inverse square of the distance from the source. An example is the force of radiation pressure due to real photons emitted from the Sun. Quantum mechanics, in general, modifies this prediction. For a virtual particle of pole mass m propagating over a large space-like interval $\Delta x > \Delta t$ the amplitude for displacements Δx, Δt (the space–time propagator) takes the form: $\exp\left[-m\sqrt{(\Delta x)^2 - (\Delta t)^2}\right]$ [13]. Thus, the range of the force is exponentially damped if m is nonzero. Since, however, a photon has vanishing pole mass, no damping occurs for the case of space-like virtual photons, so the inverse square law is expected to hold. The same argument applies to the propagator of any other massless particle, independently of its interactions; e.g., exchange of space-like virtual massless gravitons could explain the inverse square law of the gravitational force.

The above predictions of QED: an instantaneous intercharge interaction and the Coulomb inverse square law in electrostatics have been used, in conjunction with relativistic invariance and Hamilton's Principle, to formulate a classical theory of interchange forces without *a priori* introduction of any classical field concept [11]. The dynamical equations of this theory, Relativistic Classical Electrodynamics (RCED) are derived by Hamilton's Principle from the Lorentz-invariant Lagrangian:

$$L(x_1, u_1; x_2, u_2) = -\frac{m_1 u_1^2}{2} - \frac{m_2 u_2^2}{2} - \frac{j_1\, j_2}{c^2\sqrt{-(x_1 - x_2)^2}} \tag{7}$$

that describes the electromagnetic interaction between two charged objects $O1, O2$ with space–time, velocity and current 4-vectors x, u and j, respectively, and Newtonian mass m. The fields and potentials of Classical Electromagnetism (CEM), are all derived, as well as relativistic corrections to them, by mathematical substitution, in the dynamical equations obtained by inserting the Lagrangian (7) into the Lagrange equations that follow from Hamilton's Principle. Also predicted are the Faraday–Lenz law, the Biot and Savart law and the Lorentz force law. The complete relativistic description of the effect of inverse square intercharge forces between $O1$ and $O2$ is provided by the «fieldless» first-order differential equations:

$$\frac{d\mathbf{p}_1}{dt} = \frac{q_1}{c}\left[\frac{j_2^0\mathbf{r} + \boldsymbol{\beta}_1 \times (\mathbf{j}_2 \times \mathbf{r})}{r^3} - \frac{1}{cr}\frac{d\mathbf{j}_2}{dt} - \mathbf{j}_2\frac{(\mathbf{r}\cdot\boldsymbol{\beta}_2)}{r^3}\right], \quad (8)$$

$$\frac{d\mathbf{p}_2}{dt} = -\frac{q_2}{c}\left[\frac{j_1^0\mathbf{r} + \boldsymbol{\beta}_2 \times (\mathbf{j}_1 \times \mathbf{r})}{r^3} + \frac{1}{cr}\frac{d\mathbf{j}_1}{dt} - \mathbf{j}_1\frac{(\mathbf{r}\cdot\boldsymbol{\beta}_1)}{r^3}\right], \quad (9)$$

where $r \equiv |\mathbf{x}_1 - \mathbf{x}_2|$, $\beta \equiv v/c$. On neglecting relativistic corrections of $O(\beta^2)$ and higher, these equations are equivalent to the Coulomb law, the Biot and Savart law and the Lorentz force law of CEM.

Not described by (8) and (9) are the effects of *real* photons propagated at the speed of light, c. The corresponding *radiative* electric and magnetic fields, to be contrasted with the *force fields* implicit in (8) and (9), are produced by accelerated charges and have an r^{-1} dependence, instead of the r^{-2} dependence of the force fields. On solving the coupled differential equations (8) and (9), for the particular case of circular Keplerian orbits of two equal and opposite charges [11], it is found that the r^{-1}-dependent terms in (8) and (9), although also containing acceleration factors, do not describe radiation of real photons, but rather the modification of the masses of the objects due to their mutual electromagnetic interaction.

In any experiment where source charges are accelerated and the effect on test charges is observed, contributions of two distinct types are therefore to be expected:

(i) The effect of instantaneous *force fields*, mediated in QED by the exchange of space-like virtual photons, with r^{-2} dependence.

(ii) The effect of *radiation fields*, mediated by propagation at the speed of light of real (on shell) photons, with r^{-1} dependence.

Because of the rapid fall-off with distance of the force fields, special experimental arrangements are necessary for their detection. Remarkably, it is only very recently, more than a century after the Hertz experiment [14] in which «electromagnetic waves» were discovered, that the results of a new experiment showing clearly the temporal properties (instantaneous or retarded) of the force fields have been published [15]. This experiment is now briefly described.

In essence, the experiment is a repetition of the Hertz experiment using modern electronics to detect and visualize the signals, and probing small separations of the emitting and receiving antennas in order to be sensitive to the force fields with r^{-2} dependence (called by the authors of [15] as «bound fields»). The emitting (EA) and receiving (RA) antennas are essentially one-turn circular coils of radius 5 cm and depths 5 cm (EA) and 10 cm (RA) consisting of 1 mm thick copper sheet, and placed in the same horizontal plane, with centers separated by distance R. The EA is activated by discharging capacitor C, in series with the EA of inductance L, with the aid of a spark gap, to generate a pulsed harmonically varying current with angular frequency $\omega = 1/\sqrt{LC} = 7.4 \cdot 10^8$ rad/s. The associated time-varying magnetic field induces a current in the RA which is displayed as a temporal signal on a 500 MHz digital oscilloscope.

Neglecting relativistic corrections, the magnetic field produced by a small single-turn coil of surface area ΔS containing current I at a large distance R from the coil in the plane of the latter is (see Appendix of [15])

$$\mathbf{B} = \mathbf{B}_v^{\rm force} + \mathbf{B}_c^{\rm rad} = \frac{\Delta S}{4\pi\epsilon_0 c^2}\left\{\frac{[I]_v}{R^3} + \frac{c}{v}\frac{[\dot{I}]_v}{cR^2} + \frac{[\ddot{I}]_c}{c^2 R}\right\}\hat{k}, \tag{10}$$

where the dot denotes a time derivative and the unit vector $\hat{k}$ is perpendicular to the plane of the coil. The terms with R^{-3} and R^{-2} dependence describe the effects of the force field, and are derived from the Biot and Savart law, that with R^{-1} dependence is a radiation field. The square brackets indicate retardation of the enclosed quantity; that is, it is evaluated at the time $t - R/v$ for the force field and at $t - R/c$ for the radiation field. In CEM it is usually assumed that $v = c$, whereas QED predicts that $v = \infty$. In the experiment, v is assigned an arbitary value in (10) and measured by comparing the prediction of this formula with the experimental data. The Faraday–Lenz law predicts that the time-varying current signal, $\epsilon_v(t)$, in a parallel test coil, also of surface area ΔS, placed in the field $\mathbf{B}$ is

$$\epsilon_v(t) = \frac{(\Delta S)^2}{4\pi\epsilon_0 c^2}\left\{\frac{[\dot{I}]_v}{R^3} + \frac{c}{v}\frac{[\ddot{I}]_v}{cR^2} + \frac{[\dddot{I}]_c}{c^2 R}\right\}. \tag{11}$$

The harmonic variation of the magnetic field produced by the EA implies that (11) may be written as

$$\epsilon_v(t) = \epsilon_0\left[-\frac{\sin\omega(t - R/v)}{R^3} + \frac{\omega\sin[\omega(t - R/v) - \pi/2]}{vR^2} + \frac{\omega^2\sin\omega(t - R/c)}{c^2 R}\right]. \tag{12}$$

It can be seen that the phases of the force field contributions in the first two terms are different to that of the radiation field contribution in the third term on the right side of (12). Since the phase difference between the force field and radiation field contributions depends on both R and v, the value of v can be determined by measuring the R dependence of the signal function $\epsilon_v(t)$. In practice, this is done by measuring the R dependence of the time difference, Δt, between the first zero crossing points of the total signal $\epsilon_v(t)$ and a reference signal, $\epsilon^{\rm ref}(t)$, provided by measuring $\epsilon_v(t)$ at large distances where only the radiative contribution remains:

$$\epsilon^{\rm ref}(t) \equiv \epsilon^{\rm rad}(t) = \epsilon_0\frac{\omega^2\sin\omega(t - R/v)}{c^2 R} \tag{13}$$

and extrapolating it back to the short distance region. In order to reproduce correctly the experimental conditions, the elementary formula (11) was numerically integrated over the surfaces and depths of the EA and the RA.

The measured results for Δt as a function of R, in comparison with predictions for different values of v, are shown in Fig. 2. It can be seen that the case of conventionally retarded force fields, as in CEM, is completely excluded by the measurements, but that good agreement is found for the case $v \geqslant 10c$, which is essentially the same as the QED prediction $v = \infty$. Yet again a prediction of QED is in perfect agreement with experiment!

The physical paradigm of «causality» — that no physical influence can propagate faster than the speed of light — universally accepted in physics in the second half of the 19th century and the whole of the 20th — is thus in contradiction both with the prediction of QED and the experimental results shown in Fig. 2 — the force fields generated by the EA

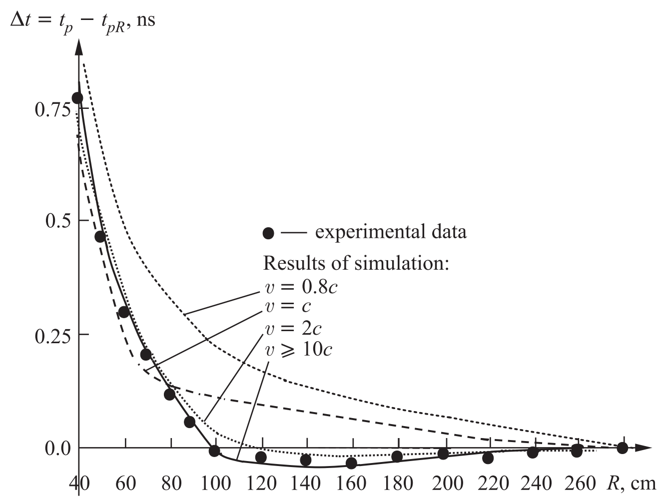

Fig. 2. Measured values of the difference, Δt, between the first zero crossing times of the signals $\epsilon_v(t)$ and $\epsilon^{\rm ref}(t)$, in comparison with simulations for different values of v from [15]. The measurement error on each experimental datum is $\simeq 0.02$ ns. The prediction $v = c$ of conventional CEM is completely excluded whereas the data are consistent with the QED prediction $v = \infty$ which is indistinguishable from the $v \geqslant 10c$ curve shown

arrive at the RA before the radiation fields; in QED the virtual photons arrive before the real ones. Einstein's argument in the special relativity paper of 1905 [10] that «causality» follows from special relativity is correct only insofar as «information» is transmitted by objects with time-like 4-momentum vectors. However, the virtual photons that manifest as the force fields of CEM or RCED have space-like, not time-like 4-vectors. Any particle with a space-like 4-momentum vector is tachyonic in nature — the speed of light is a lower, not an upper, limit on its speed.

It is interesting to notice that Hertz' paper [14] reporting the discovery of electromagnetic waves propagating with a velocity «akin to that of light» also showed data, taken close to the source, that was consistent with an infinite propagation speed for the associated signal [16,17]. The more surprising is that no comment was made on this data in the conclusions of [14], given that two types of electric force, one instantaneous, the other propagating at the speed of light were proposed in the electromagnetic theory of Hertz' mentor, Helmholtz [16].

The author thanks Professor A. L. Kholmetskii for bringing [15] to his attention, and for providing Fig. 2.

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Received on October 16, 2008.