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SENSITIVITY TO NEW PHYSICS: a_e VERSUS a_μ

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At present it is generally believed that «new physics» effects contribute to leptonic anomalous magnetic moment a_ℓ via quantum loops only and they are proportional to the squared lepton mass m_ℓ^2. An alternative mechanism for a contribution by new physics is proposed. It occurs at *the tree level* and exhibits *a linear* rather than quadratic dependence on m_ℓ. This leads to a much larger sensitivity of a_e to the new physics than was expected so far.

В настоящее время бытует широко распространенное мнение, что вклад «новой физики» в аномальный магнитный момент лептона дают лишь радиационные квантовые поправки и он пропорционален квадрату массы лептона. В данной работе предложен альтернативный механизм, который генерирует вклад новой физики на древесном уровне и который прямо пропорционален массе лептона. Это ведет к более сильной чувствительности аномального магнитного момента электрона к новой физике, чем ожидалось до сих пор.

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INTRODUCTION

Since Schwinger's one-loop calculation [1], leptonic anomalous magnetic moments have usually been used for precision tests of the Standard Model (SM). Very precise recent experimental measurements of the electron anomalous magnetic moment [2]

$$a_e^{\rm exp} = 1\,159\,652\,180.73(0.28)\cdot 10^{-12}\ (0.24\ {\rm ppb}) \qquad (1)$$

and the muon anomalous magnetic moment [3]

$$a_\mu^{\rm exp} = 1\,165\,920.80(0.63)\cdot 10^{-9}\ (0.54\ {\rm ppm}) \qquad (2)$$

give a possibility to look further for allusive «new physics».

Indeed, $a_e^{\rm exp}$ is the most precise experimental value, which provides a determination of α, the fine structure constant [4]:

$$\alpha^{-1}\left(a_e^{\rm exp}\right) = 137.035\,999\,084(051)\ (0.37\ {\rm ppb}), \qquad (3)$$

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with an accuracy of more than an order of magnitude better than the independent measurements [5, 6]

$$\alpha^{-1}\,(\mathrm{Rb}) = 137.035\,998\,78(091)\ (6.7\ \mathrm{ppb}), \tag{4}$$

$$\alpha^{-1}\,(\mathrm{Cs}) = 137.036\,000\,00(110)\ (7.7\ \mathrm{ppb}). \tag{5}$$

It is this fact that limits at present testing the a_e^{SM} prediction.

On the other hand, the a_μ^{exp} persists to show a deviation in comparison with the SM prediction [7]. To be more definitive, we choose a little bit conservative, but the most recently updated value [8]

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = +267(96)\cdot 10^{-11}, \tag{6}$$

which shows 2.8σ standard deviation.

Remarkably, this difference exceeds by an order of magnitude the biggest uncertainties from the hadronic contributions to the muon anomalous magnetic moment and it is two times larger than the SM electroweak contribution. The latter fact is apparently in some conflict with the viable at present «natural» conception that new physics contributions are induced by quantum loop effects, rather than at the tree level [9]. Thanks to the mass limits set by LEP and Tevatron, it is highly nontrivial to reconcile the observed deviation with many of the new physics scenarios. Only the $\tan\beta$ enhanced contributions in SUSY extensions of the SM for $\mu > 0$ and/or large enough $\tan\beta$ may explain the «missing contribution».

Based on this approach it is generally expected that contributions to the leptonic anomalous magnetic moment are proportional to m_ℓ^2/Λ^2 [9,10], where Λ is the scale of the new physics [1]. It leads to the conclusion that a_μ is more sensitive to new physics. The $m_\mu^2/m_e^2 \simeq 43\,000$ relative enhancement for the muon more than compensates for the factor of $\delta a_\mu^{\mathrm{exp}}/\delta a_e^{\mathrm{exp}} \simeq 2\,250$ current experimental precision advantage of a_e.

In this paper we consider a model which allows one to generate a contribution of the new physics to the leptonic anomalous magnetic moment at *the tree level*. Moreover, the contribution exhibits *a linear* rather than quadratic dependence on m_ℓ. It changes drastically the situation with the relative sensitivity to new physics of the muon versus the electron anomalous magnetic moment. The mass ratio $m_\mu/m_e \simeq 200$ cannot anymore compensate the advantage of $\delta a_e^{\mathrm{exp}}$ over $\delta a_\mu^{\mathrm{exp}}$, which results in a much larger sensitivity of a_e to the new physics than was expected so far.

1. THE MODEL

In this paper we are going to investigate the physical consequences of interacting spin-1 massive bosons described by a formalism of the second-rank antisymmetric tensor fields. The corresponding Lagrangian, which has been successfully used already during more than two decades in the chiral perturbation theory, has the form [12]

$$\mathcal{L}_0^T = -\frac{1}{2}\,\partial^\mu T_{\mu\nu}\partial_\rho T^{\rho\nu} + \frac{1}{4}\,M^2\,T_{\mu\nu}\,T^{\mu\nu}. \tag{7}$$

[1] Although, other models, which lead to a linear dependence on m_ℓ, have been discussed in the past (see [11] and references therein).

Using the canonical formalism, it can be shown [13] that the Lagrangian describes the evolution of the three physical degrees of freedom of the vector (T_{01}, T_{02}, T_{03}), while the three unphysical components of the axial vector (T_{23}, T_{31}, T_{12}) do not propagate and they are frozen.

Although on the mass shell such a description of the spin-1 massive bosons is equivalent to the usual formalism, using vector Proca fields V_μ, off shell they have different unphysical states and can, in general, lead to different physical effects. For example, the gauge-like Yukawa coupling of the vector field to the bilinear vector combination of the fermion fields

$$\mathcal{L}^V_{\rm int} = g_V\,\bar\psi\gamma^\mu\psi V_\mu \tag{8}$$

leads to the well-known static Coulomb interaction due to the exchange of the unphysical degree of freedom V_0. Therefore, the antisymmetric tensor field, possessing a richer structure of the unphysical states than the vector field, can give birth to new physical effects due to its coupling to a corresponding fermion current.

A simple generalization of the Yukawa coupling (8) in the case of the antisymmetric tensor field reads

$$\mathcal{L}^T_{\rm int} = g_T\,\bar\psi\sigma^{\mu\nu}\psi T_{\mu\nu}, \tag{9}$$

where definition of $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ provides the tensor current $\bar\psi\sigma^{\mu\nu}\psi$ to be antisymmetric and hermitian. It is interesting to note that despite intensive utilization of the original Yukawa interactions for describing the Higgs boson couplings or the gauge interactions (8), the interaction (9) still does not have broad phenomenological applications. Here we would like to discuss one of its consequences.

Since the quantum numbers of the physical degrees of freedom of the vector field V_i (here Latin indices run over $i = 1, 2, 3$) and the antisymmetric tensor field T_{0i} are the same, they can mix. Indeed, the quantum loop corrections (see Fig. 1) generate the following additional mixing term:

$$\mathcal{L}^{VT}_{\rm int} = -\frac{1}{2}\,m_\chi\,(\partial^\mu V^\nu - \partial^\nu V^\mu)\,T_{\mu\nu} \tag{10}$$

to the total Lagrangian of the interacting vector and antisymmetric tensor fields. Here

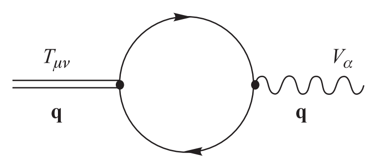

Fig. 1. Mixing between the antisymmetric tensor field and the vector field

$$m_\chi = -i\sum_f \int \frac{d^4p}{(2\pi)^4}\,\frac{8g^f_V g^f_T m_f}{(p^2 - m_f^2)[(p-q)^2 - m_f^2]} \tag{11}$$

is the effective mass parameter, which leads to the nontrivial mixing between the antisymmetric tensor field and the vector field in the case of the chiral symmetry breaking. The summation in (11) is performed by all fermion flavors f, which couple simultaneously to the tensor antisymmetric field and to the vector field, and have also nonzero mass terms $m_f \neq 0$.

An important property of such a mixing consists in the gauge-invariant form of the coupling (10) for the vector field V_μ. This allows one to preserve the gauge invariance of the free Lagrangian

$$\mathcal{L}^V_0 = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} \tag{12}$$

and the zero mass term for the vector field, where as usual $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is the gauge-invariant field strength tensor. The resulting mixing between the antisymmetric tensor field and the vector field is dynamical one, since it depends on the momentum transfer q_μ. In general, it leads to very complicated expressions for the physical states after diagonalization.

In our case it is simplified by the physical conditions of very small momentum transfers, which we are going to discuss. The second simplification comes from an assumption of a smallness of the mixing parameter m_χ in comparison with very heavy boson mass M, so that their ratio is negligibly small. In this case the only dominating term in the Lagrangian, including contributions from (7), (10) and (12), is the mass term from (7) and the procedure of diagonalization consists in a simple rearrangement of the terms

$$\mathcal{L}_0 \;=\; \frac{1}{4}\,M^2\Big(T_{\mu\nu} - \frac{m_\chi}{M^2}F_{\mu\nu}\Big)\Big(T^{\mu\nu} - \frac{m_\chi}{M^2}F^{\mu\nu}\Big) - \frac{1}{4}\,F_{\mu\nu}F^{\mu\nu}\left(1 + \frac{m_\chi^2}{M^2}\right). \qquad (13)$$

Therefore, the physical vector field

$$V'_\mu = V_\mu\sqrt{1 + \frac{m_\chi^2}{M^2}} \qquad (14)$$

is defined up to the normalization factor. However, such a transformation does not lead to a physically observable effect, since it reduces effectively to a redefinition of the coupling constant g_V. On the other hand, the physical antisymmetric tensor field

$$T'_{\mu\nu} = T_{\mu\nu} - \frac{m_\chi}{M^2}F_{\mu\nu} \qquad (15)$$

is defined by the inhomogeneous transformation, which results in the appearance of the anomalous coupling from the interaction (9) and the mixing (10) (see corresponding tree level diagram in Fig. 2)

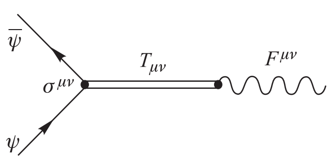

Fig. 2. The tree level diagram for generation of the anomalous magnetic moment of a fermion

$$\mathcal{L}_{\rm int}^{\rm anom} = g_T\frac{m_\chi}{M^2}\,\bar\psi\sigma^{\mu\nu}\psi F_{\mu\nu} \qquad (16)$$

and the corresponding anomalous magnetic moment for the fermion field

$$a_\psi = 4\,\frac{g_T}{g_V}\,\frac{m_\chi}{M^2}\,m_\psi. \qquad (17)$$

2. THE EXPERIMENTAL CONSEQUENCES

In the previous section we have shown that an additional contribution from the new physics to an anomalous magnetic moment of the fermion can be generated at the tree level. The role of a new physics here is played by the nontrivial coupling (9) of the massive spin-1 boson, described by the antisymmetric tensor field, to the fermion tensor current. This coupling leads inevitably to the mixing (10) between the known gauge fields, such as the photon, and the new hypothetical spin-1 heavy boson. The smallness of the mixing parameter m_χ and the heaviness of the new boson mass M could be the reasons why their effects and the direct production of such particles have not been registered up to now.

Probably the only places where such an effect could be tested in low-energy physics are the very precise measurements of the anomalous photon couplings to the leptons, namely electron and muon. Therefore, the difference (6) between the predicted and the measured anomalous magnetic moment of the muon may be explained *completely* by the new mechanism, if the following identification holds:

$$\Delta a_\mu = 4\,\frac{g_T^\mu}{e}\,\frac{m_\chi}{M^2}\,m_\mu. \tag{18}$$

Unfortunately, the only one experimentally measured value cannot fix separately each of the three new parameters g_T^μ, m_χ and M. Nevertheless, our predictions can be more definitive, if we make an additional assumption about the universality of the new Yukawa coupling constant g_T. Let us assume that by an analogy with the gauge coupling g_V, which is the same for different fermion generations, the new coupling g_T also possesses the universality condition

$$g_T = g_T^e = g_T^\mu = g_T^\tau. \tag{19}$$

In this case the contribution of the new physics to the anomalous magnetic moment of the lepton

$$\Delta a_\ell = \kappa m_\ell \tag{20}$$

depends linearly on the lepton mass, where the coefficient

$$\kappa = 4\,\frac{g_T}{e}\,\frac{m_\chi}{M^2} = (25.3 \pm 9.1)\cdot 10^{-12}\ \mathrm{MeV}^{-1} \tag{21}$$

is assumed to be universal for each lepton species.

Therefore, we are in a position now to make a definitive prediction for a new physics effect on the electron anomalous magnetic moment a_e. The linear (20) rather than quadratic dependence on m_ℓ results in a huge effect due to the new physics on the determination of the fine structure constant α via a_e. So, according to formula (20), there should be an additional contribution

$$\Delta a_e = (12.9 \pm 4.6)\cdot 10^{-12} \tag{22}$$

to the anomalous magnetic moment of the electron from the new physics, which is well above the non-QED contributions $a_e^{\mathrm{had}} = 1.671(19)\cdot 10^{-12}$, $a_e^{\mathrm{ew}} = 0.030(01)\cdot 10^{-12}$ [14] and the experimental precision $\delta a_e^{\mathrm{exp}} = 0.28\cdot 10^{-12}$ [2].

If we subtract the additional contribution (22) from the experimentally measured value a_e^{exp} (1), this results in a lower value of the fine structure constant than the extracted one (3). Indeed, we predict that the inverse value of α should be by

$$\Delta\alpha^{-1} = (1.52 \pm 0.55)\cdot 10^{-6} \tag{23}$$

greater than presently accepted (Fig. 3). This prediction will be verified soon by an independent new Cs measurement, which is now in progress. It is designed to obtain the value of α with the relative uncertainty 0.3 ppb [15].

Beside of the description of the absolute value of the difference between the predicted and measured anomalous magnetic moment of the muon, it is interesting also to predict its sign. It could be done in our framework, if we make further assumptions. Let us assume that the new massive boson interacts only with the *down*-type fermions and, by an analogy with the electric charge, all coupling constants g_T^{down} have the same sign. In this case

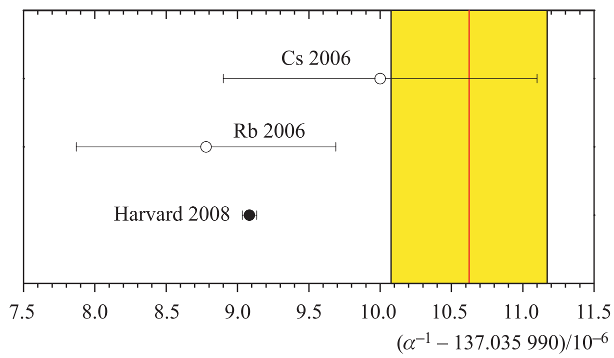

Fig. 3. The most accurate α determination and our prediction

the generated coefficient (11) in the mixing term multiplied by the ratio g_T/e results in the positive constant κ. Therefore, it confirms that the experimental value for the muon anomalous magnetic moment is higher than the predicted one. It is interesting also to note that if the new boson exists and it is not too heavy, $M < 3$ TeV, it may be observed in the Drell–Yan process at the LHC.

CONCLUSIONS

In this paper we have considered the alternative scenario for a contribution by the new physics to the leptonic anomalous magnetic moment. The key role in this scenario belongs to a new massive spin-1 boson, which is described by a second-rank antisymmetric tensor field. The latter has new nonminimal tensor interactions with fermions that lead to its mixing with the photon in the case of a chirally broken symmetry. Therefore, the initial wave functions of the antisymmetric tensor field and the photon can be expressed through linear combinations of their physical states, which results in the appearance of a direct anomalous photon coupling to the fermions at the tree level.

In the case of universality of the new tensor interactions, the contribution of the new physics to the anomalous magnetic moment of the lepton depends linearly on the lepton mass. This leads to a higher sensitivity of the electron anomalous magnetic moment to the new physics than was expected before. The latter fact may substantially affect the extraction of a real value of the fine structure constant from a_e.

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