Письма в ЭЧАЯ. 2009. Т. 6, № 5(154). С. 677–683

ФИЗИКА ТВЕРДОГО ТЕЛА И КОНДЕНСИРОВАННЫХ СРЕД

CAVITY GENERATION AND QUASI-MONOENERGETIC ELECTRON GENERATION IN LASER–PLASMA INTERACTION

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Electron cavity acceleration is one of the relativistic regimes to describe the monoenergetic electron acceleration. In this work, we introduce a new ellipsoid model that could improve the quality of the electron beam in contrast to other methods such as that using periodic plasma wake field, spherical cavity regime and plasma channel-guided acceleration. The trajectory of the electron motion can be described as hyperbola, parabola or ellipsoid path. It is influenced by the position and energy of the electrons and the electrostatic potential of the cavity. We have noticed that the electron output energy is not affected by the elongation of the transverse cavity radius in the ellipsoid regime.

Ускорение с помощью электронного резонатора является одним из релятивистских режимов описания моноэнергетического пучка электронов. В представленной работе рассматривается модель эллипсоида, в которой можно улучшить качество электронного пучка по сравнению с другими методами, такими как использование периодического возбуждения плазмы, режим сферического резонатора и канальное ускорение плазмы. Траекторию движения электрона можно описать гиперболой, параболой или эллипсоидом. Это зависит от положения и энергии электронов и электростатического потенциала резонатора. В работе отмечается, что в эллипсоидальном режиме энергия электрона на выходе не зависит от величины поперечного радиуса резонатора.

PACS: 52.38.-r, 41.75.Jv, 42.62.-b, 41.75.Lx

INTRODUCTION

In the last decade, with CPA table-top lasers, the laser intensities increased up to $I = 10^{22}$ W/cm^2, and electric field strengths of more than 10^{14} V/m were obtained [1]. Generation of laser pulses in the multi-terawatt (or even pettawatt) power range is possible with compact chirped-pulse amplification (CPA) systems, and the extreme light infrastructure (ELI) will be able to generate intensities in the range of $10^{25}-10^{26}$ W/cm^2 [2]. In these high gradient fields particles can be accelerated. One of the most important applications for such short ultra-intense laser pulses is the acceleration of charged particles, both electrons and ions [3–8]. This new generation of particle accelerators could be used for various applications,

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including transmutation of cheep and hazardous materials of long-lived radioactive wastes to valuable radioisotopes [9]. The extremely high electric field makes the laser wake field acceleration method attractive for the development of a new generation of accelerators [10]. The ponderomotive force associated in the front and the rear sides of a short laser pulse expels the plasma electrons from the regions where the laser field is the most intense [11] by the laser wake field scheme. The ponderomotive force F_p is given by $F_p \approx -\nabla a^2$, where a is the laser pulse envelope. The acceleration gradient resulting from the charge displacement is reported to be about 100 GV/cm when plasma density is 10^{18} cm^{-3} [12].

The induced charge separation between the electrons and the ions gives rise to a space charge field and a plasma wave [13]. Large amplitude plasma waves are generated by this ponderomotive force in the laser wake field accelerator (LWFA).

In the linear regime, this mechanism is more efficient when the pulse duration of the laser is of the order of the plasma frequency, and it is called resonant wake field. The resonant wake field regime creates a controllable and linear accelerating structure.

In nonlinear regimes, background plasma electrons can be trapped in the plasma wave bucket and accelerated up to GeV [14–18]. The self-modulated laser wake field [19] (SMLWF) and the forced laser wake field (FLWF) [20] are the well-known nonlinear regimes. In the SMLWF regime the envelope of the laser pulse can modulate at the plasma wave period and drives the wake field properly with its ponderomotive force via Raman forward scattering instability. In the FLWF regime, the laser pulse is compressed by group velocity dispersion during the excitation of the plasma wave and drives the plasma wave to very high amplitude. Electron injection in the correct phase of the plasma wave could improve the electron acceleration efficiency. Recently injection of the background plasma electrons is reported instead of an external electron injector (e.g., a lilac) when wave breaking occurs [21].

By using the steepened density profile, the wave-breaking injection can be fast. A well collimated, ultra short MeV electron bunch is obtained due to the transverse wave breaking [22, 23] by using a shock wave driven with the irradiation of laser prepulses [24].

In recent experiments and PIC simulations [25], generation of quasi-monoenergetic electrons has been reported. A cavity (bubble) free of cold plasma electrons behind the laser pulse is observed [26]. The following features are absent in the ordinary regime of laser wake field acceleration [27–30]: (i) a cavity free from cold plasma electrons is formed behind the laser pulse instead of a periodic plasma wave; (ii) a dense bunch of relativistic electrons with a quasi-monoenergetic spectrum is self-generated; (iii) the laser pulse propagates many Rayleigh lengths in the homogeneous plasma without a significant diffraction. The cavity behind the laser pulse is shown by shadowgraphs [31] and PIC simulation [32, 26]. Quasi-monoenergetic electron beams were generated from intense laser pulses in various gas targets [33]. We have described analytically the new ellipsoid model [34] to be used instead of the previous spherical cavity. In fact, the cavity shape is not exactly a sphere, and this is a defect in previous works [27, 28]. Some deviations between shadowgraphs, PIC simulation and analytical calculation results are reported because of the spherical estimation for the cavity shape. Appropriate conditions of forming an ellipsoid cavity are obtained. We have evaluated fields inside of this cavity and the energy spectrum for relativistic trapped electrons, and obtain energy and electron gain when self-focusing is considered.

1. ELECTROSTATIC POTENTIAL IN A PLASMA ELLIPSOID CAVITY

We have considered an electrically neutral bulk plasma ellipsoid cavity. The volume of the ellipsoid with axes $2a_e, 2b_r$ and $2c_e$ is $V = 4\pi a_e b_e c_e/3$, the total charge of the ions is enV (here $e > 0$ is the value of the electron charge, n is the number of ions per unit volume of a sample). The electrostatic potential inside a uniformly charged ellipsoid (with total charge enV), $\psi(x, y, z)$, is calculated in volt units as [35]

$$\psi(x,y,z) = \left(\frac{enV}{c_{\text{cav}}}\right) + \left[\frac{3ena_e b_e c_e V}{2\varepsilon_0[(a_e b_e)^2 + (a_e c_e)^2 + (b_e c_e)^2]}\right] \left[1 - \left(\frac{x}{a_e}\right)^2 - \left(\frac{y}{b_e}\right)^2 - \left(\frac{z}{c_e}\right)^2\right], \quad (1)$$

where c_{cav} is the electrical capacity of the plasma in vacuum and ε is the cavity dielectric constant. One can see that Eq. (1) satisfies the Poisson equation,

$$\nabla\varphi = -\frac{4\pi en}{\varepsilon}, \quad (2)$$

and all necessary boundary conditions. We normalized the potential to unity at the ellipsoid boundary,

$$\varphi(x,y,z) = 1 + \varphi_0 \left[1 - \left(\frac{x}{a_e}\right)^2 - \left(\frac{y}{b_e}\right)^2 - \left(\frac{z}{c_e}\right)^2\right], \quad (3)$$

where in this equation

$$\varphi_0 = \frac{3a_e b_e c_e c_{\text{cav}}}{2\varepsilon_0[(a_e b_e)^2 + (a_e c_e)^2 + (b_e c_e)^2]}. \quad (4)$$

If we assume $a_e b_e c_e = R^3$, for spheroid ellipsoid $(a_e > b_e = c_e)$

$$\varphi_0 = \frac{3a_e b_e^2 c_{\text{cav}}}{2\varepsilon_0[2(a_e b_e)^2 + b_e^4]} = \frac{3a_e c_{\text{cav}}}{2\varepsilon_0(2a_e^2 + b_e^2)}. \quad (5)$$

Eccentricity is

$$E^2 = \frac{a_e^2 - b_e^2}{a_e^2} \quad (6)$$

and $a_e b_e^2 = R^3$, according to [35] we have

$$C_{\text{cav}} = \frac{2RE}{(1-E^2)^{1/3}\ln[(1+E)/(1-E)]}. \quad (7)$$

Then

$$c_{\text{cav}} = \varepsilon_0 C_{\text{cav}} = \frac{2\varepsilon_0\sqrt{a_e^2 - b_e^2}}{\ln\left[\dfrac{a_e + \sqrt{a_e^2 - b_e^2}}{a_e - \sqrt{a_e^2 - b_e^2}}\right]}, \quad (8)$$

and

$$\varphi_0 = \frac{3a_e}{(2a_e^2 + b_e^2)} \frac{\sqrt{a_e^2 - b_e^2}}{2\sqrt{1 - \dfrac{b_e^2}{a_e^2}}} = \frac{3a_e^2}{(2a_e^2 + b_e^2)}. \tag{9}$$

Then we obtain

$$\varphi(x, y, z) = 1 + \left(\frac{3a_e^2}{(2a_e^2 + b_e^2)}\right) \left[1 - \left(\frac{x}{a_e}\right)^2 - \left(\frac{y}{b_e}\right)^2 - \left(\frac{z}{c_e}\right)^2\right]. \tag{10}$$

2. FIELDS INSIDE RELATIVISTIC ELLIPSOID CAVITY

We consider a cavity moving in plasma. Ions are immobile in the cavity while the cavity runs with the relativistic velocity $v_0 \approx 1$ along x axis. The ion dynamics are neglected because the cavity dimensions are assumed to be smaller than the ion response length $\approx c/\omega_{\text{pi}}$, where $\omega_{\text{pi}} = (4\pi\, e^2 n_0/M)$ the ion plasma frequency and M is the ion mass. To calculate the fields, we write the Maxwell equations in terms of potentials using the following convenient gauge $A_x = -\varphi$ and we get [30]

$$\Delta\Phi = 1 - n\left(1 - \frac{p_x}{\gamma}\right) + \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right)(\nabla A) + \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)\Phi, \tag{11}$$

$$\nabla \times \nabla \times A + n\frac{p}{\gamma} + \frac{\partial}{\partial t}\left(\frac{\partial A}{\partial t} - \frac{\nabla\Phi}{2}\right) = 0. \tag{12}$$

Here we use the wake field potential $\Phi = A_x - \varphi$ instead of the scalar one, and p is the electron momentum. We use dimensionless units, normalizing the time to ω_p^{-1}, the lengths to c/ω_p, the velocity to c, the electromagnetic fields to $mc\omega_p/|e|$ and the electron density n to the background density n_0. If we use a quasi-static approximation assuming that all quantities depend on $\xi = x - v_0 t$ instead of x and t, the Maxwell equations reduce to the form

$$\Delta\Phi = \frac{3}{2}(1 - n) - n\frac{p_x}{\gamma} - \frac{1}{2}\frac{\partial}{\partial\zeta}(\nabla_\perp \cdot A_\perp), \tag{13}$$

$$\Delta_\perp A_\perp - \nabla_\perp(\nabla_\perp \cdot A_\perp) = n\frac{p_\perp}{\gamma} + \frac{1}{2}\nabla_\perp\frac{\partial\Phi}{\partial\zeta}. \tag{14}$$

We have neglected the terms proportional to $\gamma_0^{-2} \ll 1$. Inside the cavity we have $(n = 0)$, then we get

$$\Delta\Phi = \frac{3}{2} - \frac{1}{2}\frac{\partial}{\partial\zeta}(\nabla_\perp \cdot A_\perp), \tag{15}$$

$$\Delta_\perp A_\perp - \nabla_\perp(\nabla_\perp \cdot A_\perp) = \frac{1}{2}\nabla_\perp\frac{\delta\Phi}{\delta\zeta}. \tag{16}$$

We have solved Eqs. (15) and (16) with spherical symmetry and obtained

$$\Phi = 1 - \Phi_0\left(1 - \frac{\zeta^2}{a_e^2} - \frac{y^2}{b_e^2} - \frac{z^2}{c_e^2}\right), \tag{17}$$

where $\Phi_0 = \dfrac{3}{\dfrac{4}{a_e^2}+\dfrac{4}{b_e^2}+\dfrac{4}{c_e^2}}$, when $A_\perp = 0$ and $A_x = -\varphi = \frac{\Phi}{2}$.

So we obtain

$$\Phi = 1 - \left[\frac{3}{\dfrac{4}{a_e^2}+\dfrac{4}{b_e^2}+\dfrac{4}{c_e^2}}\left(1-\frac{\zeta^2}{a_e^2}-\frac{y^2}{b_e^2}-\frac{z^2}{c_e^2}\right)\right]. \tag{18}$$

3. ENERGY OF ELECTRONS

The energy of electrons in an ellipsoid cavity can be derived using the Hamiltonian formulation. The one-dimensional Hamiltonian of a charged particle in an electromagnetic field is

$$H = \sqrt{1+(p_c+A)^2+a^2} - v_0 p_c - \varphi, \tag{19}$$

where p_c is the particle canonical momentum and φ is the scalar potential. Hamiltonian can be split into two parts by expanding it in the power of p_c^2. The first part determines the longitudinal motion and the second part determines the transverse motion. We obtain the longitudinal Hamiltonian and consider only the x dimension:

$$\bar{H}_{||} = \sqrt{1+p_x^2} - v_0 p_{\zeta,c} - \Phi \approx 0. \tag{20}$$

If we assume an ellipsoid cavity in plasma with axes $2a_e$, $2b_e$ and $2c_e$

$$H_{||} = \sqrt{1+P_x^2} - v_0 p_{\zeta,c} - 1 + \Phi_0\left(1-\frac{\zeta^2}{a_e^2}\right). \tag{21}$$

In the zeroth order we obtain the longitudinal Hamiltonian

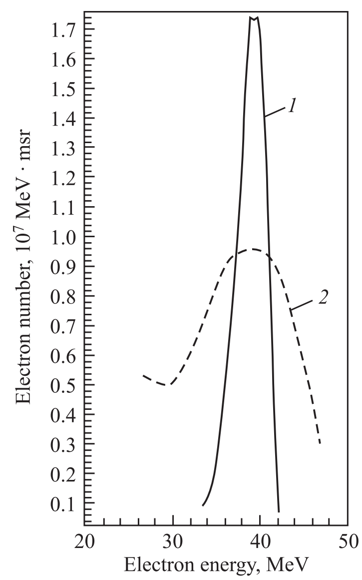
$$H_{||} = \frac{P_x}{2\gamma_0^2} - 1 - \Phi_0\left(1-\frac{\zeta^2}{a_e^2}\right) \approx 0. \tag{22}$$

With solving the Hamiltonian equation, the maximum energy of the accelerated peaks at the cavity center

$$\gamma_{\max} = 2\gamma_0^2 + 2\gamma_0^2\Phi_0\left(1-\frac{\zeta^2}{a_e^2}\right). \tag{23}$$

By substituting the $\Phi_0 = \dfrac{3}{\dfrac{4}{a_e^2}+\dfrac{4}{b_e^2}+\dfrac{4}{c_e^2}}$ in Eq. (23) for cavity center we obtain

$$\gamma_{\max} = 2\gamma_0^2 + \frac{3\gamma_0^2}{\dfrac{2}{a_e^2}+\dfrac{2}{b_e^2}+\dfrac{2}{c_e^2}}, \tag{24}$$



Comparison of ellipsoid (curve *1*, Eq. (25)) and spherical (curve *2*, Eq. (26)) cavity model. This figure shows that the electron beam in the ellipsoid model has a narrower energy distribution where for elongation case when $(a_e > b_e = c_e)$:

$$\gamma_{\max} = 2\gamma_0^2 + \frac{3\gamma_0^2 b_e^2}{4}. \tag{25}$$

We have found that cavity elongation in laser propagation direction is not effective on electron energy and for other directions; the elongations are small and can be neglected.

Initial condition to obtained ellipsoid cavity is defined by laser–plasma parameters. In conclusion it can be considered that the electrons of the bunch have equal energy and the ellipsoid cavity holds the electron bunch in quasi-monoenergetic situation better than previous spherical models. In spherical model, energy of the accelerated electrons peak is given by [30]

$$\gamma_{\max} \approx \frac{1}{2}\gamma_0^2 R^2. \tag{26}$$

In this equation R is radius of spherical bubble and γ_0 is defined by $\gamma_0 = (1 - v_0)^{-1/2}$, where v_0 is the laser pulse group velocity [14]. As Eq. (26) shows, electron bunch energy is strongly related to the transverse radius of cavity and during the laser propagation because of transverse elongation of cavity the energy peak spectrum will spread but in ellipsoid model. Equation (25) shows that the longitudinal elongation (a_e) is not effective on energy spectrum of electron bunch. The result electron beam in ellipsoid model will be quasi-monoenergetic (see figure).

CONCLUSIONS

In the present work we derived analytical expressions for the fields within an ellipsoid cavity moving at relativistic velocity in plasma. Our analytical model is in agreement with the PIC results. We derived the maximum energy of electrons. We showed that the fields linearly depend on the coordinates as in the spherical we have shown that the cavity elongation has not affected maximum electron energy so the quality of electron beam is developed.

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Received on April 12, 2008.