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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

PHOTOPRODUCTION OF SCALAR AND PSEUDOSCALAR MESONS ON A LEPTON WITHIN THE LOCAL NAMBU–JONA-LASINIO MODEL

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Using the description of the subprocess $\gamma\gamma^* \to S(P)$ in terms of local Nambu–Jona-Lasinio model, we calculate the cross sections of photoproduction of scalar and pseudoscalar mesons in high-energy photon–lepton collision processes. The dependence on the transversal momentum and the total cross sections in Weizsäcker–Williams approximation are presented.

Используя описание подпроцесса $\gamma\gamma^* \to S(P)$ в рамках локальной модели Намбу–Йона-Лазинио, мы вычисляем сечения фоторождения скалярных и псевдоскалярных мезонов в процессах столкновения фотонов с лептонами при высоких энергиях. Представлены зависимости сечений от поперечного переданного импульса, а также даны значения полных сечений фоторождения в приближении Вайцзеккера–Вильямса.

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INTRODUCTION

Photoproduction of mesons on a lepton is the cross channel for the processes of the radiative production of a single meson (scalar or pseudoscalar) in electron–positron annihilation [1]. Production of the pseudoscalar mesons in photon–lepton collisions can be considered as an experiment of kind of Primakoff one: photoproduction of the neutral pion in the Coulomb field of a heavy nuclei. It is a rather difficult problem to measure the Primakoff effect using as a target the field of proton. The reason is huge background effects connected with the hadron structure of proton [2]. When one considers the lepton as a target, the background is absent. We note that using as a target the electrons of a matter implies using very energetic photons since the threshold photon energy is $E > M_\pi^2/(2m_e) \sim 20$ GeV. Using the muon as a target permits one to use the photons with energy of several GeV when producing such heavy mesons as $a_0(980)$, $f_0(980)$, $\sigma(600)$, η'.

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1. THE LOCAL NAMBU–JONA-LASINIO MODEL

The local Nambu–Jona-Lasinio (NJL) model describes the interaction of quarks with mesons by the Lagrangian [3, 4]:

$$\mathcal{L}_{\rm int} = \bar{q}\Big[eQ\hat{A} + g_{\sigma_u}\lambda_u\sigma_u + g_{\sigma_s}\lambda_s\sigma_s + g_{\sigma_u}\lambda_3 a_0 + \\ + i\gamma_5 g_\pi\left(\lambda_{\pi^+}\pi^+ + \lambda_{\pi^-}\pi^- + \lambda_3\pi^0\right) + i\gamma_5 g_K\left(\lambda_{K^+}K^+ + \lambda_{K^-}K^-\right) + \\ + i\gamma_5\left(g_\pi\lambda_u\eta_u + g_{\eta_s}\lambda_s\eta_s\right)\Big]q, \quad (1)$$

where $\bar{q} = \left(\bar{u}, \bar{d}, \bar{s}\right)$, and u, d, s are the quark fields; $Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$ is the quark electric charge matrix; e is the elementary electric charge ($e^2/4\pi = \alpha = 1/137$), $\lambda_u = \left(\sqrt{2}\lambda_0 + \lambda_8\right)/\sqrt{3}$, $\lambda_s = \left(-\lambda_0 + \sqrt{2}\lambda_8\right)/\sqrt{3}$, $\lambda_{\pi^\pm} = (\lambda_1 \pm i\lambda_2)/\sqrt{2}$, $\lambda_{K^\pm} = (\lambda_4 \pm i\lambda_5)/\sqrt{2}$, where λ_i are the well-known Gell-Mann matrices and $\lambda_0 = \sqrt{2/3}\,\mathrm{diag}\,(1,1,1)$. The coupling constants from the Lagrangian (1) are defined in the following way [3]:

$$g_{\sigma_u} = \left(4I^\Lambda\left(m_u, m_u\right)\right)^{-1/2} = 2.43,$$
$$g_{\sigma_s} = \left(4I^\Lambda\left(m_s, m_s\right)\right)^{-1/2} = 2.99,$$
$$g_\pi = \frac{m_u}{F_\pi} = 2.84,$$
$$g_K = \frac{m_u + m_s}{2F_K} = 3.01,$$
$$g_{\eta_s} = \frac{m_s}{F_s} = 3.37,$$

where $m_u = m_d = 263$ MeV, $m_s = 406$ MeV are the constituent quark masses, for g_π and g_K constants we used the Goldberger–Treiman relation, $F_\pi = 92.5$ MeV, $F_s = 1.3\ F_\pi$ and $F_K = 1.2\ F_\pi$, and $I^\Lambda\left(m, m\right)$ is the logarithmically divergent integral which has the form

$$I(m, m) = \frac{N_c}{(2\pi)^4}\int d^4k \frac{\theta\left(\Lambda^2 - k^2\right)}{\left(k^2 + m^2\right)^2} = \frac{N_c}{(4\pi)^2}\left(\ln\left(\frac{\Lambda^2}{m^2} + 1\right) - \frac{\Lambda^2}{\Lambda^2 + m^2}\right), \quad N_c = 3.$$

This integral is written in the Euclidean space. The cut-off parameter $\Lambda = 1.27$ GeV and constituent quark masses were taken from [1, 3].

The scalar isoscalar mesons f_0, σ are the mixtures of pure u-, d-quarks scalar state σ_u and pure s-quark scalar state σ_s:

$$f_0 = \sigma_u \sin\alpha + \sigma_s \cos\alpha, \\ \sigma = \sigma_u \cos\alpha - \sigma_s \sin\alpha, \quad (2)$$

where mixing angle is $\alpha = 11.3°$ [4–6]. The pseudoscalar mesons η, η' are the mixtures of pure u-, d-quarks scalar state η_u and pure s-quark scalar state η_s:

$$\eta = -\eta_u \sin\theta + \eta_s \cos\theta, \\ \eta' = \eta_u \cos\theta + \eta_s \sin\theta, \quad (3)$$

where mixing angle is $\theta = 51.3°$ [3, 7].

2. GENERAL CONSIDERATION OF PHOTOPRODUCTION PROCESSES

We will consider the processes of photoproduction of scalar and pseudoscalar mesons on lepton target

$$\begin{aligned} &\gamma(k) + \mu(P) \to S(p_s) + \mu(P'), \qquad && S = a_0, f_0, \sigma, \\ &\gamma(k) + \mu(P) \to P(p_p) + \mu(P'), \qquad && P = \pi_0, \eta, \eta', \end{aligned} \tag{4}$$

$$k^2 = 0, \quad P^2 = P^{'2} = m^2, \quad p_s^2 = M_S^2, \quad p_p^2 = M_P^2, \quad s = 2(kP) = 2mE,$$

where E is the energy of incident photon. Within the local NJL model, matrix elements of these processes have the form

$$\mathcal{M}^{\gamma\mu\to S\mu} = \frac{(4\pi\alpha)^{3/2}}{2\pi^2} J_\alpha(P, P') \frac{g^{\alpha\mu}}{q^2} \epsilon^\nu(k) \left(g_{\mu\nu}(kq) - k_\mu q_\nu\right) \rho^S, \tag{5}$$

$$\mathcal{M}^{\gamma\mu\to P\mu} = \frac{(4\pi\alpha)^{3/2}}{2\pi^2} J_\alpha(P, P') \frac{g^{\alpha\mu}}{q^2} \epsilon^\nu(k) (\nu, k, \mu, q) \rho^P, \tag{6}$$

where we use the notation $(a, b, c, d) = \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$, $q = P - P'$ is the transferred momentum, and $J^\mu(P, P') = \bar{u}(P') \gamma^\mu u(P)$ is the electromagnetic current of lepton target. The quantities $\rho^{S,P}$ encode the contributions of Feynman diagrams with both the quark loops and meson loops and will be considered in Sec. 3.

We consider these processes in the peripheral kinematics ($s \gg q^2$), which gives the main contribution to the total cross section. For this aim we introduce the light-like auxiliary 4-vector

$$\begin{gathered} \tilde{P} = P - k\frac{m^2}{s}, \\ \tilde{P}^2 = 0, \quad 2\tilde{P}P = m^2, \quad 2\tilde{P}k = s, \end{gathered} \tag{7}$$

and use the Sudakov decomposition of transferred momentum 4-vector q:

$$q = \alpha\tilde{P} + \beta k + q_\perp, \quad q_\perp^2 = -\mathbf{q}^2 < 0, \tag{8}$$

where $\mathbf{q}$ is the Euclidean two-dimensional vector perpendicular to two light-like vectors k, $\tilde{P}$ ($q_\perp k = q_\perp \tilde{P} = 0$). The square of the 4-vector of the transfer momentum q, using the on-mass shell conditions of the target lepton, and the scalar (pseudoscalar) mesons can be written in the form

$$q^2 = -\frac{\mathbf{q}^2 + m^2\alpha^2}{1-\alpha} \approx -\left(\mathbf{q}^2 + q_{\min}^2\right), \quad q_{\min}^2 = \frac{m^2 M_{S,P}^4}{s^2}. \tag{9}$$

The phase volume of final particles then reads as

$$\begin{aligned} d\Gamma = \frac{d^3 p_{s,p}}{2E_{s,p}} \frac{d^3 P'}{2E'} \frac{1}{(2\pi)^2} \delta^4(k + P - p_{s,p} - P') = \\ = \frac{1}{(2\pi)^2} \delta^4(P + k - p_{s,p} - P') \delta^4(P - q - P') d^4 p_{s,p} d^4 P' d^4 q \times \\ \times \delta\left((k+q)^2 - M_{S,P}^2\right) \delta\left((P-q)^2 - m^2\right) = \\ = \frac{1}{(2\pi)^2} \frac{d^2 q}{2s} d(s\alpha)\, d(s\beta)\, \delta\left(s\beta(1-\alpha) + \mathbf{q}^2 + m^2\alpha\right) \delta\left(q^2 + s\alpha - M_{S,P}^2\right). \end{aligned} \tag{10}$$

After integration over the Sudakov parameters α, β we obtain

$$d\Gamma = \frac{d^2q}{8\pi^2 s}. \tag{11}$$

The next standard step is to use the Gribov representation for the Green function of the virtual photon, omitting terms which are not enhanced by s:

$$g^{\alpha\mu} \approx \frac{2}{s}\tilde{P}^{\mu}k^{\alpha}. \tag{12}$$

The other components of the metric tensor contribution vanish in the limit $s \gg M_s^2$, which is implied in peripheral kinematics. Then the square of matrix element modulus is

$$\sum \left|\mathcal{M}^{\gamma\mu\to(S,P)\mu}\right|^2 = \frac{32\alpha^3 s^2 \mathbf{q}^2}{\pi(q^2)^2}\left|\rho^{S,P}\right|^2. \tag{13}$$

The differential cross section then has the form

$$\frac{d\sigma^{\gamma\mu\to(S,P)\mu}}{d\mathbf{q}^2} = \frac{\alpha^3\mathbf{q}^2}{2\pi^2\left(\mathbf{q}^2+q_{\min}^2\right)^2}\left|\rho^{S,P}\right|^2. \tag{14}$$

3. DEFINITE CHANNELS

Now we can calculate quantities $\rho^{S,P}$ for different channels of photoproduction of different mesons. The quantities ρ^S have the contributions from the quark and the meson loops and ρ^P have only the contributions from quark loops. Let us consider first the process of a_0 meson production $\gamma+\mu\to a_0(980)+\mu$. Charge-color factor associated with the u, d quarks is $3(4/9+1/9)=5/3$. The result is

$$\rho^{a_0} = \frac{5g_{\sigma_u}}{3m_u}I_u^{a_0} + \frac{g_{a_0K^+K^-}}{M_K^2}I_K^{a_0}, \tag{15}$$

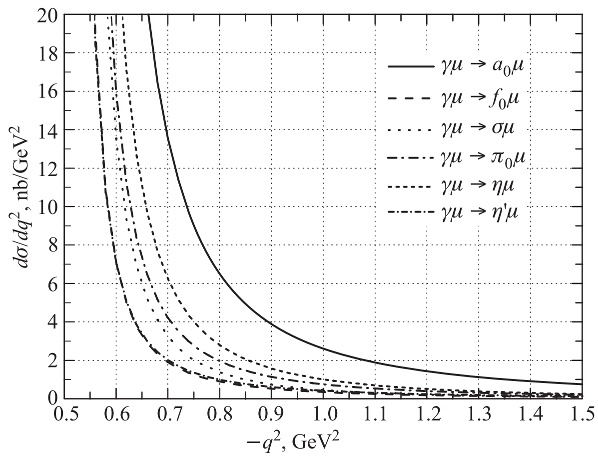
where integrals corresponding to quark and meson loops are [9]

$$\begin{gathered}
I_q^S = \mathrm{Re}\left(\int_0^1 dx\int_0^{1-x}\frac{dz\,(-1+4xz)}{1-zx\dfrac{M_S^2}{m_q^2}+xy\dfrac{\mathbf{q}^2}{m_q^2}}\right), \qquad y = 1-x-z,\\
I_M^S = \int_0^1 dx\int_0^{1-x}\frac{dz\,(-xz)}{1-zx\dfrac{M_S^2}{M^2}+xy\dfrac{\mathbf{q}^2}{M^2}},
\end{gathered} \tag{16}$$

where m_q and M are the masses of quarks and mesons circulating in the loop.

For the process $\gamma+\mu\to f_0(980)+\mu$ we obtain

$$\rho^{f_0} = \left[\frac{5g_{\sigma_u}}{3m_u}I_u^{f_0} - 4\frac{m_u}{M_\pi^2}\frac{g_\pi^2}{g_{\sigma_u}}I_\pi^{f_0} + \frac{g_{a_0K^+K^-}}{M_K^2}I_K^{f_0}\right]\sin\alpha + \\ + \left[\frac{1}{3}\frac{g_{\sigma_s}}{m_s}I_s^{f_0} + \frac{g_{\sigma_sK^+K^-}}{M_K^2}I_K^{f_0}\right]\cos\alpha \tag{17}$$



The dependence of cross sections $\frac{d\sigma^{\gamma\mu\to(S,P)\mu}}{d\mathbf{q}^2}$ (14) for different channels on transfer momentum square $\mathbf{q}^2$

and finally for $\gamma + \mu \to \sigma(600) + \mu$ we obtain

$$\rho^{\sigma(600)} = \left[\frac{5g_{\sigma_u}}{3m_u} I_u^{f_0} - 4\frac{m_u}{M_\pi^2}\frac{g_\pi^2}{g_{\sigma_u}} I_\pi^{f_0} + \frac{g_{a_0 K^+ K^-}}{M_K^2} I_K^{a_0}\right]\cos\alpha - \\ - \left[\frac{1}{3}\frac{g_{\sigma_s}}{m_s} I_s^{f_0} + \frac{g_{\sigma_s K^+ K^-}}{M_K^2} I_K^{a_0}\right]\sin\alpha. \quad (18)$$

The defined channels of the photoproduction of pseudoscalars $P = \pi^0, \eta, \eta'$ have the form

$$\rho^{\pi^0} = \frac{5}{3}\frac{1}{F_\pi} J_u^{\pi^0}, \quad (19)$$

$$\rho^{\eta} = -\sin\theta \frac{5}{3}\frac{1}{F_\pi} J_u^{\eta} + \cos\theta \frac{1}{3}\frac{1}{F_s} J_s^{\eta}, \quad (20)$$

$$\rho^{\eta'} = \cos\theta \frac{5}{3}\frac{1}{F_\pi} J_u^{\eta'} + \sin\theta \frac{1}{3}\frac{1}{F_s} J_s^{\eta'}, \quad (21)$$

where loop integrals J_q^P have the form

$$J_q^P = \mathrm{Re}\left(\int_0^1 dx \int_0^{1-x} \frac{dz}{1 - zx\frac{M_P^2}{m_q^2} + xy\frac{\mathbf{q}^2}{m_q^2}}\right), \quad y = 1 - x - z. \quad (22)$$

The dependence of cross sections for different channels on transfer momentum square $\mathbf{q}^2$ is shown in the figure.

The total cross section in the Weizsäcker–Williams approximation has the form

$$\sigma^{\gamma\mu\to(S,P)\mu} = \frac{\alpha^3 L}{2\pi^2}\left|\rho^{S,P}(0)\right|^2, \quad L = 2\ln\left(\frac{q_{\max}^2}{q_{\min}^2}\right) = 2\ln\left(\frac{s}{mM}\right) \approx 10, \quad s \approx 5\ \mathrm{GeV}^2, \quad (23)$$

where L is the big logarithm which enhances the cross section in peripheric kinematics. The matrix element square $\rho^{S,P}(0)$ in this approximation for different channels has the form

$$\begin{aligned} \rho^{\pi^0}(0) &= -9.22\ \text{GeV}^{-1}, & \rho^{a^0}(0) &= -10.6\ \text{GeV}^{-1}, \\ \rho^{\eta}(0) &= 14.4\ \text{GeV}^{-1}, & \rho^{\sigma}(0) &= -12.5\ \text{GeV}^{-1}, \\ \rho^{\eta'}(0) &= -3.52\ \text{GeV}^{-1}, & \rho^{f^0}(0) &= -5.14\ \text{GeV}^{-1}. \end{aligned} \tag{24}$$

4. RESULTS AND DISCUSSION

The total cross sections of processes $\gamma\mu \to (S,P)\mu$ in the Weizsäcker–Williams approximation for $\sqrt{s} > 3$ GeV are

$$\begin{aligned} \sigma^{\gamma\mu\to\pi^0\mu} &= 5.6\ \text{nb}, & \sigma^{\gamma\mu\to\eta\mu} &= 13.8\ \text{nb}, & \sigma^{\gamma\mu\to\eta'\mu} &= 0.8\ \text{nb}, \\ \sigma^{\gamma\mu\to a_0\mu} &= 7.6\ \text{nb}, & \sigma^{\gamma\mu\to\sigma\mu} &= 10.4\ \text{nb}, & \sigma^{\gamma\mu\to f_0\mu} &= 1.7\ \text{nb}. \end{aligned} \tag{25}$$

Similar estimations will be valid for proton or nuclei instead of muon. The question about the background events is important in this case. The background will be mainly provided by other mechanisms of meson production when the initial photon interacts directly with the target. Kinematics of the processes considered above is quite different — really the energy of produced mesons is large (of order of the initial photon) and the cross sections practically do not depend on $\sqrt{s}$. These features of these events can be used to suppress the background.

We note that our results do not agree with the asymptotic behavior of transition form factor obtained in [8]. Really in [8] the modification of loop integrals was done; i.e., the cut was introduced to avoid large logarithm $\ln\left(q^2/M^2\right)$ type contribution. In our case the characteristic momenta of loop integral are of order $\left|k^2\right| \sim \left|q^2\right|$ and we do not put any cut. For $\mathbf{q}^2 \to 0$ our results are in agreement with current algebra ones.

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