Письма в ЭЧАЯ. 2009. Т. 6, № 6(155). С. 763–768

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

CHERENKOV EFFECT IN THE WEAK INTERACTIONS GENERATED BY THE NEUTRINOS AND NEW APPROACH FOR ESTIMATION OF NEUTRINO MASS

Kh. M. Beshtoev[1]

Joint Institute for Nuclear Research, Dubna

It is shown that if weak interactions can generate masses and polarize matter, then the Cherenkov effect induced by these interactions at $v_\nu > c/n$ appears. The effect of (resonance) enhancement of neutrino oscillations in matter ($v_\nu < c/n$) and the Cherenkov ($v_\nu > c/n$) effect are competitive processes and at definite neutrino energies the effect of (resonance) enhancement of neutrino oscillations in matter will change to the Cherenkov effect. Then neutrino vacuum oscillations will regenerate and we obtain an excellent possibility of estimating neutrino masses. And knowing estimation of mass for one (electron) neutrino we can obtain masses of the rest neutrinos by using the values for mass differences for neutrinos obtained in oscillation experiments.

Показано, что если слабые взаимодействия могут генерировать массы и поляризовать вещество, то при скоростях нейтрино $v_\nu > c/n$ появляется черенковский эффект, индуцированный этими взаимодействиями. Эффект резонансного усиления осцилляций нейтрино в веществе ($v_\nu < c/n$) и черенковский эффект ($v_\nu > c/n$) являются конкурирующими процессами, и при определенных энергиях нейтрино-эффект резонансного усиления осцилляций нейтрино в веществе меняется на черенковский эффект. Тогда вакуумная осцилляция нейтрино восстанавливается и появляется возможность получения оценки массы нейтрино. Далее, зная оценку массы одного нейтрино, мы можем получить оценку на массы остальных нейтрино, используя разности масс нейтрино, полученные в осцилляционных экспериментах.

PACS: 14.60.Pq; 14.60.Lm

INTRODUCTION

At present the existence of three types of neutrinos — electron (ν_e), muon (ν_μ) and tau (ν_τ) neutrinos — is established [1]. Determination of masses of these neutrinos is of great interest. Experiments were carried out to estimate the electron neutrino mass with using the beta decay [2]. Also, experiments on neutrinoless double-beta decays were conducted to estimate neutrino masses on the assumption that neutrinos are the Majorana particles [3]. In addition, mass differences between ν_1, ν_2, ν_3 were measured in the neutrino oscillation experiments [4], but in these experiments it is impossible to establish neutrino masses.

[1]E-mail: beshtoev@cv.jinr.ru

The suggestion that, by analogy with $K^0, \bar{K}^0$ oscillations, there could be neutrino–antineutrino oscillations ($\nu \to \bar{\nu}$), was considered by Pontecorvo in 1957 [5]. It was subsequently considered by Maki et al. [6] and Pontecorvo [7] that there could be mixings (and oscillations) of neutrinos of different flavors (i.e., $\nu_e \to \nu_\mu$ transitions). Then the resonance mechanism of neutrino oscillations in matter [8] was assumed which implied that as neutrinos passed through matter, enhancement of neutrino oscillations took place since effective masses of neutrinos change and at definite matter density they can be equal.

This work is devoted to consideration of the Cherenkov radiation of neutrinos in matter and to a new approach for estimation of the neutrino mass.

The Cherenkov radiation can appear only when neutrinos move in matter, where $n_i - 1 > 0$ with velocity $v_i > c/n_i$, $i = \nu_e, \nu_\mu, \nu_\tau$, n_i is refraction index. But at the neutrino velocity $v_i < c/n_i$ resonance enhancement of neutrino oscillations in matter may take place. Therefore, before considering the Cherenkov effect, we give elements of the resonance effect.

1. ELEMENTS OF THE RESONANCE MECHANISM ENHANCEMENT OF NEUTRINO OSCILLATIONS IN MATTER

Before consideration of the resonance mechanism it is necessary to gain an understanding of the physical nature of origin of this mechanism. As neutrinos pass through matter, there can be two processes: neutrino scattering and polarization of the matter by neutrinos. Obviously, resonance enhancement of neutrino oscillations in matter will arise due to polarization of the matter by neutrinos. If the weak interaction can generate not only neutrino scattering but also polarization of matter, then the resonance effect will exist, otherwise this effect cannot exist.

In the ultrarelativistic limit, the evolution equation for the neutrino wave function $\nu_{\rm Ph}$ in matter has the following form [8]:

$$i\frac{d\nu_{\rm Ph}}{dt} = \left(p\hat{I} + \frac{\hat{M}^2}{2p} + \hat{W}_i\right)\nu_{\rm Ph}, \tag{1}$$

where p, $\hat{M}^2$, $\hat{W}_i$ are, respectively, the momentum, the (nondiagonal) square mass matrix in vacuum, and the matrix, taking into account neutrino interactions in matter,

$$\nu_{\rm Ph} = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \qquad \hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\hat{M}^2 = \begin{pmatrix} m^2_{\nu_e\nu_e} & m^2_{\nu_e\nu_\mu} \\ m^2_{\nu_\mu\nu_e} & m^2_{\nu_\mu\nu_\mu} \end{pmatrix}.$$

If we suppose that neutrinos in matter behave analogously to the photons in matter (i.e., the polarization at neutrino passing through matter arises) and the neutrino refraction indices are defined by the expression

$$n_i = 1 + \frac{2\pi n_e}{p^2} f_i(0) = 1 + 2\frac{\pi W_i}{p}, \tag{2}$$

where i is a type of neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$; n_e is the electron density in matter; $f_i(0)$ is a real part of the forward scattering amplitude which appears owing to polarization of matter by neutrino, then W_i characterizes polarization of matter by neutrinos (i.e., it is the energy of matter polarization).

The electron neutrino (ν_e) in matter interacts via $W^{\pm}$, Z^0 bosons and ν_μ, ν_τ interact only via the Z^0 boson. These differences in interactions lead to the following differences in the refraction coefficients of ν_e and ν_μ, ν_τ:

$$\Delta n = \frac{2\pi n_e}{p^2}\Delta f(0), \quad \Delta f(0) = \sqrt{2}\frac{G_F}{2\pi}p, \tag{3}$$

$$E_{\text{eff}} = \sqrt{p^2 + m^2} + \langle e\nu | H_{\text{eff}} | e\nu \rangle \approx p + \frac{m^2}{2p} + \sqrt{2}G_F n_e,$$

where G_F is the Fermi constant.

Therefore, the velocities (or effective masses) of ν_e and ν_μ, ν_τ in matter are different. And at the suitable density of matter this difference can lead to resonance enhancement of neutrino oscillations in matter [8, 9]

$$\sin^2 2\theta_m = \sin^2 2\theta \left[\left(\cos 2\theta - \frac{L_0}{L^0} \right)^2 + \sin^2 2\theta \right]^{-1}, \tag{4}$$

where $\sin^2 2\theta_m$ and $\sin^2 2\theta$ characterize neutrino mixing in matter and in vacuum, L_0 and L^0 are lengths of oscillations in vacuum and in matter.

$$L_0 = \frac{4\pi E_\nu \hbar}{\Delta m^2 c^3}, \qquad L^0 = \frac{\sqrt{2}\pi\hbar c}{G_F n_e}, \tag{5}$$

where E_ν is the neutrino energy; Δm^2 is the difference between squared neutrino masses; c is the velocity of light; $\hbar$ is the Planck constant; G_F is the Fermi constant and n_e is the electron density in matter.

At resonance

$$\cos 2\theta \cong \frac{L_0}{L^0}, \qquad \sin^2 2\theta_m \cong 1, \qquad \theta_m \cong \frac{\pi}{4}. \tag{6}$$

It is necessary to stress that this resonance enhancement of neutrino oscillations in matter is realized when the neutrino velocity is smaller than the velocity of light in matter (i.e., $v_i < c/n_i$).

What will happen when the neutrino velocity is larger than the velocity of light in matter? Now let us turn to consideration of this problem.

2. THE CHERENKOV EFFECT IN WEAK INTERACTIONS GENERATED BY NEUTRINOS IN MATTER

The specific electromagnetic radiation produced by fast electrons moving in a medium was observed by Cherenkov in 1934 [10]. Tamm and Frank [11] showed that the charged particle must radiate when its velocity exceeds the velocity of light in the medium (see also [12], where a motion of a charged particle in a medium with a constant electric permittivity (or refraction index) was considered). The Cherenkov radiation in the strong interactions was studied in [13], then indication on the Cherenkov radiation in experiment with hadrons was found out in [14]. It is obvious that analogous radiation must take place when neutrinos move in a medium with a velocity exceeding the velocity of light in the medium if $n_i - 1 > 0$.

For realization of the mechanism of resonance enhancement of neutrino oscillations in matter the following condition must be fulfilled:

$$n_i - 1 > 0, \tag{7}$$

which is equivalent to the demand that the matter polarization exists at

$$v_i < \frac{c}{n_i}. \tag{8}$$

Numerical value for $n_{\nu_e} - 1$ is

$$n_{\nu_e} - 1 = 4.76 \left(\frac{n_e}{n_0}\right) \left(\frac{p_0}{p_\nu}\right) \cdot 10^{-19}, \tag{9}$$

where n_e is the electron density in matter, $n_0 = 6.02 \cdot 10^{23}$ and $p_0 = 1$ MeV. For the Sun at $p_{\nu_e} = 10$ MeV

$$n_{\nu_e}^{\rm Sun} - 1 \approx 5 \cdot 10^{-18}.$$

For existence of the Cherenkov effect in weak interactions it is necessary to fulfil the following two conditions:

$$n_i - 1 > 0 \tag{10}$$

and

$$v_i > \frac{c}{n_i}, \tag{11}$$

where v_i, c are the velocities of the neutrino and light, respectively. The first condition coincides with the resonance existence condition. The second condition means that if the neutrino moves with the velocity which is larger than the velocity of light in matter, then it will polarize matter and go away keeping in reserve this polarization which must be radiated afterwards. What will happen after that? This polarization can be taken off in two ways:

1) via radiation of weakly interacting particles (neutrinos) or

2) via electromagnetic radiation (polarized electrons combined in atoms have electrical charges and they can give electromagnetic radiation).

Since the probability of radiation of weakly interacting particles is very small, the Cherenkov radiation will be realized mainly in the form of electromagnetic radiation but not weak interaction radiation (it is clear that if energy of matter polarization is very small, then neutrinos cannot be produced). It is related to the fact that weak interactions are slow processes while electromagnetic processes are fast. Energy of this radiation is

$$\begin{gathered} E \approx W = \sqrt{2} G_F n_e, \quad W = 7.6 \left(\frac{n_e}{n_0}\right) \cdot 10^{-14}, \\ E^{\rm Sun} \approx 10^{-13}\text{–}10^{-11}\,\text{eV}, \end{gathered} \tag{12}$$

where G_F is the Fermi constant; n_e is the electron density in matter. The radiation angle β_i is

$$\cos\beta_i = \frac{c}{v_i n_i}.$$

If

$$v_i < \frac{c}{n_i}, \tag{13}$$

then the neutrino will polarize matter and since the velocity is smaller than the velocity of light in matter, this matter polarization will move together with this neutrino and the resonance effect can be realized.

Now we see that the resonance and the Cherenkov effects are competing processes. If $v_i < c/n_i$, the resonance effect will be realized, and if $v_i > c/n_i$, the Cherenkov effect will be realized. It is very important to remark that if in reality the matter polarization in weak interactions is present, then we obtain an excellent possibility of estimating neutrino masses using the point transition between the above two mechanisms, and then the expression for the neutrino mass is

$$m_\nu = E_{\text{trans}}\sqrt{1-\left(\frac{1}{n_i}\right)^2}, \tag{14}$$

if $(n-1) \ll 1$, then

$$m_\nu \simeq E_{\text{trans}}\sqrt{2(n_i-1)}, \tag{15}$$

where E_{trans} is the neutrino energy at the point where the transition between the indicated mechanisms is realized. The numerical value of m_ν ($E_{\text{trans}} \simeq pc$, $c = 1$) is

$$m_{\nu_e} = 3.1 \cdot 10^{-10} p_0 \sqrt{\left(\frac{n_e}{n_0}\right)\left(\frac{E_{\text{trans}}}{E_0}\right)} \text{ eV}. \tag{16}$$

If we suppose that the raising of the Sun neutrino energy spectrum above 10 MeV is connected with the Cherenkov effect, then ($n_e = 120$, $E_{\text{trans}} = 10$ MeV)

$$m_{\nu_e} \approx 1.01 \cdot 10^{-2} \text{ eV}. \tag{17}$$

Using the values for $\Delta m_{12}^2 = m_1^2 - m_2^2 = 0.83 \cdot 10^{-4}$ eV^2 obtained in [4] and $m_{\nu_e} \cong m_1$ from (17) we get estimation on mass of m_2

$$m_2 \approx 0.5 \cdot 10^{-2} \text{ eV}. \tag{18}$$

And then using the values for $\Delta m_{23}^2 = m_3^2 - m_2^2 \simeq 2.5 \cdot 10^{-3}$ eV^2 obtained in [4] and m_2 from (18) we get estimation on mass of m_3

$$m_3 \approx 4.97 \cdot 10^{-2} \text{ eV}. \tag{19}$$

CONCLUSION

It has been shown that if weak interactions can generate masses and polarize matter, the Cherenkov effect at $v_\nu > c/n$ appears which is induced by these interactions. The effect of (resonance) enhancement of neutrino oscillations in matter ($v_\nu < c/n$) and the Cherenkov effect ($v_\nu > c/n$) are competitive processes and at definite neutrino energies the effect of (resonance) enhancement of neutrino oscillations in matter will change to the Cherenkov effect. Then neutrino vacuum oscillations will regenerate and we obtain an excellent possibility of estimating neutrino masses. And knowing estimation of mass for one (electron) neutrino we can obtain masses of the rest neutrinos by using the values for mass differences for neutrinos obtained in oscillation experiments.

It is necessary to remark that if the mechanism of MaVaN oscillations [15] is realized, then the Cherenkov effect generated by neutrinos will also be realized.

Acknowledgements. The author expresses sincere gratitude to Prof. G. T. Zatsepin and Prof. V. A. Kuzmin for very useful discussions.

REFERENCES

1. Rev. Part. Phys. // J. Phys. G (Nucl. Part. Phys.). 2006. V. 33. P. 35; 435.
2. *Doe P.* Report at the Intern. Conf. «Neutrino 2006», Santa Fe, June 2006;
   Barabash A. Report at the Intern. Conf. «Neutrino 2006», Santa Fe, June 2006.
3. Rev. Part. Phys. // J. Phys. G (Nucl. Part. Phys.). 2006. V. 33. P. 479.
4. Rev. Part. Phys. // Ibid. P. 156;
   Eguchi K. et al. // Phys. Rev. Lett. 2003. V. 90. P. 021802;
   Habig A. // Proc. of the Intern. Cosmic Ray Conf., Japan. 2003. V. 1. P. 1255;
   Kearns Ed. (Super-Kamiokande Collab.). Report at the Intern. Conf. «Neutrino 2004», Paris, 2004.
5. *Pontecorvo B. M.* // ZhETP. 1957. V. 33. P. 549; 1958. V. 34. P. 247.
6. *Maki Z. et al.* // Prog. Theor. Phys. 1962. V. 28. P. 870.
7. *Pontecorvo B. M.* // ZhETP. 1967. V. 53. P. 1717.
8. *Wolfenstein L.* // Phys. Rev. D. 1978. V. 17. P. 2369.
9. *Mikheyev S. P., Smirnov A. Yu.* // Sov. J. Nucl. Phys. 1985. V. 42. P. 1491;
   Mikheyev S. P., Smirnov A. Yu. // Nuono Cim. 1986. V. 9. P. 17.
10. *Cherenkov P. A.* // Dokl. Akad. Nauk SSSR. 1934. V. 2. P. 457.
11. *Frank I., Tamm I.* // Dokl. Akad. Nauk SSSR. 1937. V. 14. P. 107.
12. *Afanasiev G. N., Beshtoev Kh. M., Stepanovsky Yu. P.* // Helv. Phys. Acta. 1996. V. 62. P. 111.
13. *Dremin I. M.* // JETP Lett. 1979. V. 30. P. 140; Part. Nucl. 1987. V. 18. P. 31;
    Ion D. B., Stoker W. // Phys. Lett. B. 1991. V. 273. P. 20.
14. *Dremin I. M. et al.* // J. Nucl. Phys. 1990. V. 52. P. 840;
    Agababyan N. M. et al. // Phys. Lett. B. 1998. V. 389. P. 397.
15. *Barger V., Huber P., Marfatia D.* // Phys. Rev. Lett. 2005. V. 95. P. 211802; hep-ph/0502196. 2005.

Received on January 27, 2009.