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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

ASYMPTOTIC BEHAVIOR OF PION CLUSTERS IN DIFFERENT NUCLEAR REACTIONS

J. Karachuk [a,b], *A. I. Malakhov* [a]

[a]Joint Institute for Nuclear Research, Dubna

[b]INCDIE ICPE-CA, Bucharest, Romania

This paper is aimed to analyze nuclear processes by means of invariant variables b_{ik} based on four-velocity vectors. Pion clusters generated in different nuclear reactions are investigated in order to point out their properties able to indicate universal laws for nuclear matter behavior. Some interesting results concerning pion clusters invariant parameters are presented.

Целью данной статьи является анализ ядерных процессов с помощью инвариантных переменных b_{ik} на базе четырехмерных скоростей. Пионные кластеры, рожденные в разных ядерных реакциях, исследованы с целью выявления универсальных закономерностей поведения ядерной материи. Представлены некоторые интересные результаты относительно инвариантных параметров пионных кластеров.

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INTRODUCTION

Our experimental investigation is based on the following nuclear reactions:

1) π^-–C at $|\mathbf{p}_{\rm beam}| = 40$ GeV/c; 8791 events were registered in the propane bubble chamber; π^+ and π^- mesons were measured [3];

2) Mg–Mg at $|\mathbf{p}_{\rm beam}| = 4.5A$ GeV/c; 14 218 central interactions were registered in the streamer chamber; only π^- mesons were measured [4].

As these reactions are different from many points of view, their comparison by means of invariant variables b_{ik} based on four-velocity vectors may lead to interesting conclusions concerning nuclear matter behavior.

1. INVARIANT VARIABLES BASED ON FOUR-VELOCITY VECTORS

In this section we shall define the concepts our study is based on. First of all, we introduce four-velocity vector for every particle. These vectors enable us to construct useful invariant variables as scalar products.

1.1. Four-Velocity Vectors. Let p_i be the four-momentum of particle i:

$$p_i = (E_i,\, p_{xi},\, p_{yi},\, p_{zi}). \tag{1}$$

By dividing this vector by scalar m_i, which is the mass of particle i, we obtain a four-vector which will be called four-velocity u_i:

$$u_i = \frac{p_i}{m_i} = \left(\frac{E_i}{m_i}, \frac{\mathbf{p}_i}{m_i}\right) = \left(\frac{E_i}{m_i}, \frac{p_{xi}}{m_i}, \frac{p_{yi}}{m_i}, \frac{p_{zi}}{m_i}\right). \tag{2}$$

It is easy to prove that four-velocity vectors u_i are unitary.

Let us remember:

$$p_i^2 = E_i^2 - p_{xi}^2 - p_{yi}^2 - p_{zi}^2 = m_i^2. \tag{3}$$

Dividing (3) by m_i, we get

$$u_i^2 = 1. \tag{4}$$

This characteristic of four-velocity vectors will be often used in our following considerations.

1.2. Cumulative Coefficients *XB* and *XT*. Let us describe a nuclear process using four-momentum vectors:

$$p_{\rm I} + p_{\rm II} \to p_1 + p_2 + \ldots \tag{5}$$

The beam is represented by I index, the target — by II, and the secondary particles — by $1, 2, \ldots$ Now we focus our attention on secondary particle i and we write its four-momentum as a sum of ratios XB, XT of four-momentum of primary particles (beam and target):

$$XB_i p_{\rm I} + XT_i p_{\rm II} \to p_i + \ldots \tag{6}$$

Let us use (2) in order to replace four-momentum by four-velocity and also assume that the value of incident momentum is much larger than the nucleon mass.

$$|\mathbf{p}| \gg m. \tag{7}$$

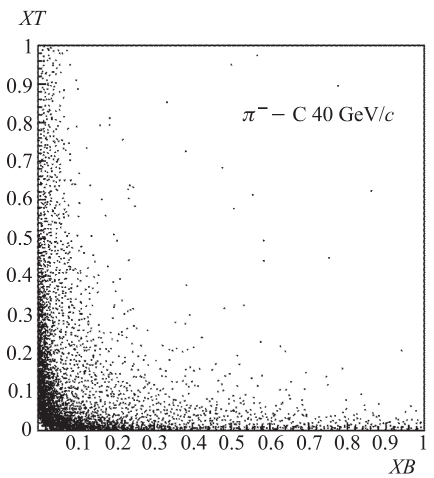

Fig. 1

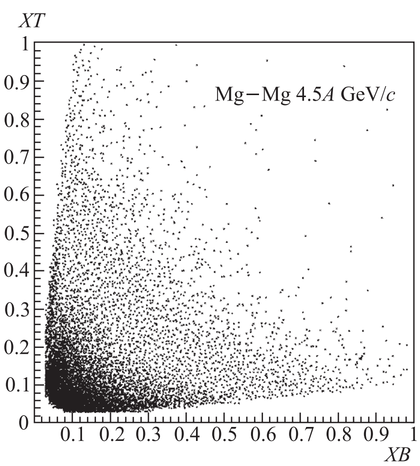

Fig. 2

Using this approximation, we get the following formulas:

$$XB_i = \frac{m_i}{m_{\rm I}}\frac{u_i u_{\rm II}}{u_{\rm I} u_{\rm II}}, \tag{8a}$$

$$XT_i = \frac{m_i}{m_{\rm II}}\frac{u_i u_{\rm I}}{u_{\rm I} u_{\rm II}}. \tag{8b}$$

The pair of variables XB_i, XT_i enables us to establish to which of colliding objects (beam or target) particle i is closer from dynamic point of view. The advantage of using these variables (instead of momentum and angle) is their independence of reference frame. Figures 1 and 2 present the distributions of these coefficients for the reactions we investigate. One can notice significant difference between these two pictures. We suggest that this is due to different steps of activation of quark–gluon freedom degrees.

1.3. Invariant Variables b_{ik}. Our investigation method is based on b_{ik} invariant variables which can be used to describe the behavior of every pair of particles i and k:

$$b_{ik} = -(u_i - u_k)^2 = 2(u_i u_k - 1). \tag{9}$$

According to [1, 5], this parameter can be used to classify nuclear processes and to reveal transition to quark–gluon degrees of freedom which correspond to large values $b_{ik} \geqslant 10$. Scalars b_{ik} can be used for separation of clusters as it will be shown in Sec. 2. However, in [8] the necessity to verify the properties of b_{ik} space was discussed.

2. INVARIANT CLUSTERS

Cluster α is a group of n_α secondary particles which can be regarded as an object. In our approach a cluster is related either to the target or to the beam. We call these clusters «invariant» because we get them using an algorithm which does not depend on the reference frame. Clusters are regarded as objects, having their own momentum and mass. For every cluster we also can calculate variables XB_α, XT_α which allow us to find out to which primary particle (beam or target) it is related.

2.1. Definition of Cluster Parameters. First of all, we have to define the axis of cluster α consisting of n_α secondary particles:

$$V_\alpha = \sum_{i_\alpha=1}^{n_\alpha} u_{i_\alpha} \Bigg/ \sqrt{\left(\sum_{i_\alpha=1}^{n_\alpha} u_{i_\alpha}\right)^2}. \tag{10}$$

As one can calculate, V_α is a unitary vector.

We regard cluster α as an object which can be identified by four-velocity vector V_α defined above. Every particle belonging to this cluster can be related to the axis by a b_{ik}-type invariant variable:

$$b_{\alpha i_\alpha} = -(V_\alpha - u_{i_\alpha})^2. \tag{11}$$

Average value of this variable is defined as width of α cluster.

In our investigation we regard clusters as groups of secondary particles related to primary objects, beam and target. So, we suppose to have two clusters in every investigated event.

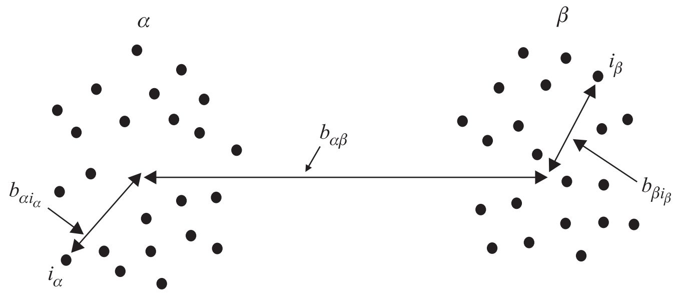

Fig. 3. Separation of particles into clusters

Clusters α and β separated in a nuclear reaction (Fig. 3) can be related by means of invariant variable:

$$b_{\alpha\beta} = -(V_\alpha - V_\beta)^2. \tag{12}$$

Variable $b_{\alpha\beta}$ is an important parameter which enables us to characterize the nuclear reaction as a whole. If values of $b_{\alpha\beta}$ parameter are much larger than the width of clusters, this means that they are clearly separated from one another. Large values of $b_{\alpha\beta}$ also indicate that interaction between colliding particles occurs at quark–gluon level.

2.2. Algorithm for Cluster Separation. Algorithm based on invariant variables which enables us to get invariant clusters is described in [1, 2]. We suppose that clusters α and β (one related to the beam and the other one to the target) include all the n secondary particles observed in the nuclear collision. They are split into two groups which contain n_α and respectively n_β particles. The algorithm is based on functional A_2^n:

$$A_2^n = \min \left[-\sum_{i_\alpha=1}^{n_\alpha} (V_\alpha - u_{i_\alpha})^2 - \sum_{i_\beta=1}^{n_\beta} (V_\beta - u_{i_\beta})^2 \right]. \tag{13}$$

From all possible ways to split secondary particles into two groups we chose the one which leads to the minimal value for functional A_2^n. This means that clusters are separated such a way that their width values reach their minimum.

Variable $b_{\alpha\beta}$ defined in (12) is used to select events for further investigation: if this value is larger than parameter b_0, clusters α and β separated in the given event are taken into consideration (b_0 is empirically determined for every nuclear reaction).

3. EXPERIMENTAL DATA CONCERNING PION CLUSTERS BEHAVIOR

By means of algorithm described in Sec. 2 we separate pion clusters in nuclear reactions π^-–C ($|\mathbf{p}_{\rm beam}| = 40$ GeV/*c*) and Mg–Mg ($|\mathbf{p}_{\rm beam}| = 4.5A$ GeV/*c*).

After having clusters separated, we can establish by means of invariant variables XB_α, XT_α whether a cluster is a beam fragment or a target one. Investigating both reactions, we found that for each event in which clusters were found, one cluster was a beam fragment and the other was a target one. This enables us to draw the conclusion that hypothesis of getting two clusters in the final state, as a result of colliding particles fragmentation, was correct and the algorithm for cluster separation was properly chosen.

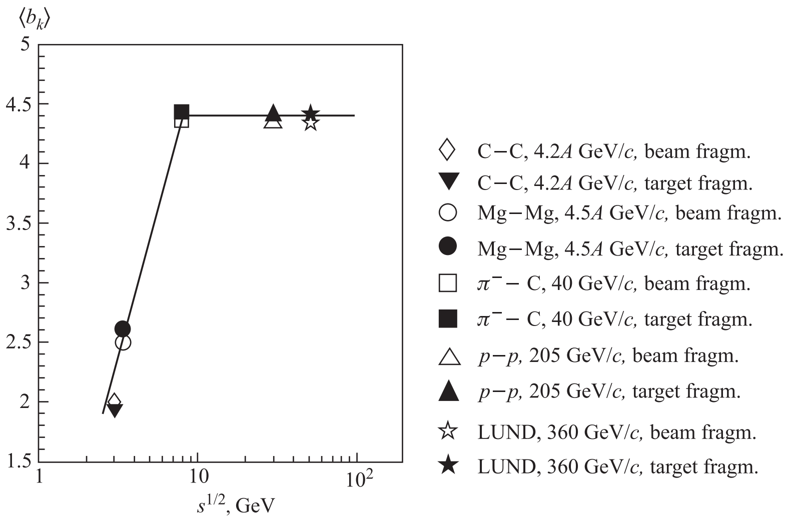

Fig. 4. Cluster width versus incident energy for different nuclear reactions

In this section we present some experimental distributions which show some properties of invariant clusters. Our analysis is focused on comparison between different nuclear reactions.

After having clusters separated, we are able to calculate the invariant variables defined in Subsec. 2.1, in order to compare distributions of variables defined in (11) and (12). If clusters are clearly separated, parameter $b_{\alpha\beta}$ is much larger than cluster width. This is what we have in the first case (π^-–C). For Mg–Mg reaction, at lower incident energy, clusters are partially overlapped. In the nuclear reactions used in this paper we have different types of pion clusters. Some experimental data concerning the behavior of different type pion clusters in the same nuclear reactions are presented in [8]. The stage of clear cluster separation depends on the activation of quark–gluon degrees of freedom [6, 7].

3.1. Asymptotic Behavior of Cluster Width. The mean value of invariant variable defined in (11) is called cluster width and represented by $\langle b_k \rangle$. It is interesting to compare this parameter for different nuclear reactions, in a wide range of incident energy. We present these results in Fig. 4.

Part of experimental data (π^-–C at $|\mathbf{p}_{\rm beam}| = 40$ GeV/c, p–p at 205 GeV/c and LUND at 360 GeV/c) can be found in [1, 2]. Pion clusters obtained in these reactions show constant value for their width: $\langle b_k \rangle \cong 4.4$. This fact enables us to speak about asymptotic behavior of pion clusters [9, 10].

We extended investigation for incident energy of a few GeV/c per nucleon and added two experimental points to this diagram (C–C at $4.2A$ GeV/c and Mg–Mg at $4.5A$ GeV/c). As one can notice, in these two cases the clusters show a lower value of their width and asymptotic value is not reached.

3.2. Distribution of Events versus $\langle b_{\alpha\beta} \rangle$ in Different Reactions. As we have already mentioned, a very important parameter which characterizes a nuclear reaction is the $b_{\alpha\beta}$ parameter which can be defined in every case where we have two clusters separated. Comparing events distributions versus $b_{\alpha\beta}$ for different nuclear reactions, one can notice that larger values of incident momentum lead to larger values of this parameter [1, 2]. In all cases, the last region of the spectrum is similar for all nuclear reactions and can be described by a step function:

$$\frac{1}{N}\frac{dN}{db_{\alpha\beta}} = \frac{A}{b^n_{\alpha\beta}}. \quad (14)$$

Parameter n measured in a wide range $20 < b_{\alpha\beta} < 10^5$ for different nuclear reactions (including $\pi^- - \mathrm{C}$ at $|\mathbf{p}_{\mathrm{beam}}| = 40$ GeV/c) proved to be equal $n = 3$ with a precision better than 10% [1, 2]. We extend the study for reaction Mg–Mg (4.5A GeV/c), at lower values of variable $b_{\alpha\beta}$.

In the Table we present the results of measurement of n for nuclear reactions this paper is based on.

Values of *n* parameter

Reaction	n
$\pi^- - \mathrm{C}$ at $\lvert\mathbf{p}_{\mathrm{beam}}\rvert = 40$ GeV/c	3.0 ± 0.3
Mg–Mg at $\lvert\mathbf{p}_{\mathrm{beam}}\rvert = 4.5A$ GeV/c	3.6 ± 0.1

As one can notice, the values of n parameter are similar for the nuclear reaction Mg–Mg (4.5A GeV/c).

CONCLUSIONS

1) The algorithm based on A_2 functional enables us to construct two invariant clusters α and β, related to the colliding particles. We can define cluster width $\langle b_k \rangle$ and parameter $b_{\alpha\beta}$ which enable us to estimate the degree of separation between them. Clusters are more clearly separated from one another as collision momentum grows.

2) For a wide range of nuclear reactions, cluster width $\langle b_k \rangle$ versus energy shows asymptotic behavior. The constant value $\langle b_k \rangle \cong 4.4$ is reached for clusters clearly separated from one another which can be obtained if collision energy is high enough (π^-– C). In the case of partially overlapped clusters, the cluster width is less than 4.4 (Mg–Mg at 4.5A GeV/c).

3) The step dependence of $b_{\alpha\beta}$ parameter shows similar value $n \cong 3$ for both types of clusters. This enables us to draw the conclusion that clusters have the same nature, irrespective of their degree of separation.

4) Significant difference between the nuclear reactions we investigate can be observed in plots XB_i, XT_i. We suggest that these invariant variables can be used for any nuclear reaction, in order to find out whether the invariant clusters which can be obtained are clearly separated or not.

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