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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

GEOMETRIZATION OF QUANTUM PHYSICS

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It is shown that the Dirac equation for free particle can be considered as a description of specific distortion of the space Euclidean geometry (space topological defect). This approach is based on possibility of interpretation of the wave function as vector realizing representation of the fundamental group of the closed topological space–time 4-manifold. Mass and spin appear to be topological invariants. Such a concept explains all so-called «strange» properties of quantum formalism: probabilities, wave-particle duality, nonlocal instantaneous correlation between noninteracting particles (EPR-paradox) and so on. Acceptance of suggested geometrical concept means rejection of atomistic concept where all matter is considered as consisting of more and more small elementary particles. There are no any particles a priori, before measurement: the notions of particles appear as a result of classical interpretation of the contact of the region of the curved space with a device.

Показано, что уравнение Дирака для свободной частицы может быть рассмотрено как описание специфического искажения эвклидовой геометрии физического пространства (пространственного топологического дефекта). Предлагаемый подход основан на возможности интерпретации волновой функции как вектора, реализующего представление фундаментальной группы замкнутого пространственно-временного топологического многообразия. Масса и спин являются топологическими инвариантами. Такой подход объясняет все иррациональные свойства квантового формализма: вероятности, корпускулярно-волновой дуализм, нелокальную мгновенную корреляцию невзаимодействующих частиц (ЭПР-парадокс) и т. д. Принятие предлагаемой геометрической концепции означает отказ от атомизма — физической парадигмы, согласно которой материя состоит из все более и более мелких элементарных частиц. В рамках новой парадигмы частицы a priori, до измерения, не существуют: представление о них возникает как результат интерпретации в рамках классических понятий контакта искривленной области пространства с измерительным прибором.

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1. TOPOLOGICAL INTERPRETATION OF THE DIRAC EQUATION

This equation has the following symbolic form (see, e.g., [1]):

$$i\gamma^\mu \partial_\mu \psi = m\psi, \tag{1}$$

where $\partial_\mu = \partial/\partial x_\mu, \mu = 1, 2, 3, 4$, $\psi(x)$ is the four-component Dirac bispinor, $x_1 = t, x_2 = x, x_3 = y$, $x_4 = z$, and γ^μ are four-row Dirac matrices. The summation in Eq. (1) goes over

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the repeating indices with a signature $(1,-1,-1,-1)$. Here, $\hbar = c = 1$. For definite values of 4-momentum p_μ, the solution to Eq. (1) has the form of the plane wave

$$\psi = u(p_\mu)\exp(-ip_\mu x^\mu), \tag{2}$$

where $u(p_\mu)$ is a normalized bispinor. Substitution of (2) in Eq. (1) gives

$$p_1^2 - p_2^2 - p_3^2 - p_4^2 = m^2. \tag{3}$$

Let us consider transformation properties of the Dirac bispinor. It is known that there may be established correspondence between every kind of tensors and some class of geometrical objects in the sense that these tensors define invariant properties of the above objects. For example, usual vectors correspond to simplest geometrical objects — to points [2], and this is the reason why Newtonian mechanics uses vectors within its formalism. From this point of view, spinors correspond to nonorientable geometrical object (see, e.g., [3]). Therefore, we suppose that spinors are used in Eq. (1), because this equation describes some nonorientable geometrical object and «$spin = 1/2$» is a formal expression of the nonorientable property of the object.

To define properties of the proposed geometrical object more exactly we consider more precisely the symmetry properties of the solution of Eq. (1). We rewrite function (2) and relation (3) in the form

$$\psi = u(p_\mu)\exp(-2\pi i x^\mu \lambda_\mu^{-1}), \tag{4}$$

$$\lambda_1^{-2} - \lambda_2^{-2} - \lambda_3^{-2} - \lambda_4^{-2} = \lambda_m^{-2}, \quad \lambda_\mu = 2\pi p_\mu^{-1}, \quad \lambda_m = 2\pi m^{-1}. \tag{5}$$

Function (4) is an invariant with respect to coordinates transformations

$$x'_\mu = x_\mu + n_\mu \lambda_\mu, \quad n_\mu = 0, \pm1, \pm2, \ldots \tag{6}$$

Transformations (6) can be considered as elements of the discrete group of translations operating in the 4-space where wave function (4) is defined. Then function (4) can be considered as a vector realizing this group representation.

As a bispinor, function (4) realizes representation of one more group of symmetry transformations of 4-space. Being a four-component spinor, $\psi(x)$ is related to the matrices γ^μ by the equation (see, e.g., [4])

$$\psi'(x') = \gamma^\mu \psi(x),$$

where $x \equiv (x_1, x_2, x_3, x_4)$, and $x' \equiv (x_1, -x_2, -x_3, -x_4)$ for $\mu = 1, x' \equiv (-x_1, x_2, -x_3, -x_4)$ for $\mu = 2$, and so on. This means that the matrices γ^μ are the matrix representation of the group of reflections along three axes perpendicular to the x_μ axis, and the Dirac bispinors realize this representation.

Taken together, the above two groups form a group of four sliding symmetries with perpendicular axes (sliding symmetry means translations plus corresponding reflections). The physical space–time does not have such a symmetry. So, this group may operate only in some auxiliary space. On the other hand, it is known that discrete groups operating in some space can reflect a symmetry of geometrical objects that have nothing in common with this space. It will be the case when such a space is a universal covering space of some closed topological manifold. Universal covering spaces are auxiliary spaces that are used in topology for the description of closed manifolds, because discrete groups operating in these spaces are isomorphic to fundamental groups of manifolds — groups whose elements are different classes of closed paths on manifolds (so-called π_1 group [5, 6]). We assume that function (4) realizes a representation of the fundamental group of some closed nonorientable topological 4-manifold — a specific curved part of the space–time. Equation (1) imposes limitations (5) on the possible values of the fundamental group parameters λ_μ. Space–time plays also the role of a universal covering space for the above manifold.

At the present time, only two-dimensional Euclidean closed manifolds are classified in detail, and their fundamental groups and universal covering planes are identified [5]. Therefore, we have no opportunity for rigorous consideration of specific properties of suggested pseudo-Euclidean 4-manifold. But qualitative properties, explaining main ideas of new interpretation, can be investigated using one of the advantages of geometrical approach — possibility of employment of low-dimensional analogies. Using these analogies we will show within elementary topology that the above-mentioned 4-manifold can represent propagation of the space topological defect that can demonstrate stochastic properties of a single quantum particle.

2. STOCHASTIC BEHAVIOR

Consider the simplest example of closed topological manifolds — one-dimensional manifold homeomorphic to a circle with given perimeter length λ. The closed topological manifold is representable by any of its possible deformations (without pasting) that conserve manifold's continuity, and we will see that just this property explains appearance of probabilities in quantum formalism. For simplicity we consider plane deformations of the circle (some of the possible deformations are shown in Fig. 1).

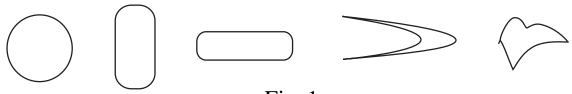

Fig. 1

To use concrete simple calculations, we consider only all possible manifold's deformations that have a shape of ellipse with perimeter length λ. The equation for the ellipse on a Euclidean plane has the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{7}$$

where all possible values of the semiaxes a and b are connected with the perimeter length λ by the known approximate relation

$$\lambda \simeq \pi[1,5(a+b) - (ab)^{1/2}]. \tag{8}$$

This means that the range of all possible values of a is defined by the inequality $a_{\min} \leqslant a \leqslant a_{\max} \simeq \lambda/1,5\pi, \quad a_{\min} \ll a_{\max}$.

In the pseudo-Euclidean two-dimensional «space–time», the equation for our ellipses has the form of the equation for hyperbola (after substitution $y = it$)

$$\frac{x^2}{a^2} - \frac{t^2}{b^2} = 1, \tag{9}$$

and this equation defines the dependence on time t for a position of point x of the manifold corresponding to definite a. At $t = 0, x = \pm a$; that is, our manifold is represented by the two point sets in one-dimensional Euclidean space, and the dimensions of these point sets are defined by all possible values of a. So, at $t = 0$, the manifold is represented by two regions of the one-dimensional Euclidean space $a_{\min} \leqslant |x| = a \leqslant a_{\max}$. It can easily be shown that at $t \neq 0$ these regions increase and move along the x axis in opposite directions.

All another possible deformations of our circle will be obviously represented by points of the same region, and every such a point can be considered as a possible position of the «quantum object» described by our manifold. All manifold's deformations are realized with some probabilities depending on external conditions. In the considered case of free particle all deformations are realized with equal probabilities — there are no reasons for another suggestion. Therefore, all possible positions of the point-like object into the region are realized with equal probabilities. So, this example shows the possibility of the consideration of the above object as a point with probability distribution of its positions in closed region of the space as it is suggested within standard representation of quantum particles.

It should be stressed that within suggested approach stochastic behavior is a property of a single quantum particle: the role of statistical ensemble plays here the ensemble of all possible topological realization of the same particle.

3. TOPOLOGICAL DEFECT. WAVE-CORPUSCULAR PROPERTIES

The above example does not explain what geometrical properties allow us to differ points of the moving region from neighbour points of the Euclidean space making them observable. To answer this question we consider more complex analogy of the closed 4-manifold — two-dimensional torus. In Euclidean 3-space such a torus is denoted as topological production of two circles — $S^1 \times S^1$. The role of different manifold's deformations as a reason for stochastic behavior was considered in Sec. 2. Therefore, now we restrict our consideration to one simplest configuration when one of S^1 is a circle in the XY plane and another is a circle in the plane ZX (we denote it as S_1').

In pseudo-Euclidean space this torus looks like a hyperboloid that appears if we replace the circle S_1' by a hyperbola (as it was done in Sec. 2). Positions of the geometrical object described by our pseudo-Euclidean torus are defined by time cross sections of the hyperboloid. These positions form an expanding circles in the two-dimensional Euclidean plane (Fig. 2).

But we need to have in mind that two-dimensional pseudo-Euclidean torus describes the object existing in two-dimensional space–time with one-dimensional Euclidean «physical» space. This means that an observable part of the object is represented in our example by the points of intersections of the above circle with $0X$ axis though, as a whole, the circle is «embedded» into two-dimensional, «external» space. This circle can be considered as a topological defect of the physical one-dimensional Euclidean space. Just an affiliation of the intersection points to the topological object differs these points geometrically from neighboring points of the one-dimensional Euclidean space. So, in pseudo-Euclidean four-dimensional physical space–time the suggested

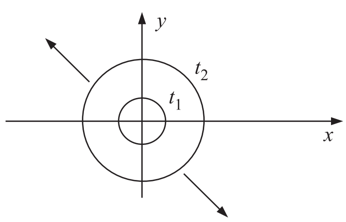

Fig. 2

object described by the Dirac equation looks like a topological defect of physical Euclidean 3-space that is embedded into 5-dimensional Euclidean space, and its intersection with physical space represents an observable quantum object.

Note, that expanding circles in Fig. 2 can be considered as a model for propagation in opposite directions of two identical noninteracting particles. Being the intersection points of the same defect with the physical space, these particles can correlate one with another without any interaction in physical space — the channel for information is provided by their common defect embedded in the outer space. This can be considered as an explanation for the paradox of Einstein, Podolsky, Rosen.

The above analogy with torus does not yet demonstrate appearance of any wave-corpuscular properties of the object, represented in «physical» one-dimensional space by the moving intersection point — properties that could be expressed by wave function (5) and relation (4). In the case of considered two-dimensional «space–time» this solution has the form

$$\psi = u(p) \exp\left(-2\pi i x^1 \lambda_1^{-1} + 2\pi i x^2 \lambda_2\right). \tag{10}$$

Topological defect represented by the expanding circle does not demonstrate any periodical movements when the intersection point (physical object) propagates along one-dimensional Euclidean $0X$ space.

Appearance of observable wave-corpuscular properties is a consequence of nonorientable character of the topological defect. Torus is an orientable two-dimensional closed manifold and, therefore, we need to use some nonorientable low-dimensional analogy. The nonorientable Klein bottle could be such a two-dimensional analogy [5, 7]. In the case with torus, topological defect was represented by cross sections of pseudo-Euclidean torus-plane circles. The Klein bottle is a manifold that is obtained by gluing of two Mobius strips [7]. Therefore, the Klein bottle cross section is an edge of the Mobius strip. This edge cannot be placed in the two-dimensional XY plane without intersections, and it means that corresponding topological defect is now a closed curve embedded into three-dimensional XYZ space.

In this case the position of the topological defect relative to its intersection with $0X$ axis (physical object) can change periodically. Such a periodical process can be expressed by function (10). It leads to the new interpretation of the wave function as a description of periodical movement of the topological defect relative to its projection on the physical space. Corpuscular properties of the above periodical movement appear as a result of the definition for classical notion of 4-momentum through the wave characteristic of the topological object, namely

$$p_\mu = 2\pi/\lambda_\mu. \tag{11}$$

Substitution of these relations into (18) leads to the Dirac solution (2)

$$\psi = u(p) \exp\left(-ip_1 x^1 + ip_2 x^2\right). \tag{12}$$

It is important to note that within suggested geometrical interpretation the notions of the less general, macroscopic theory (4-momentums) are defined by (11) through the notions of more general microscopic theory (wave parameters of the defect periodical movement). This looks more natural than the opposite definitions (4) within traditional interpretation. And this means rejection of atomistic concept where all matter is considered as consisting of more and more small elementary particles. There are no any particles a priori, before measurement: the notions of particles appear as a result of classical interpretation of the contact of the region of the curved space with a device. Experimentally this fact was established in investigations of effects of quantum nonlocality (see, e.g., [8]).

Preliminary results see in [9].

REFERENCES

1. *Bjorken J. D., Drell S. D.* Relativistic Quantum Mechanics and Relativistic Quantum Fields. N.Y.: McGraw-Hill, 1964.

2. *Rachevski P. L.* Riemannian Geometry and Tensor Analysis. M.: Nauka, 1966. § 55.

3. *Jelnorovitch V. A.* Theory of Spinors and Its Applications. M.: August-Print, 2001. § 1.3.

4. *Achiezer A. I., Peletminski S. V.* Fields and Fundamental Interactions. Kiev: Nauk. Dumka, 1986. Ch.1.

5. *Dubrovin B. A., Novikov S. P., Fomenko A. T.* Modern Geometry. M.: Nauka, 1986.

6. *Schvartz A. S.* Quantum Field Theory and Topology. M.: Nauka, 1989.

7. *Gilbert D., Kon-Fossen S.* Nagladnaya Geometriya. M.: Nauka, 1981 (in Russian).

8. *Belinsky A. V.* Quantum Nonlocality and Absence of a priori Values of Measuring Quantities in Experiments with Photons // Usp. Fiz. Nauk. 2003. V. 173, No. 8. P. 905.

9. *Olkhov O. A.* Geometrization of Quantum Mechanics // J. Phys.: Conf. Ser. 2007. V. 67. P. 012037.