Письма в ЭЧАЯ. 2009. Т. 6, № 7(156). С. 109–114

EPR LIGHT BEAMS AND NON-GAUSSIAN QUANTUM DISTRIBUTIONS IN THE TIME DOMAIN

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We investigate time-dependent properties of Einstein–Podolsky–Rosen (EPR) light beams generated in nondegenerate optical parametric oscillator (NOPO) driven by a sequence of laser pulses with Gaussian time-dependent envelopes. The peculiarities of EPR beams are discussed on the basis of quadrature squeezing and also in the framework of phase-space Wigner functions for EPR beams which are combined on a half beam splitter. We also investigate the Wigner functions of intensity-correlated twin beams following the conditional photon state-preparation scheme. It is demonstrated that the Wigner functions involve negative values in parts of the phase space for the schemes with one, two, and three photons.

Мы исследуем динамику световых пучков Эйнштейна–Подольского–Розена (ЭПР), полученных при генерации невырожденного оптического параметрического осциллятора под действием лазерных импульсов с временной гауссовской огибающей. Исследованы свойства ЭПР-пучков на основе квадратурного сжатия, а также в рамках квантовых распределений. Мы также исследуем функции Вигнера коррелированных световых пучков, следуя схеме приготовления фотонных состояний. Показано, что функции Вигнера содержат отрицательные части в фазовом пространстве для схем приготовления одно-, двух- и трехфотонных состояний.

PACS: 03.65.Ud, 01.30.Cc, 03.67.-a

INTRODUCTION

It is recognized that nondegenerate optical parametric oscillator (NOPO) is one of the effective systems generating Einstein–Podolsky–Rosen (EPR) entangled light beams [1–3]. In the standard treatment of NOPO, the pump field is considered as a monochromatic beam and the calculations are performed in the frequency domain. Nevertheless, recent experimental achievements in quantum optics initiate investigations of EPR entanglement also in the time domain. Such investigations may open a way for new applications in many areas of time-resolved quantum information and communications in addition to the well-known protocols already elaborated in the spectral domain. In this area generation and characterization of quadrature-squeezed pulses as well as entangled light pulses in the time domain have been recently performed (see [4] and the references therein). The experimental generation and characterization of a two-mode squeezed vacuum state in a time-gated way has also been recently demonstrated in [5].

Thus, for applications with nonclassical states, particularly with EPR states, in the time domain a rigorous study of NOPO operating in various time-dependent regimes is needed.

As a step in this direction, a periodically pulsed NOPO, i.e., a NOPO under time-modulated pumping field, has been proposed and studied theoretically [6] in application to generation of EPR entangled light beams in the time domain.

In this paper we continue investigation of periodically pulsed NOPO. In one part of the present paper we extend our previous results [6] regarding NOPO above threshold under the action of a sequence of the Gaussian pulses. The other part of the paper is devoted to calculation of dynamics of the Wigner functions for the pulsed regime of NOPO, since Wigner functions give a complete description of the states of quantum systems.

1. EPR ENTANGLEMENT IN THE PULSED REGIME

We consider below a NOPO driven by Gaussian pulses separated by the time intervals τ. The corresponding time-dependent field is

$$E_L(t,z) = E_{0L} f(t)\ \mathrm{e}^{-i(\omega_L t - k_L z)}, \tag{1}$$

$$f(t) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-(t-t_0-n\tau)^2/T^2}. \tag{2}$$

We assume wideband collinear phase matching which can be more effectively realized in a periodically poled crystal. The basic energy conservation for the central frequencies and perfect phase matching imply that $\omega_L \to \frac{w_L}{2}(\uparrow) + \frac{w_L}{2}(\to)$ and $\Delta k = k_L(\omega_L) - k_1(\omega_1) - k_2(\omega_2) - k_g = 0$, where k_g is the poling wave vector. We also allow ω_1 and ω_2 vary from the degenerate frequency $\omega_1 = \omega_2 = \frac{\omega_L}{2}$ as $\omega_1 = \frac{\omega_L}{2} + \delta\omega_1$ and $\omega_2 = \frac{\omega_L}{2} + \delta\omega_2$. The corresponding interaction Hamiltonian within the framework of the rotating wave approximation reads

$$H = i\hbar\chi f(t)\left(\mathrm{e}^{i\Phi_L} b_3^+ - \mathrm{e}^{-i\Phi_L} b_3\right) + i\hbar k\left(\mathrm{e}^{i\Phi_k} b_3 b_1^+ b_2^+ - \mathrm{e}^{-i\Phi_k} b_3^+ b_1 b_2\right), \tag{3}$$

where χ is the coupling constant of pump field with the ω_3-intracavity mode which is proportional to the amplitude E_{0L} of the pump field and constant $k\ \mathrm{e}^{i\Phi_k}$ determines the efficiency of the parametric process. The operators b_n are discrete-nonmonochromatic mode annihilation operators.

The analysis shows [6] that like the standard NOPO with stationary pump field amplitude, the periodically pulsed NOPO also exhibits threshold behavior, which is easily described through the period averaged pump field amplitude $\overline{f(t)} = \frac{1}{\tau}\int\limits_0^\tau f(t)dt$. For the case of Gaussian pulses (2), the above threshold regime is realized if $\chi > \chi_{\mathrm{th}} = \frac{\gamma\gamma_3}{\sqrt{\pi}k}\frac{\tau}{T}$, where γ and γ_3 are the damping rates of the modes ω_1, ω_2 and ω_3.

The criterion of two-mode squeezing or EPR entanglement is formulated as $V = \frac{1}{2}(V(x_1 - x_2) + V(y_1 - y_2)) < 1$ in terms of the variances of the quadrature amplitudes of two subharmonic modes.

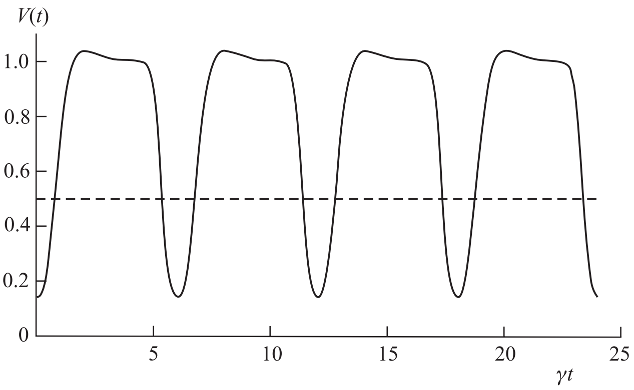

Fig. 1. Degree of two-mode squeezing versus dimensionless time for the following parameters: $k^2/\gamma\gamma_3 = 10^{-8}, \chi = 1.1\chi_{\rm th}, \tau = 6\gamma^{-1}, T = 0.6\gamma^{-1}$

The dependence of V on the scaled time is shown in Fig. 1. The dashed line in Fig. 1 indicates the degree of two-mode squeezing for the stationary regimes: $T \gg \gamma^{-1}, \tau \to 0$ and $\chi = \chi_{\rm th}$. In the nonstationary regime (see solid curve, pulses of duration $T = 0.6\gamma^{-1}$ separated by the time interval $\tau = 6\gamma^{-1}$) the modulation of the quadrature variance repeats the periodicity of the pump laser. It is clearly seen that the variance for pulsed dynamics obeys the EPR criterion $V^2 < 1/4$ for the definite time intervals. We also found a remarkable result that the variance goes below the stationary limit of 0.5 in the ranges where photon number is maximal for appropriate chosen parameters. Particularly, comparing the results of Fig. 1 and calculations of the mean photon number, we conclude that for time intervals leading to the maximal photon number $n_{\max} = 6.5 \cdot 10^7$, the corresponding variance equals $V = 0.35$. On the other hand, the maximal variance $V_{\min} = 0.146$ takes place for the main photon number $n = 2.5 \cdot 10^6$.

2. QUANTUM DISTRIBUTIONS IN THE TIME DOMAIN

In this section we present the results of numerical calculations of the Wigner functions in the phase space. At first, we consider the Wigner function for EPR beams by combining the correlated output modes (1) and (2) with a half beam splitter. This procedure is proposed here for verification of EPR entanglement in the time domain as a two-mode squeezing. Note that the opposite procedure is usually used for generation of CV entangled light beams.

We consider the output behavior of NOPO assuming that all losses occur through the output couples [6]. In this case the output fields of subharmonics are $b_i^{\rm out}(t) = \sqrt{2\gamma} b_i\,(t)$ $(i = 1, 2)$, and the output modes (b_A, b_B) from the half beam splitter can be expressed as

$$b_A = \sqrt{2\gamma}(b_1 + b_2), \quad b_B = \sqrt{2\gamma}(b_1 - b_2). \tag{4}$$

We present below the result for the Wigner functions $W_A(\alpha)$ of the combined, dimensionless modes $\alpha = \alpha_1 + \alpha_2$, corresponding to the operators $b_A/\sqrt{2\gamma}$. A qualitative demonstration of strong EPR entanglement that is below the stationary limit is provided in Fig. 2 that shows the Wigner function of the combined mode (A) for the time interval corresponding to the maximal squeezing. Indeed, the qualitative measurement of the time-dependent squeezing effect can be revealed from our results by considering the quadrature amplitude probability distributions $P(x), P(y)$ $(x = \mathrm{Re}\,(\alpha), y = \mathrm{Im}\,(\alpha))$. Those are plotted in the backgrounds of Fig. 2. Note that probability distribution $P(x, \Phi)$ for any quadrature amplitude operator $X(\Phi) = \frac{1}{\sqrt{2}}\left(b\,\mathrm{e}^{-i\Phi} + b^{+}\mathrm{e}^{i\Phi}\right)$ can be obtained by integrating the Wigner function over the conjugate quadrature:

$$P(x, \Phi) = \int\limits_{-\infty}^{\infty} dp\, W(x\cos\Phi - p\sin\Phi, x\sin\Phi + p\cos\Phi). \tag{5}$$

We next consider Wigner functions of intensity-correlated twin beams following the conditional state-preparation scheme. According to the method of conditional measurement, counting n photons in one of the correlated mode projects the other mode in an n-photon Fock state, which can then be analysed using quantum homodyne tomography. These measurements were recently demonstrated for one-photon Fock state ($n = 1$) [7] as well as for two-photon Fock state ($n = 2$) [8] by using pulsed nondegenerate amplifier producing a pure two-mode squeezed state. Here we consider this problem for the more general case that includes the full description of dissipative and pump field effects in the framework of the theory of periodically pulsed NOPO. The single-photon conditional measurement ($n = 1$), as well as both two-photon ($n = 2$) and three-photon ($n = 3$) measurement schemes are considered. We assume that for the multiphoton cases, $n = 2$ or $n = 3$, the detection of coincidences by the photodiodes operating on a photon-counting regime means that at least two-photon or three-photon states are created by the same pulse.

For this goal we calculate the conditional Wigner functions for light pulses if one of the modes (labeled trigger) is prepared in n-photon Fock state ($n = 1, 2, 3$). When n-photon Fock state $|\psi_n(2)\rangle$ of the trigger mode (2) is detected, the signal mode (1) is prepared in a quantum state whose density operator $\rho_1(n)$ reads as

$$\rho_1(n) = \frac{\langle\psi_n(2)|\,\rho\,|\psi_n(2)\rangle}{\mathrm{Sp}_1\,\langle\psi_n(2)|\,\rho\,|\psi_n(2)\rangle}. \tag{6}$$

We analyze the conditional Wigner functions of the output signal mode (1): $W(1; \rho, \theta)$, $W(2; \rho, \theta)$ and $W(3; \rho, \theta)$ corresponding to the various conditional measurement schemes with $n = 1$, $n = 2$ and $n = 3$ photon Fock states. As we see, all the three Wigner functions clearly display negative regions in the phase space that reflect a highly nonclassical character of quantum states. All Wigner functions are rotationally symmetric and hence the conditional mixed states are phase-independent. In Fig. 2 we show the radial dependence of the Wigner functions.

We stress that these results are in agreement with the experimental results on conditional Wigner functions $W(1)$ and $W(2)$ presented in [7, 8]. Thus, the corresponding conditional mixed states are very close to one-photon and two-photon Fock states. The nonclassicality of the mixed states depicted in Fig. 2 is displayed as quantum interference effects. It is quite reasonable that these effects for the state-preparation schemes based on pulsed NOPO are less than for the case of photon pair generated in the process of parametric down-conversion or

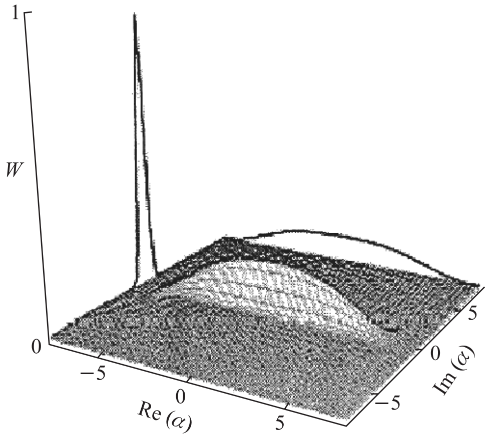

Fig. 2. The Wigner function for the transformed coordinates. The parameters are: $k^2/\gamma\gamma_3 = 0.05, \chi = 1.3\chi_{\mathrm{th}}, \tau = 4\gamma^{-1}, T = 1\gamma^{-1}$

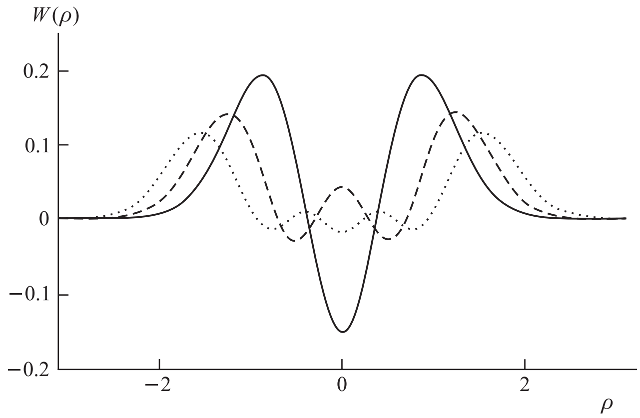

Fig. 3. Radial dependence of Wigner functions for conditional measurement for the parameters $k^2/\gamma\gamma_3 = 0.05, \chi = 0.5\chi_{\mathrm{th}}, \tau = 4\gamma^{-1}, T = 1\gamma^{-1}$: solid curve — $n = 1$; dashed curve — $n = 2$; dotted curve — $n = 3$

in an ideal nondegenerate amplifier producing pure two-mode squeezed states. The negative values of the Wigner functions $W(1)$, $W(2)$ and $W(3)$ decrease with increasing of the photon number of the signal mode. Nevertheless, as our analysis shows, the negativity of Wigner functions also takes place for the pulsed NOPO above threshold. However, in this regime the time intervals when the Wigner functions involve negative values correspond to small number of photons. With increasing of the parameter χ (the intensity of the pump field) such time intervals become shorter. Thus, the time intervals where the Wigner functions contain negative values become shorter as well.

These results demonstrate a highly nonclassical character of time-dependent mixed states generated in periodically pulsed NOPO.

Acknowledgements. The authors acknowledge helpful discussions with Olivier Pfister. This work was supported by NFSAT/CRDF Grant No. UCEP-02/07 and ANSEF Grant No. 666-PS-Opt.

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