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CORRELATIONS OF POLARIZATIONS AND ENTANGLED STATES IN THE TWO-PHOTON SYSTEM

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The correlations of the linear and circular polarizations in the system of two photons have been theoretically investigated. The polarization of a two-photon state is described by the one-photon Stokes parameters and by the components of the correlation «tensor» in the Stokes space. It is shown that in the case of two-photon decays $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, $K_L^0 \to 2\gamma$, $K_S^0 \to 2\gamma$ and the cascade process $|0\rangle \to |1\rangle + \gamma \to |0\rangle + 2\gamma$ ($|0\rangle$ and $|1\rangle$ are states with the spin 0 and 1, respectively) the final two-photon state represents a characteristic example of the entangled (nonfactorizable) state, and the correlations between the Stokes parameters in all these decays have the purely quantum character: the incoherence inequalities of the Bell type for the components of the correlation «tensor», established previously for the case of classical «mixtures», are violated. The general analysis of the registration procedure for two correlated photons by two one-photon detectors is performed.

Теоретически исследованы корреляции линейных и круговых поляризаций в системе двух фотонов. Поляризация двухфотонного состояния описывается через однофотонные параметры Стокса и компоненты корреляционного «тензора» в пространстве Стокса. Показано, что в случае двухфотонных распадов $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, $K_L^0 \to 2\gamma$, $K_S^0 \to 2\gamma$ и каскадного процесса $|0\rangle \to |1\rangle + \gamma \to |0\rangle + 2\gamma$ ($|0\rangle$ и $|1\rangle$ — состояния со спином 0 и 1 соответственно) конечное состояние двух фотонов представляет собой характерный пример запутанного (нефакторизуемого) состояния, и корреляции между параметрами Стокса во всех этих распадах имеют чисто квантовый характер: неравенства некогерентности типа Белла для компонент корреляционного «тензора», установленные ранее для классических смесей, нарушаются. Выполнен общий анализ процедуры регистрации двух коррелированных фотонов двумя однофотонными детекторами.

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INTRODUCTION

Previously, in the works [1–5] the spin correlations of two free particles with spin 1/2 [1–4], as well as the angular correlations between the flight directions of decay products of two particles or resonances [5], reflecting the spin correlations in the system of two unstable particles with arbitrary spin, have been analyzed in detail. In doing so, the spin states of each of the particles were set in the respective rest frames, which is possible only at nonzero masses of both particles.

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In the present work we study the correlation properties of the system of two photons. Since the photon mass is equal to zero, the introduction of spin as the internal angular momentum in the rest frame is inapplicable in this case, and thus for describing the photon polarization the special consideration is required.

1. NONFACTORIZABLE TWO-PARTICLE STATES

We will discuss the nonfactorizable (entangled) states of two photons.

The nonfactorizable (entangled) states of two particles cannot be reduced to the simple direct product of two one-particle states, and they represent coherent superpositions of pairs of one-particle states:

$$|\Phi\rangle^{(1,2)} = \sum_i \sum_k c_{ik} |i\rangle^{(1)} |k\rangle^{(2)}, \tag{1}$$

where c_{ik} are constants, $\sum\limits_i \sum\limits_k |c_{ik}|^2 = 1$.

Generally, correlations at the registration of nonfactorizable two-particle states by one-particle detectors should be considered as the manifestation of the quantum-mechanical effect predicted, at first, by Einstein, Podolsky and Rosen [6]. The essence of this effect is as follows. If a two-particle state is not factorizable, the character of measurements performed for the first particle determines the readings of the detector that analyzes the state of the second particle, although the particles may prove to be at a large distance from each other after their creation. In this case the amplitude of the registration of a two-particle state (1) by two one-particle detectors, selecting the states $|L\rangle^{(1)}$ and $|M\rangle^{(2)}$, is a result of the interference of pairs of one-particle states:

$$A_{LM} = \sum_i \sum_k c_{ik} \langle L|i\rangle^{(1)} \langle M|k\rangle^{(2)}. \tag{2}$$

With this, due to the correlations, the selection of different states $|L\rangle^{(1)}$ and $|M\rangle^{(1)}$ only for the first particle leads to the different states of the second particle:

$$|\Psi\rangle_L^{(2)} = \sum_i \sum_k c_{ik} \langle L|i\rangle |k\rangle^{(2)}, \qquad |\Psi\rangle_M^{(2)} = \sum_i \sum_k c_{ik} \langle M|i\rangle |k\rangle^{(2)}. \tag{3}$$

Let us note that the states $|\Psi\rangle_L^{(2)}$ and $|\Psi\rangle_M^{(2)}$ can be the eigenfunctions of noncommuting operators. As a result, due to the correlations, in the case of a pure entangled two-particle state the corresponding one-particle states for the first and second particles are «mixed»: they should be described by the one-particle density matrices but not by the vectors of state (wave functions). We deal with the «management» by the state of one of two particles without the direct force action on it. A. Einstein considered this situation as a paradox testifying to the incompleteness of the quantum-mechanical description [6].

If a two-particle system itself is a part of a more complicated system, which is described by the two-particle density matrix, the nonfactorizability means that this density matrix cannot be represented as a sum of direct products of one-particle density matrices with non-negative coefficients. In this case we have a «mixed» entangled two-particle state [2, 4].

2. DENSITY MATRIX OF THE TWO-PHOTON SYSTEM

Let us consider the system of two photons with the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$. We introduce two systems of coordinate axes: (x, y, z) with the axis z parallel to the momentum $\mathbf{k}_1$ of the first photon, and $(\tilde{x}, \tilde{y}, \tilde{z})$ with the axis $\tilde{z}$ parallel to the momentum $\mathbf{k}_2$ of the second photon. Let us choose the axes x and $\tilde{x}$ so that they were parallel to each other and perpendicular to the plane passing through the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$. Analogously to the spin density matrix of two particles with spin 1/2 (see, for example, [4]), we can represent the polarization density matrix of two photons in the form:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\left[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \sum_{i=1}^{3} \epsilon_i^{(1)} \hat{\sigma}_i^{(1)} \otimes \hat{I}^{(2)} + \sum_{k=1}^{3} \epsilon_k^{(2)} \hat{I}^{(1)} \otimes \hat{\sigma}_k^{(2)} + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik} \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\right]. \tag{4}$$

Here $\hat{\sigma}_i^{(1)}$, $\hat{\sigma}_k^{(2)}$ are the Pauli matrices; $\epsilon_i^{(1)}$ denotes the Stokes parameters of the first photon [6–8], defined in the system of axes (x, y, z), $\epsilon_k^{(2)}$ denotes the Stokes parameters of the second photon, defined in the system of axes $(\tilde{x}, \tilde{y}, \tilde{z})$, T_{ik} is the correlation «tensor» in the Stokes space, describing the correlation of polarizations of the first and second photons. For independent photons we have $T_{ik} = \epsilon_i^{(1)} \epsilon_k^{(2)}$. In the general case such an equality does not hold.

Let $|3,+\rangle$ and $|3,-\rangle$ be the one-photon states with the full linear polarization along the axes x and y, respectively; let $|2,+\rangle$ and $|2,-\rangle$ be the one-photon states with the right (helicity $+1$) and left (helicity -1) circular polarization, respectively, and let $|1,+\rangle$ and $|1,-\rangle$ be the one-photon states with the full linear polarization along the axis directed at the angles $\pi/4$ and $3\pi/4$, respectively, with respect to the axis x. Then, by definition, the one-photon Stokes parameters are as follows: $\epsilon_i = W_i^{(+)} - W_i^{(-)}$, where $W_i^{(+)}$ and $W_i^{(-)}$ are the probabilities of registering the photon in the states $|i,+\rangle$ and $|i,-\rangle$, respectively ($W_i^{(+)} + W_i^{(-)} = 1$). In doing so, $r = \sqrt{\epsilon_1^2 + \epsilon_3^2}$ is the degree of linear polarization and ϵ_2 is the degree of circular polarization, which are invariant with respect to rotations in the plane (x, y).

Components of the «tensor» T_{ik} can be determined by using the following probabilistic formula (compare with [4]):

$$T_{ik} = W_{i,k}^{(+,+)} - W_{i,k}^{(-,+)} - W_{i,k}^{(+,-)} + W_{i,k}^{(-,-)}. \tag{5}$$

Here $i = 1, 2, 3$, $k = 1, 2, 3$; $W_{i,k}^{(+,+)}$ is the probability of registering the first photon in the state $|i,+\rangle$ and the second photon — in the state $|k,+\rangle$; $W_{i,k}^{(-,+)}$ is the probability of registering the first photon in the state $|i,-\rangle$ and the second photon — in the state $|k,+\rangle$; $W_{i,k}^{(+,-)}$ is the probability of registering the first photon in the state $|i,+\rangle$ and the second photon — in the state $|k,-\rangle$; $W_{i,k}^{(-,-)}$ is the probability of registering the first photon in the state $|i,-\rangle$ and the second photon — in the state $|k,-\rangle$. In accordance with the normalization condition, $W_{i,k}^{(+,+)} + W_{i,k}^{(-,+)} + W_{i,k}^{(+,-)} + W_{i,k}^{(-,-)} = 1$.

In the case of the entirely unpolarized photons we have: $\epsilon_i^{(1)} = \epsilon_i^{(2)} = 0$, $T_{ik} = 0$ (all the Stokes parameters and all components of the correlation tensor equal zero).

3. CORRELATIONS BETWEEN THE STOKES PARAMETERS OF TWO PHOTONS

When defining the two-photon density matrix, we have chosen the pair of transverse unit vectors of polarization of the first and second photons (let us denote them as $\boldsymbol{\chi}_1$ and $\widetilde{\boldsymbol{\chi}}_1$) in the same direction $[\mathbf{k}_1\mathbf{k}_2]$, which is perpendicular to the plane passing through the momenta of two photons $\mathbf{k}_1$ and $\mathbf{k}_2$ ($\boldsymbol{\chi}_1 = \widetilde{\boldsymbol{\chi}}_1$). Two other unit vectors of polarization of the first and second photons $\boldsymbol{\chi}_2$ and $\widetilde{\boldsymbol{\chi}}_2$ satisfy the equalities: $\boldsymbol{\chi}_2\boldsymbol{\chi}_1 = \boldsymbol{\chi}_2\mathbf{k}_1 = 0$, $\widetilde{\boldsymbol{\chi}}_2\widetilde{\boldsymbol{\chi}}_1 = \widetilde{\boldsymbol{\chi}}_2\mathbf{k}_2 = 0$, $\widetilde{\boldsymbol{\chi}}_2\boldsymbol{\chi}_2 = \cos\beta$, where β is the angle between the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$.

We will consider the transverse unit vectors as spatial parts of the unit 4-vectors χ_1 and χ_2, $\widetilde{\chi}_1$ and $\widetilde{\chi}_2$; let us introduce further the gradient transformations at the transition to the frame moving with the 4-velocity u [7]:

$$\chi_1' = \chi_1 - k_1\frac{\chi_1 u}{k_1 u}, \quad \chi_2' = \chi_2 - k_1\frac{\chi_2 u}{k_1 u}; \quad \widetilde{\chi}_1' = \widetilde{\chi}_1 - k_2\frac{\widetilde{\chi}_1 u}{k_2 u}, \quad \widetilde{\chi}_2' = \widetilde{\chi}_2 - k_2\frac{\widetilde{\chi}_2 u}{k_2 u}, \tag{6}$$

where k_1 and k_2 are 4-momenta of the first and second photons.

In the basis of the 4-vectors (6) the polarization density matrix of two photons (4) is invariant with respect to the Lorentz transformations. In accordance with this, the Stokes parameters of the first and second photons $\epsilon_i^{(1)}, \epsilon_k^{(2)}$ [7–9] and all the components of the correlation tensor T_{ik} ($i, k = 1, 2, 3$) are Lorentz-invariant.

Due to the transversality of polarization unit vectors in any frame, at the transition from the initial frame 1 to the frame 2, moving with the velocity $\mathbf{v}$ with respect to the frame 1, their spatial orientation changes: the unit vectors of polarization of the first photon $\boldsymbol{\chi}_1$ and $\boldsymbol{\chi}_2$ and those for the second photon $\widetilde{\boldsymbol{\chi}}_1$ and $\widetilde{\boldsymbol{\chi}}_2$ turn around the vectors $[\mathbf{v}\,\mathbf{k}_1]$ and $[\mathbf{v}\,\mathbf{k}_2]$, respectively, by the positive aberration angles $\theta_1 = \arcsin(|\,[\mathbf{k}_1\mathbf{k}_1']\,|/(|\mathbf{k}_1|\,|\mathbf{k}_1'|))$ and $\theta_2 = \arcsin(|\,[\mathbf{k}_2\mathbf{k}_2']\,|/(|\mathbf{k}_2|\,|\mathbf{k}_2'|))$, where $\mathbf{k}_1'$ and $\mathbf{k}_2'$ are the momenta of the first and second photons in the frame 2.

Let us introduce now the frame of the center-of-inertia of two photons. This frame always exists, if the momenta of two photons are not parallel to each other. The velocity of the center-of-inertia frame with respect to the given frame is determined by the formula $\mathbf{v} = (\mathbf{k}_1 + \mathbf{k}_2)/(|\mathbf{k}_1| + |\mathbf{k}_2|)$. It is clear that at the transition to the c.i. frame the unit vectors of polarization of the first and second photons turn in opposite directions around the axis being parallel to the vector $[\mathbf{k}_1\,\mathbf{k}_2]$. In doing so, in the c.i. frame $\mathbf{k}_1 = -\mathbf{k}_2$, $\boldsymbol{\chi}_1' = \widetilde{\boldsymbol{\chi}}_1'$, $\boldsymbol{\chi}_2' = -\widetilde{\boldsymbol{\chi}}_2'$.

Let us consider, as an example, the decay $\pi^0 \to 2\gamma$. In the π^0-meson rest frame (coinciding with the c.i. frame of two γ quanta) the decay amplitude has the structure: $A_{\gamma\gamma} \sim ([\mathbf{e}^{(1)*}\mathbf{e}^{(2)*}]\mathbf{n})$, where $\mathbf{n}$ is the unit vector directed along the momentum of one of the photons, $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are complex unit vectors of polarization of the first and second photons, respectively, being perpendicular to the vector $\mathbf{n}$. So, we find that in this case all the Stokes parameters of the first and second photons are equal to zero (thus, the single-photon states are unpolarized: $\epsilon_i^{(1)} = \epsilon_k^{(2)} = 0$, $i, k = 1, 2, 3$). Meantime, according to (5), the two-photon system is correlated: $T_{11} = +1$, $T_{22} = +1$, $T_{33} = -1$, all the nondiagonal components of the correlation tensor T_{ik} equaling zero. Let us remark that the equality $T_{22} = +1$, according to which the helicities of two photons at the decay $\pi^0 \to 2\gamma$ coincide, follows from the fact that the π^0 meson has zero spin. Meantime, the equality $T_{33} = -1$, according to which the linear polarizations of two

γ quanta are mutually perpendicular, is the consequence of the negative internal parity of the π^0 meson.

Taking into account the above-considered changes of spatial orientation of polarization unit vectors, the values of polarization parameters of two γ quanta at the decay $\pi^0 \to 2\gamma$, indicated above, remain valid in any frame (in particular, in the laboratory frame, where the decaying π^0 meson is moving). It is clear that the same holds also for the decays $\eta \to 2\gamma$, $K_L^0 \to 2\gamma$[1], as well as for the para-positronium decay into two γ quanta.

4. REGISTRATION OF THE SYSTEM OF TWO CORRELATED PHOTONS

The probability of registration of a system of two photons with two one-photon detectors, selecting the state of the first photon with the Stokes parameters $\xi_1^{(1)}, \xi_2^{(1)}, \xi_3^{(1)}$, being specified in the representation of the above-indicated unit vectors $\boldsymbol{\chi}_1$ and $\boldsymbol{\chi}_2$ and the state of the second photon with the Stokes parameters $\xi_1^{(2)}, \xi_2^{(2)}, \xi_3^{(2)}$, being specified in the representation of the unit vectors $\widetilde{\boldsymbol{\chi}}_1$ and $\widetilde{\boldsymbol{\chi}}_2$, is described, according to the density matrix (4), by the following correlation formula:

$$W \sim 1 + \sum_{i=1}^{3} \epsilon_i^{(1)} \xi_i^{(1)} + \sum_{k=1}^{3} \epsilon_k^{(2)} \xi_k^{(2)} + \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik}\, \xi_i^{(1)} \xi_k^{(2)}. \tag{7}$$

The «final» Stokes parameters have the meaning of analyzing powers. In particular, the Compton scattering on an unpolarized electron, selecting the states with the polarization vector being perpendicular to the scattering plane and the states with the polarization vector lying in the scattering plane, is a characteristic analyzer of the photon linear polarization. In the representation of these states the analyzing power is determined by one parameter, namely, by the coefficient of left–right azimuthal asymmetry at the Compton scattering of a linearly polarized photon:

$$r(\omega, \theta_{\rm sc}) = \frac{\sin^2 \theta_{\rm sc}}{(\omega_f/\omega) + (\omega/\omega_f) - \sin^2 \theta_{\rm sc}}, \tag{8}$$

where $\theta_{\rm sc}$ is the angle of the photon scattering in the laboratory frame; ω and ω_f are the photon energies before and after the Compton scattering, respectively. In the representation of the polarization unit vectors $\boldsymbol{\chi}_1$, $\boldsymbol{\chi}_2$ and $\widetilde{\boldsymbol{\chi}}_1$, $\widetilde{\boldsymbol{\chi}}_2$, which have been introduced earlier for describing the polarization properties of the system of two γ quanta, the analyzing powers are related to the «vectors» in the Stokes space $\boldsymbol{\xi}^{(1)} = (\xi_1^{(1)}, 0, \xi_3^{(1)})$ and $\boldsymbol{\xi}^{(2)} = (\xi_1^{(2)}, 0, \xi_3^{(2)})$, where $(j = 1, 2)$:

$$\xi_1^{(j)} = r(\omega_j, \theta_{\rm sc}^{(j)})\, \sin 2\psi_{\rm sc}^{(j)}, \quad \xi_3^{(j)} = r(\omega_j, \theta_{\rm sc}^{(j)})\, \cos 2\psi_{\rm sc}^{(j)}. \tag{9}$$

[1]Neglecting the effects of CP-invariance violation, the CP-parity of the long-lived neutral kaon K_L^0 is negative. Meantime, the amplitude of the two-photon decay of the short-lived neutral kaon K_S^0 with the positive CP-parity has the structure: $A_{K_S^0 \to 2\gamma} \sim (\mathbf{e}^{(1)*}\, \mathbf{e}^{(2)*})$. In this case the linear polarizations of the first and second photons, as well as their helicities, are mutually equal: $T_{11} = -1, T_{22} = +1, T_{33} = +1$.

Here $\psi_{\rm sc}^{(1)}$ ($\psi_{\rm sc}^{(2)}$) is the angle between the plane of Compton scattering of the first (second) photon and the plane $(\mathbf{k}_1, \mathbf{k}_2)$, passing through the momenta of two photons. Taking into account the values of components of the correlation tensor (see above), it follows from the relations (7) and (9) that the correlation of the planes of Compton scattering of two γ quanta, produced in the decay $\pi^0 \to 2\gamma$, will have the form:

$$d^2W = \frac{d\psi_{\rm sc}^{(1)} d\psi_{\rm sc}^{(2)}}{4\pi^2} \left[1 - r(\omega_1, \theta_{\rm sc}^{(1)})\, r(\omega_2, \theta_{\rm sc}^{(2)})\, \cos 2(\psi_{\rm sc}^{(1)} + \psi_{\rm sc}^{(2)})\right]. \tag{10}$$

In the c.i. frame of two photons ($\mathbf{k}_1 = -\mathbf{k}_2$) the angle $\psi = \psi_{\rm sc}^{(1)} + \psi_{\rm sc}^{(2)}$ is equal to the angle between the planes of Compton scattering of two photons, and we have

$$dW = \frac{1}{2\pi} \left[1 - r(\omega_1, \theta_{\rm sc}^{(1)})\, r(\omega_2, \theta_{\rm sc}^{(2)}) \cos 2\psi\right] d\psi. \tag{11}$$

5. QUANTUM CHARACTER OF THE TWO-PHOTON CORRELATIONS

According to the results of the works [2, 4], in the case of incoherent («classical») mixtures of factorizable states of two spin-1/2 particles the modulus of the sum of any two diagonal components of the correlation tensor cannot exceed unity. The same incoherence inequalities for diagonal components of the correlation «tensor» in the Stokes space should hold for incoherent mixtures of factorizable two-photon states.

However, for nonfactorizable two-photon states the incoherence inequalities can be essentially violated. Indeed, in the case of the above-considered decays $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$ and $K_L^0 \to 2\gamma$ the final two-photon system is produced in the following entangled state (in the representation of polarization unit vectors):

$\Phi^{(1,2)} = \frac{1}{\sqrt{2}}(|\boldsymbol{\chi}_1\rangle \otimes |\widetilde{\boldsymbol{\chi}}_2\rangle + |\boldsymbol{\chi}_2\rangle \otimes |\widetilde{\boldsymbol{\chi}}_1\rangle)$, and we see that one of the incoherence inequalities is violated: in the c.i. frame of two γ quanta we have: $T_{11} + T_{22} = 2 > 1$.

In the decay $K_S^0 \to 2\gamma$ the final two-photon system is generated in the entangled state of other structure:

$\Phi'^{(1,2)} = \frac{1}{\sqrt{2}}(|\boldsymbol{\chi}_1\rangle \otimes |\widetilde{\boldsymbol{\chi}}_1\rangle - |\boldsymbol{\chi}_2\rangle \otimes |\widetilde{\boldsymbol{\chi}}_2\rangle)$, and here we have: $T_{22} + T_{33} = 2 > 1$, i.e., again one of the incoherence inequalities is violated.

Thus, the correlations of polarizations of two photons in all these decays have the strongly pronounced quantum character.

It is interesting to consider also, from this viewpoint, the cascade decay $|0\rangle \to |1\rangle + \gamma$; $|1\rangle \to |0\rangle + \gamma$ with the emission of two photons (the spins of the initial and final states equal zero, and the spin of the intermediate state equals 1). Let us denote by $\mathbf{B}_m$ the complex vector, normalized to unity, corresponding to the intermediate state with the spin projection m onto the quantization axis. The amplitude of cascade transition has the structure:

$$A_{\gamma\gamma} \sim \sum_{m=0,\pm 1} (\mathbf{e}^{(1)*}\mathbf{B}_m^*)(\mathbf{B}_m \mathbf{e}^{(2)*}) \sim (\mathbf{e}^{(1)*}\mathbf{e}^{(2)*}),$$

where $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are the vectors of polarization of two cascade photons, respectively. Within the same representation of polarization unit vectors as before, the Stokes parameters and the components of the correlation «tensor» (5) have the values:

$$\begin{gathered}\epsilon_1^{(1)} = \epsilon_2^{(1)} = \epsilon_3^{(1)} = 0; \quad \epsilon_1^{(2)} = \epsilon_2^{(2)} = \epsilon_3^{(2)} = 0; \\ T_{12} = T_{21} = T_{13} = T_{31} = T_{23} = T_{32} = 0; \\ T_{11} = \frac{2\cos\theta}{1+\cos^2\theta}, \quad T_{22} = -\frac{2\cos\theta}{1+\cos^2\theta}, \quad T_{33} = 1,\end{gathered} \tag{12}$$

where θ is the angle between the momenta of two photons, as before.

At $\theta = 0$, when the photon momenta are parallel, we have: $T_{22} = -1$ (the photon helicities are mutually opposite, which follows directly from the fact of conservation of the projection of angular momentum onto the coordinate axis in the cascade decay). At $\theta = \pi$, when the photon momenta are antiparallel, $T_{22} = +1$ (the photon helicities are the same).

According to (12), within the interval of angles $\pi/2 > \theta > 0$ $T_{33} + T_{11} > 1$, and within the interval of angles $\pi > \theta > \pi/2$ $T_{33} + T_{22} > 1$. So, in this case one of the incoherence inequalities is also violated.

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