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PION DISSOCIATION AND LEVINSON'S THEOREM IN HOT **PNJL** QUARK MATTER

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Вергилюк А. и др. E2-2013-19
Диссоциация пиона и теорема Левинсона
в теории Намбу–Иона-Лазинио с петлей Полякова

В рамках обобщенного подхода Бета–Уленбека, основанного на модели Намбу–Иона-Лазинио с петлей Полякова, позволяющего единым образом описать связанные, резонансные состояния и состояния рассеяния, проведено описание диссоциации пиона в кварковой материи. На первом шаге используется форма спектральной функции в виде функции Брейта–Вигнера. Учет теоремы Левинсона приводит к тому, что выше точки Мотта при диссоциации пиона происходит уширение спектральной функции и уменьшение роли пиона. Таким образом, показано, что состояния рассеяния в непрерывном спектре играют важную роль в термодинамике диссоциации пиона.

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Pion dissociation by the Mott effect in quark plasma is described within the generalized Beth–Uhlenbeck approach on the basis of the PNJL model, which allows for a unified description of bound, resonant and scattering states. As a first approximation, we utilize the Breit–Wigner ansatz for the spectral function and clarify its relation to the complex mass pole solution of the pion Bethe–Salpeter equation. Application of the Levinson theorem proves that describing the pion Mott dissociation solely by means of spectral broadening of the pion bound state beyond T_{Mott} leaves out a significant aspect. Thus, we acknowledge the importance of the continuum of scattering states and show its role for the thermodynamics of pion dissociation.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

We investigate the thermodynamics of mesonic bound states in hot medium consisting of a nonideal quark plasma with correlations in the pion and sigma meson interaction channels. A special emphasis is put on a correct description of the bound states' dissociation in the vicinity of and beyond the Mott temperature. To this end, we utilize the Polyakov-loop-extended Nambu–Jona-Lasinio model at finite temperature.

The model has two order parameters: the chiral condensate, determining the value of the dynamically generated quark mass $m(T)$ related to the chiral symmetry breaking/restoration transition, and the Polyakov loop variable $\Phi(T)$ which is an order parameter for deconfinement. Their values are obtained from self-consistent solutions of the coupled gap equations for these parameters which correspond to the location of the minimum of the thermodynamical potential $\Omega(m, \Phi; T)$ in the mean-field approximation (MFA). Going beyond the MFA, we evaluate the contributions from pion and sigma meson fluctuations within the Gaussian approximation to the path-integral representation of the thermodynamic potential. Above the Mott temperature, one observes the spectral broadening of bound states leading to the appearance of complex mass poles in the mesonic propagators. A proper analysis leads to coupled Bethe–Salpeter equations from which meson masses and corresponding spectral widths are obtained. Next, the equation of state for the quark–meson system is obtained in the form of a generalized Beth–Uhlenbeck equation [1] which describes the effects that chiral symmetry restoration and deconfinement have on the contributions from pions and sigma mesons and allows for an adequate description of bound, resonant and scattering states on equal footing. The spectral functions in the mesonic channels are evaluated from the complex-valued polarization loop integrals. An examination of the Levinson theorem [2] proves that in order to correctly describe the Mott effect, the continnum of mesonic correlations (scattering states) has to be taken into the account. As a result of this analysis, we obtain a description of pion dissociation where the effect of the vanishing bound state is exactly compensated by the occurrence of a resonance in the continuum of scattering states.

The present work improves on previous works within the NJL model [3, 4], where unphysical quark degrees of freedom appeared in the hadronic phase due

to the lack of the coupling to the Polyakov loop and the Stefan–Boltzmann limit was not obtained due to a misplaced momentum cutoff. We also improve the recent work [5] by properly discussing the role of Levinson's theorem in the PNJL model approach to the generalized Beth–Uhlenbeck equation of state for the quark–meson plasma. The model presented joins both exact limits of finite-temperature QCD — the pion gas at low temperatures and the quark–gluon Stefan–Boltzmann limit at high temperatures — within a microscopic chiral quark model approach.

2. QUARKS AND LIGHT MESONS IN THE PNJL MODEL

The definition of the Nambu–Jona-Lasinio model [6–12] with Polyakov loop [13–17] is given in the Appendix A. Here we start from the expansion of the thermodynamic potential

$$\Omega\left(T,\mu\right)=-\frac{T}{V}\ln\mathcal{Z}\left[T,V,\mu\right] \tag{1}$$

around homogeneous mean field values, which leads to the decomposition of the auxiliary fields σ' and $\vec{\pi}'$ into their mean field and fluctuation parts:

$$\sigma'=\sigma_{\rm MF}+\sigma,\quad \vec{\pi}'=\vec{\pi}\quad(\vec{\pi}_{\rm MF}=0). \tag{2}$$

Utilizing the decomposed fields results in the factorization of the partition function into the respective parts describing mean-field and fluctuation contributions:

$$\mathcal{Z}_{\rm MF}[T,V,\mu]=\exp\left\{-\frac{V}{T}\left(\frac{\sigma_{\rm MF}^2}{4G_S}+U(\Phi,\overline{\Phi};T)\right)+{\rm Tr}\ln\left[\beta S_{\rm MF}^{-1}[m]\right]\right\}, \tag{3}$$

$$\mathcal{Z}_{\rm FL}[T,V,\mu]=\int\mathcal{D}\sigma\mathcal{D}\vec{\pi}\exp\left\{-\left[\int\limits_0^\beta d\tau\int\limits_V d^3x\,\frac{2\sigma\sigma_{\rm MF}+\sigma^2+\vec{\pi}^2}{4G_S}\right]+\right.$$
$$\left.+{\rm Tr}\ln\left[1-S_{\rm MF}[m]\left(\sigma+i\gamma_5\vec{\tau}\vec{\pi}\right)\right]\right\}. \tag{4}$$

Here the mean-field inverse propagator is

$$S_{\rm MF}^{-1}=\gamma^0(i\omega_n-\mu+A_0)-\vec{\gamma}\cdot\vec{p}-m_0-\sigma_{\rm MF}=$$
$$=\gamma^0(i\omega_n-\mu+A_0)-\vec{\gamma}\cdot\vec{p}-m. \tag{5}$$

The thermodynamic potential in the mean-field approximation of the PNJL model is evaluated to be given by the following expression [17]:

$$\Omega_{\rm MF} = \frac{\sigma^2_{\rm MF}}{4G_S} + U(\Phi,\overline{\Phi};T) - \\ - 2N_f \int \frac{d^3p}{(2\pi)^3} \left\{ N_c E_p + T\big[\ln N^-_\Phi(E_p) + \ln N^+_\Phi(E_p) \big] \right\}, \quad (6)$$

where factors N_f, N_c originate from performing the trace operation and are a consequence of isospin and color symmetry. The quark energy is given by $E_p = \sqrt{\mathbf{p}^2 + m^2}$, $E_p^\mp$ are defined as $E_p^\mp = E_p \mp \mu$, and

$$N^-_\Phi(E_p) = \left[1 + 3\left(\Phi + \overline{\Phi} \mathrm{e}^{-\beta E_p^-} \right) \mathrm{e}^{-\beta E_p^-} + \mathrm{e}^{-3\beta E_p^-} \right], \quad (7)$$

$$N^+_\Phi(E_p) = \left[1 + 3\left(\overline{\Phi} + \Phi \mathrm{e}^{-\beta E_p^+} \right) \mathrm{e}^{-\beta E_p^+} + \mathrm{e}^{-3\beta E_p^+} \right]. \quad (8)$$

In the mean-field approximation of the PNJL model, the values of the constituent quark mass m and the Polyakov-loop variable Φ, along with its complex conjugate $\overline{\Phi}$, are obtained from the condition that the thermodynamic potential should be minimized with respect to these parameters, which is augmented by the stability conditions. For $\mu = 0$, we have $\Phi = \overline{\Phi}$ and thus the minimizing conditions are given by

$$\frac{\partial \Omega_{\rm MF}}{\partial \sigma_{\rm MF}} = 0, \quad \frac{\partial \Omega_{\rm MF}}{\partial \Phi} = 0. \quad (9)$$

These conditions are equivalent to a set of coupled gap equations [15, 17]. For the mass gap equation we get

$$m \;\; = \;\; m_0 + 4N_f N_c G_S \int^\Lambda \frac{d^3p}{(2\pi)^3} \, \frac{m}{E_p} \big[1 - f^-_\Phi(E_p) - f^+_\Phi(E_p) \big], \quad (10)$$

where

$$f^\mp_\Phi(E_p) = \big[\Phi \, \mathrm{e}^{-\beta(E_p \mp \mu)} + 2\Phi \, \mathrm{e}^{-2\beta(E_p \mp \mu)} + \mathrm{e}^{-3\beta(E_p \mp \mu)} \big] / N^\mp_\Phi(E_p) \quad (11)$$

are the so-called generalized Fermi functions, characteristic for the PNJL model. One should note that if $\Phi \to 1$, the expression (11) reduces to the standard NJL-model Fermi functions. For PNJL calculations we should additionally find the values of Φ from corresponding gap equation [17] at given T and μ.

In order to solve (10), a set of model parameters has to be determined: the cutoff parameter Λ, the current quark mass m_0 (in the chiral limit $m_0 = 0$), and the coupling constant G_S. These parameters are fitted at $T = 0$ to reproduce

physical quantities: the pion mass $M_\pi = 135$ MeV, the pion decay constant $F_\pi = 92.4$ MeV, and the quark condensate $\langle\overline{q}q\rangle^{1/3} = -240.772$ MeV. The used parameters [18] are shown in Table 1.

Table 1. The set of model parameters reproducing observable quantities (in brackets) and $\langle\overline{q}q\rangle^{1/3} = -240.772$ MeV [18]

m_0, MeV	Λ, MeV	$G_S\Lambda^2$	F_π, MeV	M_π, MeV
5.495	602.472	2.318	(92.4)	(135)

Since NJL-type models are non-renormalizable, it is necessary to introduce a regularization, e.g., by a cutoff Λ in the momentum integration. Following [17], in this study, in case of thermodynamic quantities, we use the three-dimensional momentum cutoff for vacuum terms and extend the integration till infinity for finite temperatures. A comprehensive study of the differences between the two regularization procedures (with and without cutoff on the quark momentum states at finite temperature) has been performed in [19].

Solutions of the gap equation (10) and the corresponding gap equation for the Polyakov-loop variable Φ at zero chemical potential and nonzero T are presented in Fig. 1. Above the critical temperature, which is equal to $T_c = 237$ MeV in the chiral limit and $T_c = 251$ MeV for a finite current quark mass, one observes chiral symmetry restoration indicated by the rapid decrease of the constituent quark mass and the Polyakov-loop variable becoming close to 1. The corresponding mean-field contribution to the pressure, given by $P = -\Omega_{\rm MF}$, is shown in Fig. 6 (scaled

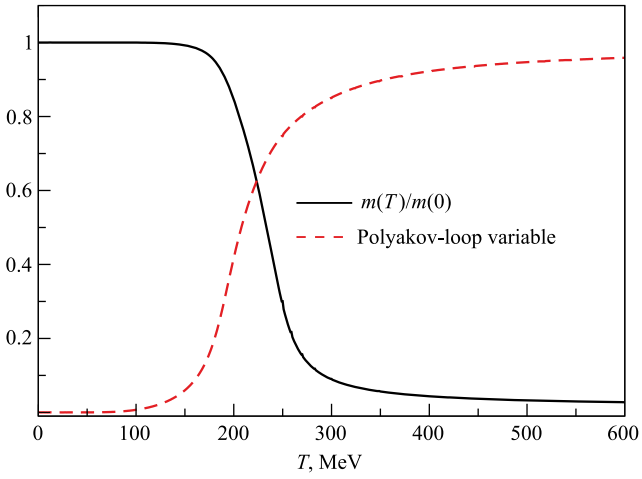

Fig. 1. Temperature dependence of the quark masses $m(T)$ and Polyakov-loop variable $\Phi(T)$ at $\mu = 0$. Results for quark mass are scaled by $m(0) = 367.5$ MeV

by a factor T^4) in Section 5. One observes that, in opposition to classical NJL models (e.g., [3, 4]), quark degrees of freedom are suppressed below the critical temperature in the PNJL model. Moreover, above the critical temperature, the gluonic degrees of freedom are accounted for correctly.

The contribution to the thermodynamics stemming from the fluctuations described by (4) is evaluated in a scheme where we expand the logarithm up to the second (Gaussian) order according to

$$\ln(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k} = -x - \frac{1}{2}x^2 + \ldots, \quad |x| < 1, \tag{12}$$

to get

$$\mathcal{Z}^{(2)}_{\mathrm{FL}}[T,V,\mu] = \int \mathcal{D}\sigma\mathcal{D}\vec{\pi} \exp\left\{ -\left[\int_0^\beta d\tau \int_V d^3x \, \frac{\sigma^2+\vec{\pi}^2}{4G_S}\right] - \right.$$
$$\left. - \frac{1}{2}\mathrm{Tr}\left(S_{\mathrm{MF}}[m]\Sigma[\sigma,\vec{\pi}]S_{\mathrm{MF}}[m]\Sigma[\sigma,\vec{\pi}]\right)\right\}, \tag{13}$$

where we have introduced $\Sigma[\sigma,\vec{\pi}] = \sigma + i\gamma_5\vec{\tau}\vec{\pi}$.

Performing the calculation leads to the subsequent factorization of the thermodynamic potential into parts describing the contribution from mesonic correlations corresponding to σ and $\vec{\pi}$ channels of interaction

$$\mathcal{Z}^{(2)}_{\mathrm{FL}}[T,V,\mu] = \left[\det\left(\frac{1}{2G_S} - \Pi_\sigma(q_0,\vec{q})\right)\right]^{-1/2} \times$$
$$\times \left[\det\left(\frac{1}{2G_S} - \Pi_{\vec{\pi}}(q_0,\vec{q})\right)\right]^{-3/2} \tag{14}$$

with the polarization loop $\Pi_M(q_0,\vec{q})$ given explicitly by

$$\Pi_M(q_0,\vec{q}) = -N_cN_f \sum_{s,s'=\pm1} \int \frac{d^3p}{(2\pi)^3} \frac{1 - f^-_\Phi(-s'E_k) - f^+_\Phi(sE_p)}{q_0 + s'E_k - sE_p} \times$$
$$\times \left(1 - ss' \frac{\mathbf{p}\cdot(\mathbf{p}-\mathbf{q}) \mp m^2}{E_pE_{p-q}}\right). \tag{15}$$

One easily obtains the thermodynamic potential corresponding to the chosen meson part of the partition function up to the Gaussian order given by

$$\Omega^{(2)}_M(T,\mu) = \frac{d_M}{2}\frac{T}{V}\,\mathrm{Tr}\ln\, S_M^{-1}, \tag{16}$$

where the degeneracy factor d_M equals 1 for sigma mesons and 2 for pions.

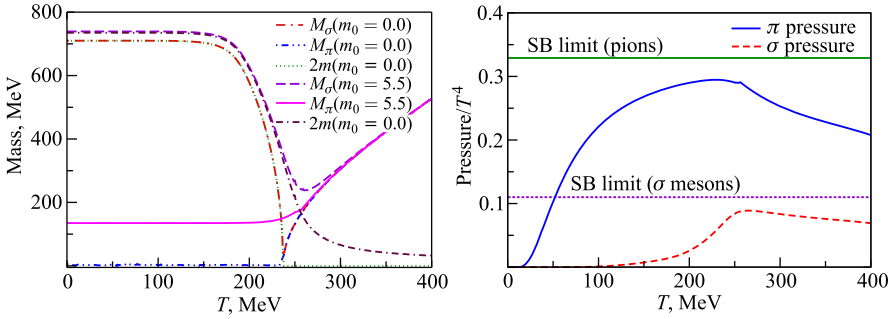

Fig. 2. Temperature dependence of the meson masses M_M (left panel) and meson pressure for $\vec{\pi}$ (solid lines) and σ (dotted lines)

From the point of view of the polarization operators, the pseudoscalar and scalar meson masses can be defined by the condition that for $q^2 = M_M^2$ the corresponding polarization operator $\Pi_M(M_M^2)$ leads to a bound-state pole in the meson correlation function [17]. For mesons at rest ($\mathbf{q} = 0$) these conditions correspond to the Bethe–Salpeter equations

$$1 + 4GN_cN_f \int \frac{d^3p}{(2\pi)^3} \frac{4E_p}{M_\pi^2 - 4E_p^2}(1 - f_\Phi^- - f_\Phi^+) = 0, \tag{17}$$

$$1 + 4GN_cN_f \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^2}{E_p^2} \frac{4E_p}{M_\sigma^2 - 4E_p^2}(1 - f_\Phi^- - f_\Phi^+) = 0. \tag{18}$$

Solutions of the two Bethe–Salpeter equations (17) and (18) constitute the set of meson masses and are presented in Fig. 2 (left panel). The Mott temperature, which is defined by the condition $M_\pi(T_{\rm Mott}) = 2m_q(T_{\rm Mott})$, is for given parameters $T_{\rm Mott} \simeq 231$ MeV in the chiral limit and $T_{\rm Mott} \simeq 256$ MeV away from it. The modification of quasiparticle properties is clearly visible: up to the Mott temperature $T_{\rm Mott}$, the σ mass practically follows the behaviour of $2m_q(T)$, with a drop towards the pion mass, signalling chiral symmetry restoration (in the chiral limit the σ mass exactly coincides with twice the quark mass up to $T_{\rm Mott}$). In the same region, the pion mass remains practically constant (and equals zero in the chiral limit). At $T \simeq T_{\rm Mott}$, however, the masses of chiral partners become approximately degenerate, $M_\sigma \approx M_\pi$, and then both masses increase linearly with temperature.

The corresponding meson pressure is shown in Fig. 2 (right panel).

3. GENERALIZED BETH–UHLENBECK APPROACH TO THE QUARK–MESON PLASMA

Deriving the thermodynamic equation of state for a quark–meson system in the Beth and Uhlenbeck form reduces to the introduction of scattering phase

shifts into the formula for the thermodynamic potential. This requires that we analytically continue the propagator into the complex plane.

In a first step, we introduce meson spectral functions $A_M^g(\omega,\vec{q})$ by utilizing the integral representation of the logarithm in (16), followed by the usual expression of the propagator by means of $A_M^g(\omega,\vec{q})$ [20], i.e.,

$$\ln S_M^{-1} = -\int_0^{G_S} dg \frac{1}{2g^2}\frac{1}{\frac{1}{2g}-\Pi_M(q_0,\vec{q})} = -\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{1}{q_0-\omega}\int_0^{G_S}\frac{dg}{2g^2}A_M^g(\omega,\vec{q}). \tag{19}$$

Using the fact that the spectral density is given by the discontinuity of the propagator at the real axis, we arrive at

$$\int_0^{G_S}\frac{dg}{2g^2}A_M^g(\omega,\vec{q}) = -i\int_0^{G_S}\frac{dg}{2g^2}\big(S_M^g(\omega+i\eta,\vec{q})-S_M^g(\omega-i\eta,\vec{q})\big) = \\ = -i\ln\left(\frac{1-2G_S\Pi_M(\omega-i\eta,\vec{q})}{1-2G_S\Pi_M(\omega+i\eta,\vec{q})}\right), \tag{20}$$

where the argument of the logarithm is by definition the scattering matrix $\$_M(\omega,\vec{q})$ in the Jost representation [3]. This normalized complex function can also be represented by means of a scattering phase shift $\$_M(\omega,\vec{q}) = \exp[2i\Phi_M(\omega,\vec{q})]$, which allows us to identify

$$\int_0^{G_S}\frac{dg}{2g^2}A_M^g(\omega,\vec{q}) = -i\ln\$_M(\omega,\vec{q}) = 2\Phi_M(\omega,\vec{q}). \tag{21}$$

Utilizing the above identity leads, after performing the trace operation, to the following result:

$$\Omega_M^{(2)}(T,\mu) = -\frac{N_M}{2}\int\frac{d^3q}{(2\pi)^3}\left(\int_0^{+\infty}\frac{d\omega}{\pi}\Big[\omega+2T\ln\big(1-\mathrm{e}^{-\beta\omega}\big)\Big]\frac{d\Phi_M(\omega,\vec{q})}{d\omega}\right) = \\ = -\frac{N_M}{2}\int\frac{d^3q}{(2\pi)^3}\left(\int_{-q^2}^{+\infty}\frac{ds}{\pi}\left[\sqrt{q^2+s}+2T\ln\big(1-\mathrm{e}^{-\beta\sqrt{q^2+s}}\big)\right]\frac{d\Phi_M(s)}{ds}\right) = \\ = -\frac{N_M}{2}\int\frac{d^3q}{(2\pi)^3}\left(\int_{-q^2}^{+\infty}ds\left[\sqrt{q^2+s}+2T\ln\big(1-\mathrm{e}^{-\beta\sqrt{q^2+s}}\big)\right]D_M(s)\right), \tag{22}$$

where $D_M(s)$ is a generalized mass distribution (density of states) containing all the dynamics of the system. The above expression is the so-called generalized Beth–Uhlenbeck form of the thermodynamic potential. The connection to the original Beth–Uhlenbeck expression for the second virial coefficient can be found in [3] in detail.

For an analysis of meson masses and corresponding widths, necessary to obtain a description of mesonic correlations as bound states dissolving into resonant states above the Mott temperature, one splits the polarization function $\Pi_M(q_0, \vec{q})$ according to

$$\Pi_M\left(q_0, \vec{0}\right) = 4N_c N_f\, I_1 - 2N_c N_f P_M\ I_2(q_0) = \widetilde{I}_1 - P_M \widetilde{I}_2(q_0), \tag{23}$$

where, in the limit $\vec{q} = 0$ that we utilize, the integrals I_1 and I_2 are given by

$$I_1 = \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{2E_p} \big(1 - f_\Phi^-(E_p) - f_\Phi^+(E_p)\big) \right], \tag{24}$$

$$I_2(q_0) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{4E_p^2} \left[\frac{1 - f_\Phi^-(E_p) - f_\Phi^+(E_p)}{2E_p - q_0} + \frac{1 - f_\Phi^-(E_p) - f_\Phi^+(E_p)}{2E_p + q_0} \right], \tag{25}$$

with $P_M = -q_0^2$ for pions and $P_M = -q_0^2 + 4m^2$ for sigma mesons. By identifying $q_0 = M_M - i\frac{1}{2}\Gamma_M$, one can perform the complex mass pole analysis leading to

$$P_M = -\frac{\dfrac{1}{4N_c N_f G_S} - 2I_1}{\left| I_2\left(q_0 = M_M - i\dfrac{1}{2}\Gamma_M\right)\right|^2} (\mathrm{Re}\, I_2(M_M) - i\, \mathrm{Im}\, I_2(M_M)), \tag{26}$$

which decomposes into coupled Bethe–Salpeter equations for meson mass and meson spectral width.

In the first departure beyond the pole approximation (Appendix B), it is justified above $T_{\rm Mott}$ to consider $D_M(s)$ to be described by a Breit–Wigner type function

$$A_R(s,T) = a_R \frac{M_M \Gamma_M}{\big(s - M_M^2\big)^2 + (M_M \Gamma_M)^2}, \tag{27}$$

where M_M is the meson pole mass, Γ_M is the corresponding meson width, and a_R is a normalization factor. Below $T_{\rm Mott}$, where the spectral broadening $\Gamma(T)$ of the states vanishes, the above expression becomes the delta function typical

for the spectral function of a mesonic bound state. The meson phase shift Φ_M corresponding to (27) should be of the form

$$\Phi_M(s) \approx \phi_R(s) = \frac{\pi}{\frac{\pi}{2} - \arctan\left(\frac{4m^2 - M_M^2}{M_M\Gamma_M}\right)} \times \\ \times \left(\arctan\left[\frac{s - M_M^2}{M_M\Gamma_M}\right] - \arctan\left[\frac{4m^2 - M_M^2}{M_M\Gamma_M}\right]\right), \quad (28)$$

where, by introducing the notation $\phi_R(s)$, we acknowledge the fact that the above phase shift is connected with resonant properties of the mesonic correlations.

4. LEVINSON'S THEOREM FOR QUARK–MESON THERMODYNAMICS

In order to inspect the validity of the approach so far presented, in our analysis we consider the Levinson theorem

$$\int_{4m^2}^{+\infty} ds \; \frac{d\Phi_M}{ds} = n\pi, \quad (29)$$

where n denotes the number of bound states below the threshold $4m^2$. Indeed, it is easy to check that the resonant phase shift $\phi_R(s)$ alone does not fulfill (29). This implies that the scattering phase shift should be composed of at least two parts. In fact, as was demonstrated in [4], it is appropriate to decompose the scattering phase shift Φ_M into a part corresponding to the mesonic correlation and a part describing quark–antiquark scattering,

$$\Phi_M = \phi_R + \phi_{\rm sc}. \quad (30)$$

Namely, using (21), we can represent the total scattering phase shift Φ_M as

$$\Phi_M = \frac{i}{2} \ln \frac{1 - 2G_S\Pi_M(\omega + i\eta, \vec{q})}{1 - 2G_S\Pi_M(\omega - i\eta, \vec{q})}. \quad (31)$$

Then it is straightforward, using (23) and the relation between logarithm and arctan functions, to show that

$$\Phi_M = -\arctan\left[\frac{2G_S P_M \,\mathrm{Im}\,\widetilde{I}_2}{1 - 2G_S\widetilde{I}_1 + 2G_S P_M \,\mathrm{Re}\,\widetilde{I}_2}\right] \quad (32)$$

and, by several more consecutive steps, to obtain

$$\Phi_M = -\arctan\left[\frac{\dfrac{\operatorname{Im}\widetilde{I}_2}{\operatorname{Re}\widetilde{I}_2} - \dfrac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\dfrac{\operatorname{Im}\widetilde{I}_2}{P_M + \dfrac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\operatorname{Re}\widetilde{I}_2}}{1 + \dfrac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\dfrac{\operatorname{Im}\widetilde{I}_2^2}{P_M\operatorname{Re}\widetilde{I}_2 + \dfrac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\operatorname{Re}\widetilde{I}_2^2}}\right]. \tag{33}$$

At this point it is enough to recognize the above expression as the sum of arctans to finally obtain

$$\Phi_M = -\arctan\left(\frac{\operatorname{Im}\widetilde{I}_2}{\operatorname{Re}\widetilde{I}_2}\right) + \\ + \arctan\left(\frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\frac{\operatorname{Im}\widetilde{I}_2}{P_M + \dfrac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\operatorname{Re}\widetilde{I}_2}\right), \tag{34}$$

which proves the formula (30), where

$$\phi_{\mathrm{sc}} = -\arctan\left(\frac{\operatorname{Im}\widetilde{I}_2}{\operatorname{Re}\widetilde{I}_2}\right) \tag{35}$$

and

$$\phi_R = \arctan\left(\frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\frac{\operatorname{Im}\widetilde{I}_2}{P_M + \dfrac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\operatorname{Re}\widetilde{I}_2}\right). \tag{36}$$

Using the conditions (26), in the above equation the masses and widths could be identified in accordance with (28).

As the first part of this decomposition is independent of mesonic properties, we presume that it is connected with the scattering states' input to the thermodynamics. On the other hand, the second part of (34) describes solely the behavior of dissolving mesons. Thus, we are assured that the correct description of mesonic correlations accounts not only for bound and resonant states' contribution, but also for the input from the scattering states.

In our analysis we will use a combined approach, where the scattering part of the phase shift Φ_M is defined according to (34) and the resonant part is given

by a delta function below T_{Mott} and by the Breit–Wigner ansatz (27) beyond it. Explicitly, we take

$$D_M(s) = \frac{1}{\pi}\frac{d\phi_M(s)}{ds} = \\ = \begin{cases} \delta(s - M_M^2) + \dfrac{1}{\pi}\dfrac{d}{ds}\phi_{\mathrm{sc}}(s), & T < T_{\mathrm{Mott}}, \\ \dfrac{a_R}{\pi}\dfrac{\Gamma_M M_M}{(s - M_M^2)^2 + \Gamma_M^2 M_M^2} + \dfrac{1}{\pi}\dfrac{d}{ds}\phi_{\mathrm{sc}}(s), & T > T_{\mathrm{Mott}}. \end{cases} \tag{37}$$

We will in the following regard the states for positive real $s \geqslant 4m^2$ only.

Finally, the scattering states' contribution to the density of states is normalized and exactly compensates the contribution from the resonance (or the bound state, resp.). This accordance with the Levinson theorem confirms the validity of the presented approach.

5. RESULTS AND DISCUSSION

The above-described complex mass pole analysis leads to the results for the meson masses M_M and widths Γ_M as presented in Fig. 3. We observe that the sigma-meson width is non-zero for all the temperatures considered, although below T_{Mott} it is not significant and therefore allows us to consider sigma to be a quasi-bound state. Above the Mott temperature, pion and sigma masses quickly become equal and so do their spectral widths. This is the imprint of the chiral

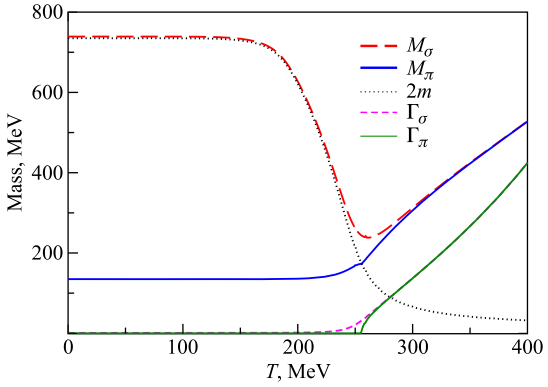

Fig. 3. Temperature dependence of meson masses M_M and corresponding spectral widths Γ_M

symmetry restoration where the σ and π meson, being chiral partners, become degenerate.

In what follows we will concentrate on the discussion of the π meson, because it undergoes the Mott transition from the bound state to the resonant correlation in the continuum, accompanied by a jump of the scattering phase shift at threshold from π to 0 in accordance with the Levinson theorem.

In Fig. 4 we show the phase shift Φ_M (lower panel) in the decomposition (30) into its resonant (upper panel) and scattering continuum (middle panel) parts, obtained from the solution of Eqs. (35) and (36).

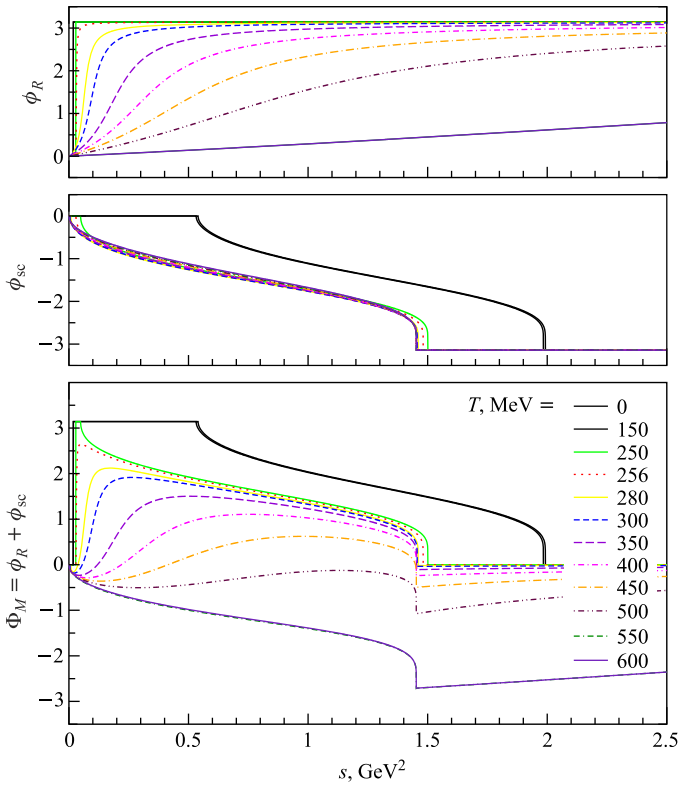

Fig. 4. Dependence of phase shift $\Phi = \phi_R + \phi_{\rm sc}$ and its components ϕ_R and $\phi_{\rm sc}$ in the pion channel on the center-of-mass energy

The pion pressure resulting from utilizing the Breit–Wigner spectral function alone and together with the input from the scattering states is shown in Fig. 5 along with the pressure of the massive pion gas. Figure 6 shows the pion pressure as compared with the quark pressure obtained from the mean-field approximation of

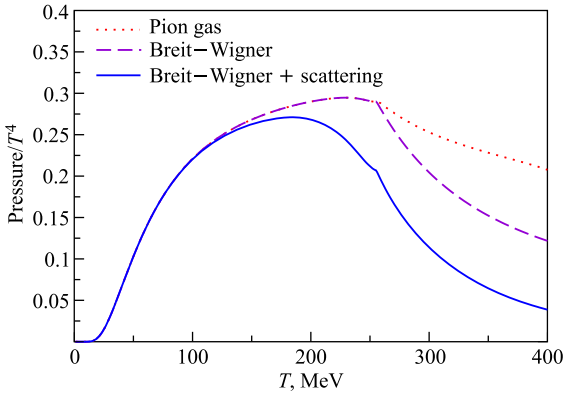

Fig. 5. Temperature dependence of the pion pressure in the approximated GBU approach

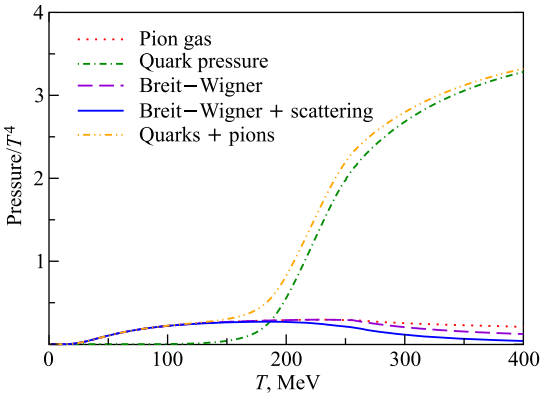

Fig. 6. Comparison of the pion pressure obtained with the input from the MFA approximation to thermodynamic potential

the PNJL thermodynamic potential. We notice that including the description of the scattering states results in a correct description of pion thermodynamics in the vicinity of and beyond the Mott temperature.

6. CONCLUSIONS

Pion dissociation by the Mott effect in quark plasma was described within the generalized Beth–Uhlenbeck approach on the basis of a PNJL model whereby a unified description of bound, resonant and scattering states was given. As a first approximation, we utilized the Breit–Wigner ansatz for the spectral function and clarified its relation to the complex mass-pole solution of the pion Bethe–Salpeter equation. It has been demonstrated that a description of the pion Mott

dissociation solely by a spectral broadening of the pion bound state when it enters the continuum of unbound states for temperatures beyond $T_{\rm Mott}$ necessarily entails a violation of Levinson's theorem. In order to solve this problem, we have extended the approach beyond the complex mass pole approximation and solved the scattering phases in the pion channel of quark–antiquark interaction. The account for the scattering continuum in accordance with the Levinson theorem leads to a strong reduction of the pion pressure above the Mott dissociation temperature. We suggest that the behavior of the scatterinng phase shift in the pion channel and its temperature dependence across the Mott transition, as obtained in the present work, can be used to develop a generic ansatz for the behavior of hadronic densities of states to be used in a generalized PNJL-hadron resonance gas model [21] that embodies the Mott dissociation of hadrons.

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APPENDIX A
NAMBU–JONA-LASINIO MODEL WITH POLYAKOV LOOP

Confinement in pure $SU(N_c)$ gauge theory can be simulated by introducing an effective potential for a complex Polyakov-loop field. The PNJL Lagrangian [13–17] is

$$\mathcal{L}_{\rm PNJL} = \overline{q}\left(i\gamma^\mu D_\mu - m_0 - \gamma^0\mu\right)q + \\ + \sum_{M=\sigma',\vec{\pi}'} G_M\left(\overline{q}\Gamma_M q\right)^2 - U(\Phi[A],\overline{\Phi}[A];T). \quad \text{(A1)}$$

The quark fields are coupled to the gauge field A_μ through the covariant derivative $D_\mu = \partial_\mu - iA_\mu$. The gauge field is $A^\mu = \delta^\mu_0 A^0 = i\delta^\mu_4 A_4$ (the Polyakov gauge). The field Φ is determined by the trace of the Polyakov loop $L(\vec{x})$ [15]:

$$\Phi[A] = \frac{1}{N_c}\,{\rm Tr}_c\, L(\vec{x}), \quad \text{(A2)}$$

where $L(\vec{x}) = \mathcal{P}\,\exp\left[-i\int\limits_0^\beta d\tau A_4(\vec{x},\tau)\right]$. Γ_M are the vertices for the scalar (σ') and pseudoscalar ($\vec{\pi}'$) four-fermion interaction channels. The gauge sector of the Lagrangian density (A1) is described by an effective potential $\mathcal{U}(\Phi[A],\overline{\Phi}[A];T)$

fitted to the lattice QCD simulation results in pure $SU(3)$ gauge theory at finite T [15, 16] with

$$\frac{\mathcal{U}(\Phi,\overline{\Phi};T)}{T^4} = -\frac{b_2(T)}{2}\overline{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\overline{\Phi}^3) + \frac{b_4}{4}(\overline{\Phi}\Phi)^2, \tag{A3}$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3. \tag{A4}$$

The parameters of the effective potential (A3) and (A4) are summarized in Table 2.

Table 2. Parameters of the effective potential $\mathcal{U}[A]$

a_0	a_1	a_2	a_3	b_3	b_4
6.75	–1.95	2.625	–7.44	0.75	7.5

In general, the parameter T_0 depends on the number of active flavors and the chemical potential. In the present work we use $T_0 = 208$ MeV, as has been proposed in [22].

The partition function in the path-integral representation is then given by

$$\begin{aligned}\mathcal{Z}_{\rm PNJL}[T,V,\mu] = \\ = \int \mathcal{D}\overline{q}\mathcal{D}q \exp\Bigg\{\int_0^\beta d\tau \int_V d^3x \Big[\overline{q}(i\gamma^\mu(\partial_\mu - iA_\mu) - m_0 - \gamma^0\mu)q + \\ + G_S(\overline{q}\Gamma_{\sigma'}q)^2 + G_S(\overline{q}\vec{\Gamma}_{\pi'}q)^2 - U(\Phi[A],\overline{\Phi}[A];T)\Big]\Bigg\},\end{aligned} \tag{A5}$$

where the interaction vertices are written explicitly. By means of the Hubbard–Stratonovich transformation, we are able to integrate out the quark degrees of freedom to arrive at the partition function written solely in terms of collective fields

$$\begin{aligned}\mathcal{Z}_{\rm PNJL}[T,V,\mu] = \int \mathcal{D}\sigma'\mathcal{D}\vec{\pi}' \times \\ \times\exp\left\{-\left[\int_0^\beta d\tau \int_V d^3x \left(\frac{\sigma'^2+\vec{\pi}'^2}{4G_S} + U(\Phi[A];T)\right)\right] + \mathrm{Tr}\,\ln\left[\beta S^{-1}[\sigma',\vec{\pi}']\right]\right\},\end{aligned} \tag{A6}$$

where $S^{-1}[\sigma',\vec{\pi}']$ is the inverse propagator given explicitly as

$$S^{-1}[\sigma',\vec{\pi}'] = \gamma^0(i\omega_n - \mu + A_0) - \vec{\gamma}\cdot\vec{p} - m_0 - \sigma'\Gamma_{\sigma'} - \vec{\pi}'\vec{\Gamma}_{\pi'} \tag{A7}$$

and the operation Tr is taken over color, flavor, Dirac and momentum indices of quark fields.

APPENDIX B
THE BREIT–WIGNER ANSATZ

We introduce the following pion propagator:

$$S_M(p^2) = \frac{1}{1 - 2G\Pi(p^2)}. \tag{B1}$$

In the first step we expand the polarization function around the mass pole:

$$\Pi(p^2) = \Pi(p^2 = M^2) + (p^2 - M^2)\left.\frac{d\Pi(p^2)}{dp^2}\right|_{(p^2=M^2)} + \dots \tag{B2}$$

and we insert this expansion into the defined propagator

$$S_M(p^2) = \frac{1}{\underbrace{1 - 2G\Pi(p^2 = M^2)}_{=0 \text{ (by definition)}} - 2G(p^2 - M^2)\left.\frac{d\Pi}{dp^2}\right|_{p^2=M^2}} =$$

$$= -\frac{1}{2G(p^2 - M^2)\left.\frac{d\Pi}{dp^2}\right|_{p^2=M^2}} = \frac{g^2_{Mq\overline{q}}}{p^2 - M^2}, \tag{B3}$$

where in the last step we simply defined $g^2_{Mq\overline{q}}$. By means of the wave function renormalization, we define a normalized propagator in the following way:

$$S_M(p^2) = g^2_{Mq\overline{q}}\widetilde{S}_M(p^2). \tag{B4}$$

Now $\widetilde{S}_M(p^2)$ is a propagator for renormalized mass fields $\widetilde{\phi} = g_{Mq\overline{q}}\phi$. At this point we consider a complex mass pole solution

$$p^2 = \left(M \pm i\frac{\Gamma}{2}\right)^2, \tag{B5}$$

where for small Γ we get $p^2 \approx M^2 \pm iM\Gamma$ and the corresponding propagator, in a similar way,

$$\widetilde{S}_M(p^2) = \frac{1}{p^2 - M^2 \mp iM\Gamma} = \frac{p^2 - M^2 \pm iM\Gamma}{(p^2 - M^2)^2 + (M\Gamma)^2}. \tag{B6}$$

We introduce the spectral function by its definition to get

$$A(s = p^2) = 2i\,\mathrm{Im}\,\widetilde{S}_M \sim \pm\frac{M\Gamma}{(s - M^2)^2 + (M\Gamma)^2}. \tag{B7}$$

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